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A NAVIER STOKES SOLVER FOR AXISYMMETRIC TURBOMACHINERY ANALYSIS

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Abstract. *An axysimmetric Navier Stokes solver is presented and applied to the analysis of multi-stage turbomachinery. As in classical through flow computations, some 3D effects are taken into account by means of appropriate source terms. In particular, the effect of the blade is modeled introducing a blade blockage and blade force, which has both a component normal to the flow path and one along the flow path, the latter introducing the viscous losses. The use of full Navier Stokes equations, however, allows to implicitly take into account some effects related to end wall boundary layers, as well as entropy radial redistribution and tip leakage flows, and nicely fits into a coherent workflow which couple axysimmetric quick preliminar design step to a complete 3d analysis for the best solutions. Despite the use of a rather crude loss and deviation model, the code demonstrates satisfying accuracy through comparison with experimental and computational data for a subsonic, four stage turbine.*

Nomenclature

e	total energy per unit volume
$\tilde{F}$	Flux vector
f	blade force
J	coordinate transformation Jacobian
k	arbitrary time relaxation coefficient
$\boldsymbol{n}$	stream surface unit normal vector
p	pressure
$\tilde{q}$	Conservative variable vector
Rg	gas constant
r	radius
S_0	Source term
s	Entropy

T	Temperature
x,y,z	cartesian coordinates
U	contravariant velocity
u	axial (x) velocity
v	radial (y) velocity
w	tangential (z) velocity
W	relative velocity vector
Z	number of blades
ξ,η,ζ	curvilinear coordinates
ϑ	Time
θ	circumferential angle
ρ	Density
Ψ	blade thickness
ω	rotation speed

Subscripts

i	Inviscid
v	Viscous
x,y,z	x,y,z component or derivative
ξ,η,ζ	ξ,η,ζ component or derivative

1 INTRODUCTION

Due to the availability of cheap and powerful computational resources, and due to the high level of reliability of modern CFD tools, fully three dimensional Navier Stokes computations have become the state of the art for the final design even in complex turbomachinery geometries. Nonetheless, there is still need for fast, yet accurate simplified modelling for quicker parametrical analysis during the preliminary design of multi stage compressors and turbines, as well as for coupling with optimization procedures. Meridional plane analysis has been historically one of the most popular approach for such applications[1], with a large number of codes developed from the early seventies, mostly using streamline curvature or similar highly simplified approaches, such as, as example, in the popular Denton code[2]. However, some authors proposed the use of fully Navier Stokes solutions on the meridional plane[3,4,5], although usually for compressor analysis.

The general theoretical framework for such models has been defined in terms of subsequent averages following Adamczyk[6] and Simon et al.[7]. Such approach allows for the identification of the proper terms to be modeled due to the different level of approximation.

A number of advantages can derive from the use of full Navier Stokes equations for axysimmetric models: in particular, complex off-design conditions including, as an example, recirculation in the exit diffuser of axial compressor, can be handled in a much easier and robust way. This happens also for transonic blockage, although the shock wave handling require careful attention[8]. Tip leakage may also be introduced in a consistent manner. Furthermore, the use of the same formulation for the preliminary meridional analysis and the final fully 3D computation limited to a few number of nearly-final geometries allows for a more efficient and coherent design work flow.

Here, an axysimmetric, compressible, finite volume Navier Stokes solver is applied to the meridional analysis of axial flow turbines. The adopted solver is a classical implicit ADI code on structured grids, developed for turbomachinery analysis[9] including heat transfer effect[10].

The circumferential average of the fully 3D Navier Stokes analysis yields a straightforward formulation of the meridional plane problem, which has the same form of the standard viscous equations for axisymmetric flow, with the addition of a few source terms arising from the integration process. Thus, the resulting equation are in some way 'exact' equations, provided that the proper closure relationships are identified for those extra terms, namely the blade force terms. These blade force terms are mainly related to the flow deviation and the corresponding angular momentum change and to the friction along the blades. Thus, they require extra information, either from blade to blade or fully 3D computations, or from empirical correlations, in addition to the geometrical details of the blade itself.

