REDUCED-BASIS APPROXIMATION AND A POSTERIORI ERROR ESTIMATION FOR THE PARAMETRIZED INCOMPRESSIBLE NAVIER-STOKES EQUATIONS

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ABSTRACT

We present reduced basis approximations and associated rigorous a posteriori error bounds for the parametrized Navier-Stokes equations [1-5]. The first component is *real-time prediction*: rapidly, uniformly convergent reduced basis approximations. The dimension reduction is achieved by Galerkin projection onto a low-dimensional space associated with a smooth parametric solution manifold, and by efficient sampling for identification of optimal and numerically stable approximations. The second component is *certainty*: rigorous a posteriori error bounds for the errors in the reduced basis approximation and associated outputs. This requires efficient calculation of the dual norm of the residual and the necessary stability factors. In both components we exploit affine parametric structure and offline-online computational decompositions to provide real-time (online) response.

The method is applied to a Boussinesq natural convection problem in a two-dimensional enclosure. Numerical results confirm the rapid convergence of the reduced-basis approximation, and that the rigorous a posteriori error bounds remain practicable for problems of physical interest.

In general, the reduced basis approach permits rather general geometric parametrizations [3]. However, for the nonlinear Navier-Stokes equations, geometric variation induces considerable difficulties both in theory and implementation [1,4]. We therefore also discuss strategies for dealing with geometric parametrizations of the Navier-Stokes equations, and thereby for increasing the method's applicability.

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