

# ITERATIVE SOLUTION OF DISCONTINUOUS GALERKIN FORMULATIONS OF THE EULER EQUATIONS

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## ABSTRACT

In this work, we investigate iterative solution methods for solving Discontinuous Galerkin discretizations of the Euler equations. Several iterative methods are compared, some based on  $p$ -multigrid methods and others using GMRES. The main permutations investigated are the relaxation scheme used and the simulation conditions. The relaxation schemes investigated are all block matrix schemes where the blocks are associated with the unknowns on each element. The schemes investigated are block equivalents of the most commonly used finite volume iterations such as the alternating direction implicit (ADI) scheme, the lower-upper symmetric Gauss-Seidel (LU-SGS) scheme, and incomplete lower-upper factorization (ILU(0)). The main parameters varied are: the resolution of the mesh, the flow angle relative to the mesh, the mach number, and the polynomial order. As the performance of almost all of the schemes degrade at low Mach number, the effect of modifying the upwinding flux evaluation based on low mach number preconditioners (see [1]) is also investigated. In all cases, periodic versions of these schemes are formulated so that Fourier analysis can be applied as a theoretical analysis tool in addition to direct numerical tests. This allows us to understand the strengths and weaknesses of Fourier analysis in examining problems having a convective nature. In many cases, because the system has non-orthogonal eigenvectors, the Fourier eigenvalue spectrum is not representative of the behavior of the scheme. Lastly, for each iterative scheme, we calculate the number of floating point operations and CPU time required per iteration and degree of freedom so that these results can be compared to other numerical implementations.

## REFERENCES

- [1] E.Turkel. Preconditioning techniques in computational fluid dynamics. *Annu. Rev. Fluid Mech.* **31**, pp.385-416, (1999).