A first approach in the introduction of such blade forces was introduced, in the framework of the present FVM solver, in[11]. In such paper, blade forces were imposed such that the flow was forced to follow exactly the flow path derived from the blade angles and the chosen correlations; the correction was instantaneous, and thus imposed, in the transient, a severe constraint on the solution, thus inducing slower convergence rates. In a following paper[12], we followed a different approach, with a pseudo-unsteady correction as described in[5]. The approach allowed quick convergence, of the same order of the convergence of standard 2D solutions. Here, tip leakage and blade blockage are introduced in the same framework. The code, although the loss and angle correlation adopted in the present study are not yet optimized for this approach, offer satisfying accuracy in the comparison with experimental data for a 4-stage turbine.

2 NUMERICAL METHOD

The numerical method employed in this application is derived from a 3D viscous solver for turbomachinery analysis[9]. The Navier Stokes equations, dicretised with finite differences, are solved by means of an implicit scheme[13] that assures inconditionate stability for the solution.

Starting from the fully 3D equations in general curvilinear coordinates ξ, η, ζ:

$$\frac{\partial \tilde{q}}{\partial \vartheta} + \frac{\partial\left(\tilde{F}_x - \tilde{F}_{vx}\right)}{\partial \xi} + \frac{\partial\left(\tilde{F}_y - \tilde{F}_{vy}\right)}{\partial \eta} + \frac{\partial\left(\tilde{F}_z - \tilde{F}_{vz}\right)}{\partial \zeta} = 0 \tag{1}$$

the axysimmetric formulation is straightforward, if we define an axysimmetric curvilinear mesh in which the surfaces at constant ζ are meridional planes, namely assuming $\zeta = r\theta$, with θ circumferential angle. Thus, at $\theta = 0$, w is the tangential velocity, u and v the axial and radial components respectively.

Finally, we obtain

$$\zeta_x = \zeta_y = \xi_z = \eta_z = 0\ ; \quad \zeta_z = \frac{1}{r} \tag{2}$$

and all of the derivatives in the ζ direction are either zero or easily computed from the axysimmetric hypothesis. In particular, considering also the blockage factor b due to the blade thickness, defined as

$$b = 1 - \frac{\psi}{2\pi/Z} \tag{3}$$

we arrive to the final formulation

$$\frac{\partial \tilde{q}}{\partial \vartheta} + \frac{\partial b\left(\tilde{F}_x - \tilde{F}_{vx}\right)}{\partial \xi} + \frac{\partial b\left(\tilde{F}_y - \tilde{F}_{vy}\right)}{\partial \eta} = bS \tag{4}$$

where the conservative variable vector is:

$$\tilde{q} = J^{-1}\begin{pmatrix} b\rho \\ b\rho u \\ b\rho v \\ b\rho w \\ be \end{pmatrix} \tag{5}$$

and the inviscid flux vector $\tilde{F}_x$ and $\tilde{F}_y$ can be expressed as:

$$\tilde{F}_x = J^{-1}\begin{pmatrix} \rho U \\ \rho u U + p\xi_x \\ \rho v U + p\xi_y \\ \rho w U + p\xi_z \\ U\left(e+p\right) \end{pmatrix} \qquad \tilde{F}_y = J^{-1}\begin{pmatrix} \rho V \\ \rho u V + p\eta_x \\ \rho v V + p\eta_y \\ \rho w V + p\eta_z \\ V\left(e+p\right) \end{pmatrix} \tag{6}$$

$$U = \xi_x u + \xi_y v + \xi_z w \qquad V = \eta_x u + \eta_y v + \eta_z w \tag{7}$$

The source term that appear in Eq. 4 takes also into account the effects of the blades and can be considered as composed by two different contributions: inviscid and viscous blade force. The first one can be further divided into blade blockage term S_{bl} and inviscid blade force S_{b_inv}.

