A POSTERIORI ERROR BOUNDS FOR REDUCED BASIS APPROXIMATIONS OF NONLINEAR PARABOLIC PARTIAL DIFFERENTIAL EQUATIONS

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ABSTRACT

We present *a posteriori* error bounds for reduced basis approximations of nonlinear parametrized parabolic partial differential equations. In particular, we extend the empirical interpolation method introduced in [1] for non-affine function approximations to efficiently treat certain classes of nonlinear parabolic partial differential equations [2].

In the recent past, reduced basis methods and associated *a posteriori* error bounds based on standard Galerkin approaches have been introduced for time-dependent problems with quadratic nonlinearities, e.g., Burgers' Equation [4] and Navier-Stokes Equations [3]. For greater-than-quadratic nonlinearities, however, these methods are not efficient; the online operation count for quadratically nonlinear problems is $O(N^4)$, where N is the dimension of the reduced-basis space. For a cubic nonlinearity the associated cost would be $O(N^6)$ — prohibitive even for small N. Furthermore, reduced basis methods based on standard Galerkin approaches are particularly inefficient for non-polynomial nonlinearities since the online complexity scales with the dimension of the underlying finite element "truth" approximation space, $\mathcal{N}$.

We present an "empirical interpolation" approach which recovers $\mathcal{N}$-independence in the online stage even for general nonlinearities. The method replaces the nonlinear terms with a coefficient function approximation which then permits an efficient offline-online computational decomposition. The essential ingredients are (i) a good collateral reduced basis approximation space, and (ii) a stable and inexpensive interpolation procedure. We will also discuss strategies for further improving the rigor of the *a posteriori* error bounds and for increasing the online efficiency of our approach by decreasing the required number of terms in the function approximation to obtain a desired accuracy.

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