$$S = S_0 + S_{bl} + S_{b\_inv} + S_{b\_vis} \tag{8}$$

$$S_0 = J^{-1}\begin{pmatrix} 0 \\ 0 \\ \zeta_z\left(p + \rho w^2\right) \\ -\rho v w \zeta_z \\ 0 \end{pmatrix} \qquad S_{bl} = J^{-1}\begin{pmatrix} 0 \\ \dfrac{p}{b}\left(\dfrac{\partial b}{\partial \xi}\xi_x + \dfrac{\partial b}{\partial \eta}\eta_x\right) \\ \dfrac{p}{b}\left(\dfrac{\partial b}{\partial \xi}\xi_y + \dfrac{\partial b}{\partial \eta}\eta_y\right) \\ 0 \\ 0 \end{pmatrix} \tag{9}$$

$$S_{b\_inv} = J^{-1}\begin{pmatrix} 0 \\ f_{bx} \\ f_{by} \\ f_{bz} \\ f_{bz}\omega r \end{pmatrix} \qquad S_{b\_vis} = J^{-1}\begin{pmatrix} 0 \\ f_{fx} \\ f_{fy} \\ f_{fz} \\ f_{fz}\omega r \end{pmatrix} \tag{10}$$

Since the axisymmetric assumption neglects tangential gradient of any quantity, the effects of blade deviation have to be introduced by means of external informations that yeld to an appropriate blade force model as described in the following paragraph.

3 BLADE FORCE

Due to the axisymmetric assumption, the blade force cannot be derived from the solution and the blade effects have to be modeled employing also external informations.

Such forces will have two effect: bending the flow (inviscid contribution) and producing losses via shear and drag.

All the other non axisymmetric effects, such as 3D secondary flows, can be taken into account only through the proper definition of blade forces.

The tangential component of the inviscid blade force S_{b_vis} can be evaluated if the flow direction is known and imposed. The geometry of the stream surfaces, as defined by Wu, can be evaluated by means of blade to blade computations or through semi-empirical correlations. It is clear therefore that this method rely on the reliability of correlation employed; however, the wealth of available details of the flow fields may suggest the possibility of the tuning of local correlations, based on local blade curvatures or load, rather than the traditional global correlation for a whole cascade.

Since the evaluation of the blade force can introduce instabilities, especially during the possibly unphysical pseudo-unsteady transient calculation required to attain steady state solution, its contribution has to be damped and relaxed in time. The following equation has been used:

$$\frac{\partial \left|\mathbf{f}_i\right|}{\partial \vartheta} = k\rho \mathbf{W}\cdot\mathbf{n} \tag{11}$$

Eq.(11) links the temporal variation of the inviscid blade force with the relative velocity **W** component along the unit vector **n** normal to the flow path surface through a proportionality constant k that is an arbitrary relaxation coefficient.

The contribution of the inviscid blade borce can be written in the absolute frame of reference, as in the following expression:

$$\frac{\partial f_i}{\partial \vartheta} = k\rho\left(un_x + vn_y + \left(w - \omega r\right)n_z\right) \tag{12}$$

The single force components are derived from the assumption that the inviscid force is normal to the flow path. If steady state is attained, left hand side of eq.(12) vanishes and we get no flow crossing the streamsurface, as required.

Since correlation for deviation angle are defined only at the blade trailing edge, we introduce a sort of smoothing to obtain a deviation distribution along the blade, and we introduce a further damping factor to avoid unphysical abrupt changes at blade leading edge. Since the inviscid component of the blade force is normal to the stream surface, in a relative frame of reference relative to each blade, it will not generate losses. Blade force work will be zero in statoric blades, different from zero in rotoric blades, where the absolute velocity is not aligned with the blade, and thus the scalar product of absolute velocity and actual blade force will give a non zero power production.

Thus, in order to take into account the losses, we need a blade force component parallel and opposite to the (relative) flow. Such terms should take into account all of the non-axisymmetric loss sources, including profile boundary layer, 3D secondary flows, non axisymmetric perturbation of endwall boundary layers.

In the most general case, this component can be related to entropy production along the meridional relative velocity[5]:

$$\left|\mathbf{f}_v\right| = \rho T \frac{W_m \Delta s}{\left|\mathbf{W}\right|} \tag{13}$$

The work done by this viscous force, in fact, has to be considered as completely converted in heat with increasing entropy. We can use popular total pressure “loss coefficient”, defined by several literature sources, in order to compute the entropy production:

$$\Delta s = R_g \log \frac{p_1}{p_2} \tag{14}$$

In the present work the classical Traupel correlation[17] has been implemented to evaluate the loss coefficient, while the flow angle deviations required in the inviscid components were derived from[11]. The final expression for the total blade force is then added to the source term:

$$S_t = S_0 + J^{-1}\left[0, f_{ix} - f_{vx}, f_{iy} - f_{vy}, f_{iz} - f_{vz}, \left(f_{iz} - f_{vz}\right)\omega r\right] \tag{15}$$

Leakage flow at the rotor tips can be automatically modelled, although in a rather crude fashion, by cancelling the blade forces in the tip gap..

4 APPLICATIONS

The experimental turbine of Hannover University[14] was chosen for the validation, due to the wealth of both experimental and open literature numerical data, obtained with both traditional inviscid based through flow computation and with state of the art 3D viscous CFD codes as in Gerolymos and Hanisch[15] and Petrovic and Reiss[16]. It is a four stage gas turbine, with swirled rotor blades and high aspect-ratios. The experimental data, however, are not complete: in particular measurements between stator and rotor rows of each stage are not provided. On the other hand, experimental or numerical data offers comparisons for both design and off-design conditions, including a critical low mass flow configuration in which massive hub separation are expected[15,16].

The geometry of the meridional plane is shown in Fig.1. Following a grid independence analysis, the turbine was modelled via a structured grid of around 8000 nodes, allowing for very quick convergence (in the order of several minutes) on a state of the art desktop PC.

Results will be compared to experimental and numerical results from a CFD commercial code. Due to the wealth of details of CFD computations, such results are considered as reference case in all those stations where measurements are not available. NUMECA, a finite volume commercial code was chosen. Turbulence was simulated via Favre-Reynolds averaging and one equation Spalart-Allmaras model. A 4-millions nodes grid has been employed after a grid dependency check has been carried out with coarser grid levels.

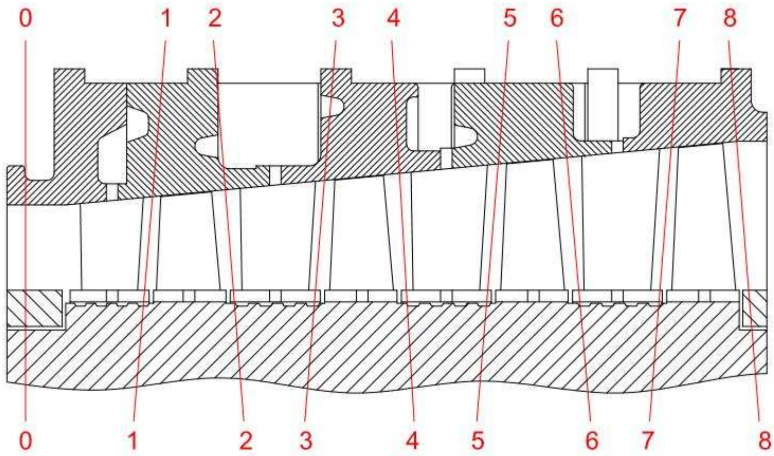

Figure 1: Experimental turbine meridional plane with calculation sections

The gas turbine considered has been tested in different work condition, both in design and off design. In the present work two different operating condition has been analysed with the method proposed in order to show its potential. The first one is the design condition and the second one has a smaller rotational speed and mass flow as indicated in the following table 1:

Condition	Mass flow	Rot. Speed
Design	7.5 kg s^{-1}	7500 RPM
Off Design	3.1 kg s^{-1}	5625 RPM

Table 1: Computed cases

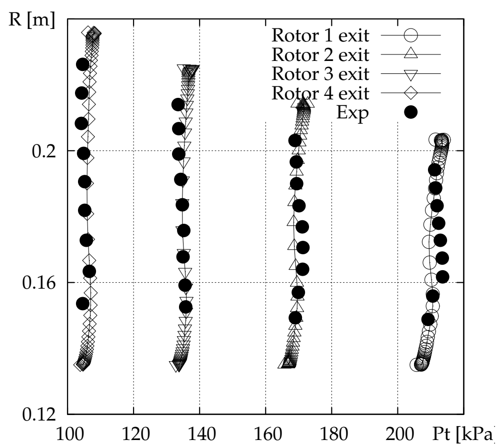

Figure 2: Computed and experimental absolute total pressure at stages exit

At design conditions, the axisymmetric code, despite its still rather simple correlation adopted, appears to offer good accuracy: in Fig.2 the computed results for absolute total pressure along the radial direction at the stages exit section are compared to the experimental data. The total pressure drop, resulting from both rotors mechanical work and shear losses, is well predicted in all of the considered sections. It is worth of notice that only profile losses (and losses related to 3D flow structures) are evaluated by empirical correlation, while other contribution from hub and tip boundary layers are straightforwardly evaluated by the solver.

The design case has been deeply investigated in the past, and the attention of the authors has often been focused on the interaction of the inviscid blade force with hub and tip boundary layers.

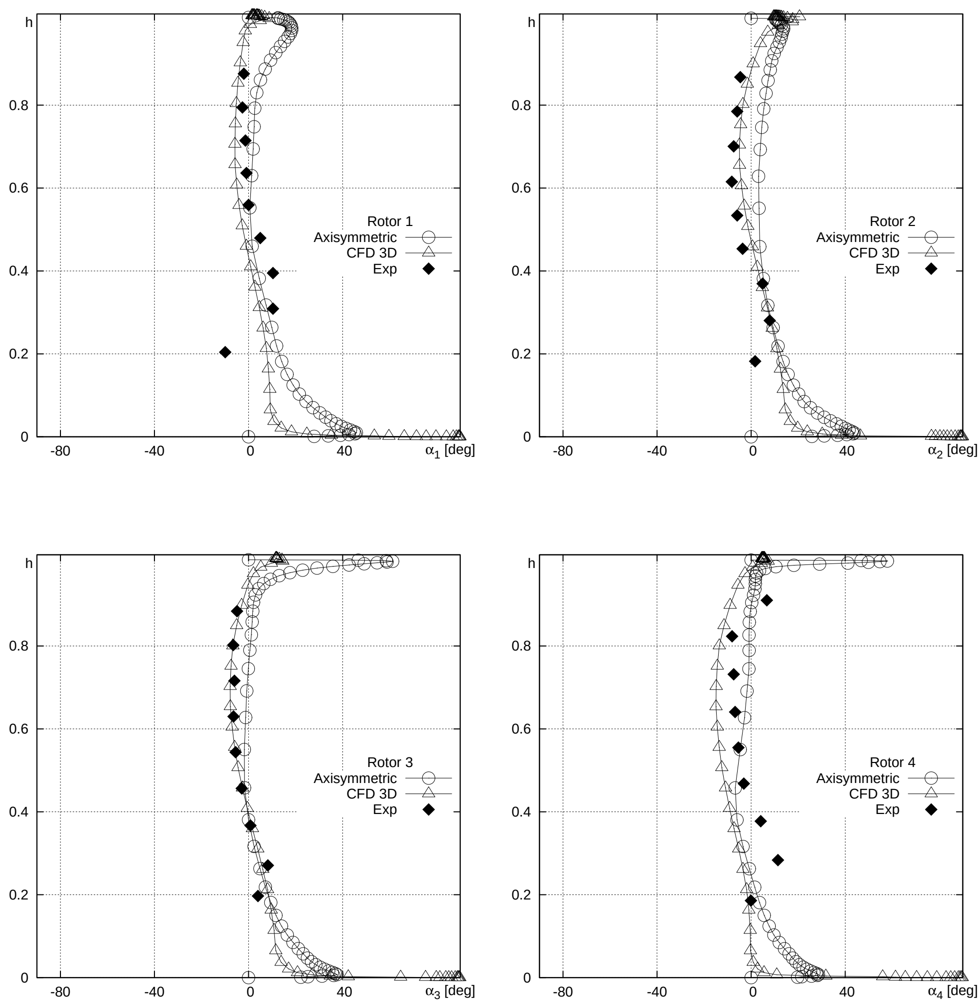

Figure 3: Absolute flow angle downstream of the rotors, compared with experimental data

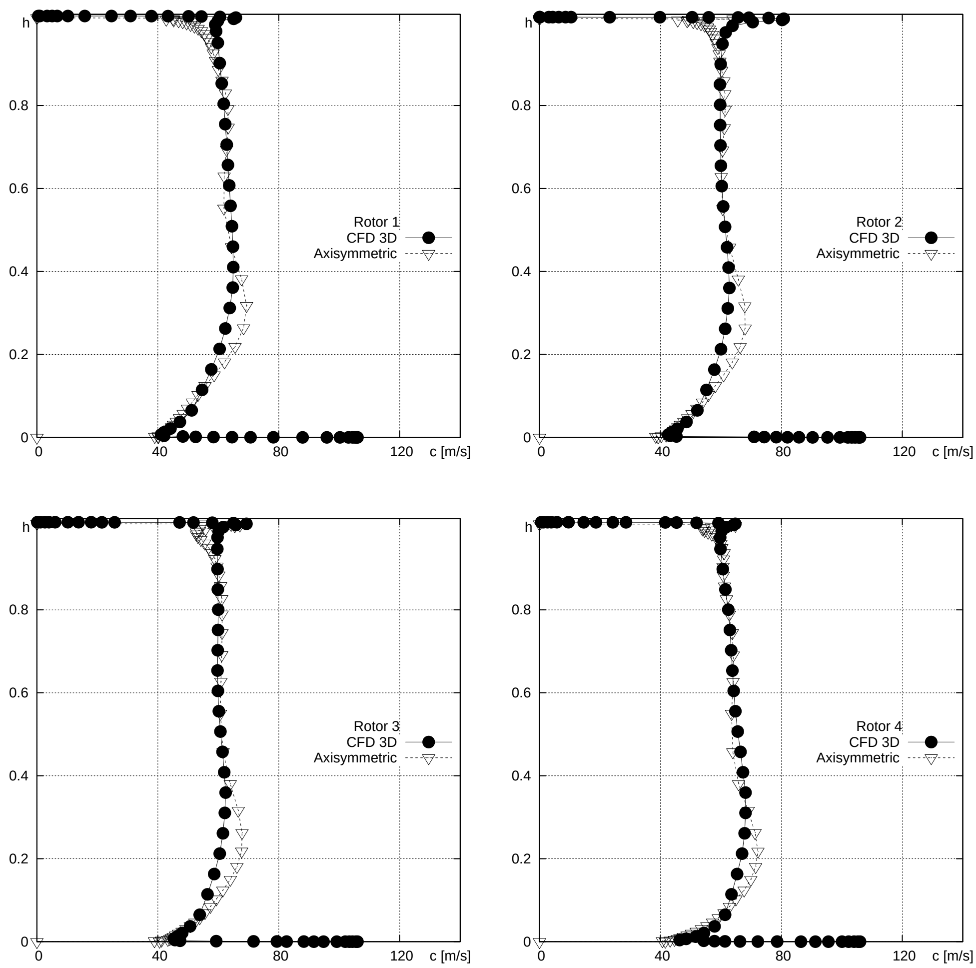

Figure 4: Absolute velocity, rotor exits, compared with fully 3D CFD

In an axisymmetric computation, the prediction of flow angle actually define most of the features of the final solution. Here, angles from the fully 3D CFD calculation, and predicted by Satta and Massardo[18] correlation were considered. Since they showed a good agreement, the Satta and Massardo results will be shown in the following results.

In fig.3 the absolute angle downstream of the rotors is compared with both experimental data and fully 3D computations. Again, the agreement is usually satisfactory.

Absolute velocities on the same sections are compared with CFD 3D data, due to the lack of experimental reliable data, in the following Fig.4. Again, we note good agreement, except for a slight overestimate of velocity on the second rotor, as expected from the discrepancy in the absolute angles.

However, the analysis of the off-design case is probably most significant that the nominal one. In the latter case, in fact, provided that proper correlations are selected, even traditional inviscid based through flow computations can offer good prediction.

The extreme off-design set up, at 45% mass flow and reduced rotation speed, triggers a massive separation downstream of the last rotor[15,16] which is not easily handled by such methods. In Fig.5 we compare the axial velocity downstream of the last rotor computed with our axisymmetric solver for the design and off design conditions. The expected recirculation actually appears, thus confirming the robustness and reliability of the code.

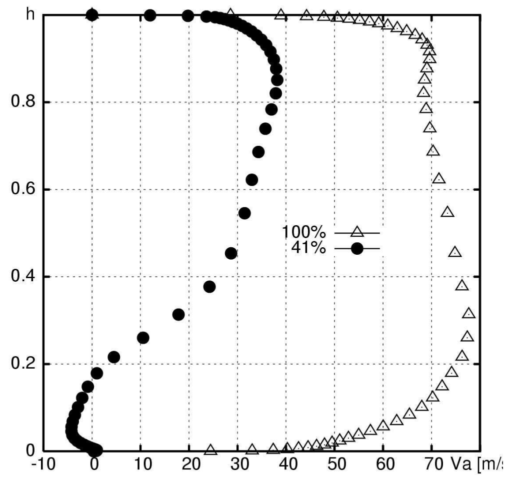

Figure 5: Axial velocity downstream of rotor 4, design and off design conditions

Actually, with respect to results in[15,16] such recirculation area, clearly detectable in the streamline plots of Fig.6, is overestimated. However, it is important that such separation is detected: the correct estimate of its magnitude is a tough task for an axisymmetric code, since it heavily interacts with the 3D blade flow structure; furthermore, such critical conditions obviously exceed the limit of application of the chosen correlations.

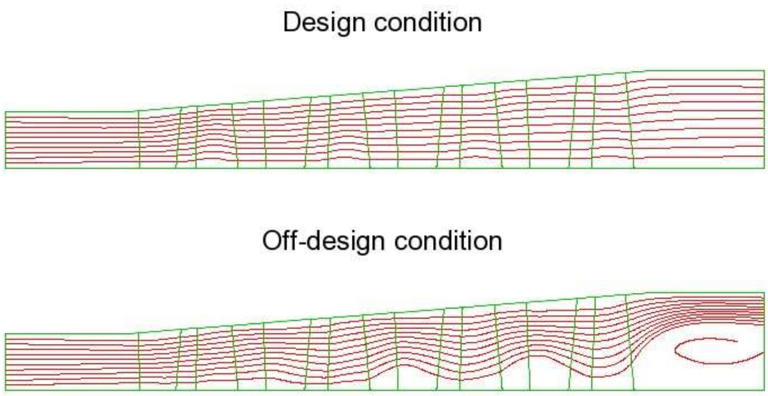

Figure 6: Streamlines of two different operating conditions

As stated in the theoretical section, tip leakage was introduced in a straightforward way by neglecting the blade forces in the tip gap. Fig.7 and Fig.8 show the effect of leakage flow at rotor tips, as well as introducing the wall velocity at the rotor hub. Flow angles are little affected, as shown in fig.7; however, Fig.8 demonstrate that the leakage model allows for the proper prediction of the velocity increase near the rotor tip, while the hub sliding wall allows for a better description of the lower high velocity region.

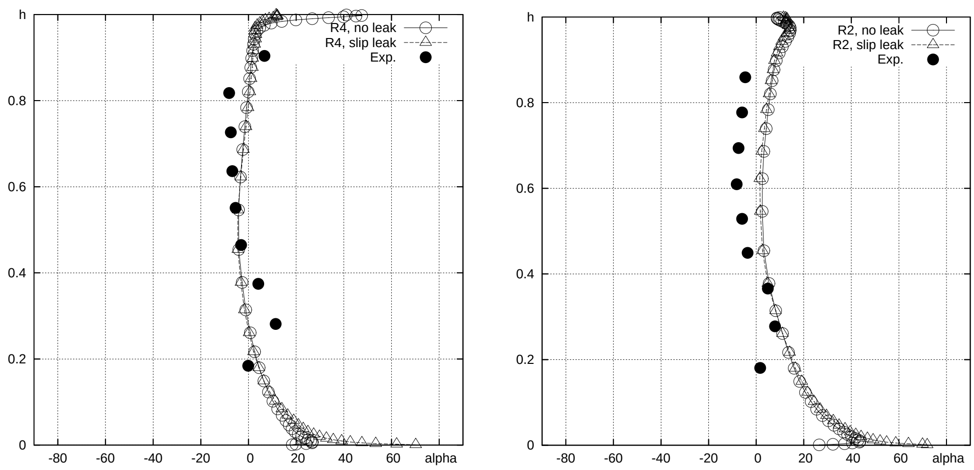

Figure 7: Absolute flow angle downstream of the rotors, including or neglecting rotor tip leakage and hub rotation

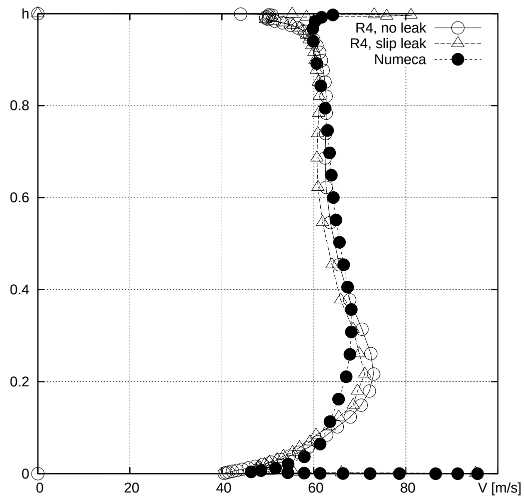

Figure 8: Absolute flow angle downstream of rotor 4, including or neglecting rotor tip leakage and hub rotation

5 CONCLUSIONS

A Navier Stokes axisymmetric solver has been applied to the analysis of flow in the meridional plane of an axial subsonic turbine. The comparison with experimental and fully 3D CFD results demonstrated that the approach can offer reliable predictions of the actual flow in the turbine, with a minimum amount of computational time and resources.

An application to a severe off-design case showed also the robustness of the method, which can easily handle and correctly predict the onset of massive separations. Furthermore, tip leakage flows were easily incorporated without need for empirical correlations. Much further investigation is needed, however, to clarify the most suitable correlations for loss and deviation prediction, and in particular to clarify their interaction and possible redundancy with the viscous dissipation computed by the code. In particular, due to the wealth of details offered by the numerical solution, it would be interesting to try to develop locally-based correlation.

Nonetheless, the approach appears already promising for a quick preliminary analysis in the early stage of the design process, as well as for possible coupling with optimization tools.

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