

CONJUGATE PROBLEM OF FOREST FIRE INITIATION AND SPREAD IN THREE DIMENSIONAL SETTING

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Abstract. *In this paper the theoretical investigations of the problems of forest fire initiation was carried out. Mathematical model of forest fire was based on an analysis of experimental data and using concept and methods from reactive media mechanics. The research was based on numerical solution of three dimensional Reynolds equations. The boundary-value problem is solved numerically using the method of splitting according to physical processes..*

1 INTRODUCTION

The forest fires are very complex phenomena. At present, fire services can forecast the danger rating of, or the specific weather elements relating to, forest fire. There is need to understand and predict forest fire initiation, behaviour and spread. This paper's purposes are the improvement of knowledge on the fundamental physical mechanisms that control forest fire initiation and spread. A great deal of work has been done on the theoretical problem of how forest fire initiation. Crown fires are initiated by convective and radiative heat transfer from surface fires. However, convection is the main heat transfer mechanism. Crown fires are more difficult to control than surface. The first accepted method for prediction of surface fires was given by Rothermel [1] and crown fires behaviour Rothermel [2] and Van Wagner [3], but not for predicting interacting between them. In this paper the model proposed there links settings of surface and crown forest fire initiation and spread. The semi-empirical models [1-3] allow to obtain a quite good data of the forest fire rate of spread as a function of fuel bulk and moisture, wind velocity and the terrain slope. But these models use data for particular cases and do not give results for general fire conditions. Also crown fire initiation and hazard have been studied and modeled in detail (eg: Alexander [4], Van Wagner [5], Xanthopoulos, [6], Van Wagner, [7], Cruz [8], Albini [9], Scott, J. H. and Reinhardt, E. D. [10]. The discussion of the problem of modeling forest fires is provided by a cycle of works produced by a group of co-workers at Tomsk University (Grishin [11], Grishin and Perminov [12], Perminov [13-15]). In particular, a mathematical model of forest fires was obtained by Grishin [11] based on an analysis of known and original experimental data [11,16], and using concepts and methods from reactive media mechanics. The physical two-phase models used in [17-18] may be considered as a continuation and extension of the formulation proposed by Grishin and Perminov [11-14]. However, the investigation of crown fires initiation has been limited mainly to cases studied of forest fires initiation without take into account the mutual interaction of surface and crown forest fires and three dimensional atmosphere flows. This paper's purpose is to demonstrate the necessity and the utility of conjugate crown-surface fire model.

2 PHYSICAL AND MATHEMATICAL FORMULATION

The main assumptions adopted during the deduction of mathematical setting: 1) the forest represents a multi-phase, spatially heterogeneous medium; 2) in the fire zone the forest is a porous-dispersed, two-temperature, single-velocity, reactive medium; 3) the forest canopy is supposed to be non - deformed medium (trunks, large branches, small twigs and needles), affects only the magnitude of the force of resistance in the equation of conservation of momentum in the gas phase, i.e., the medium is assumed to be quasi-solid (almost non-deformable during wind gusts); 4) let there be a so-called "ventilated" forest massif, in which the volume of fractions of condensed forest fuel phases, consisting of dry organic matter, water in liquid state, solid pyrolysis products, and ash, can be neglected compared to the volume fraction of gas phase (components of air and gaseous pyrolysis products); 5) the flow has a developed turbulent nature and molecular transfer is neglected; 6) gaseous phase density doesn't depend on the pressure because of the low velocities of the flow in comparison with the velocity of the sound. Let the coordinate reference point $x_1, x_2, x_3 = 0$ be situated at the centre of the surface forest fire source at the height of the roughness level, axis Ox_1 directed parallel to the Earth's surface to the right in the direction of the unperturbed wind speed, axis Ox_2 directed perpendicular to Ox_1 and axis Ox_3 directed upward (Fig. 1).

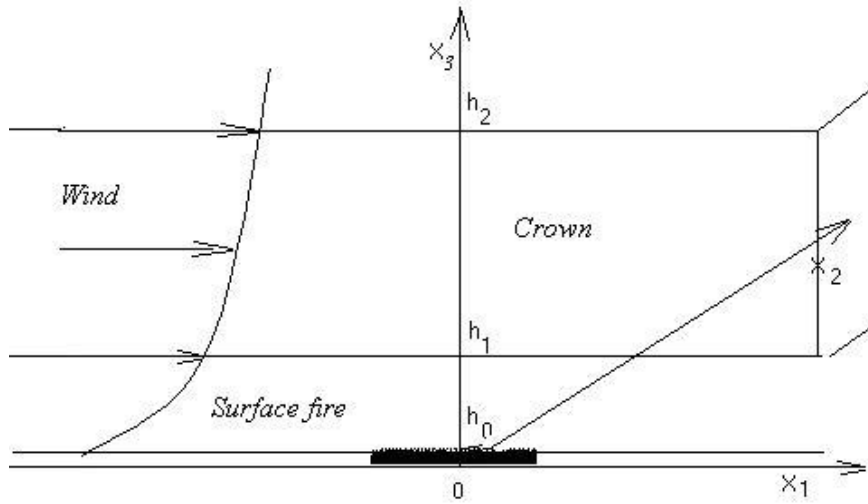


Figure 1.

Using the results of [11-14], known experimental data [11,16] and adopted above assumptions, the following sufficiently general equations, which define the state of the medium in the forest fire zone, written using tensor notation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j} (\rho v_j) = \dot{m}, \quad j=1,2,3, \quad i=1,2,3; \quad (1)$$

$$\rho \frac{dv_i}{dt} = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} (-\rho \bar{v}_i \bar{v}_j) - \rho s c_d v_i |\bar{v}| - \rho g_i - \dot{m} v_i; \quad i=1,2,3; \quad (2)$$

$$\rho c_p \frac{dT}{dt} = \frac{\partial}{\partial x_j} (-\rho c_p \bar{v}_j \bar{T}') + q_5 R_5 - \alpha_v (T - T_s) + k_g (c U_R - 4 \chi \sigma T^4); \quad (3)$$

$$\rho \frac{dc_\alpha}{dt} = \frac{\partial}{\partial x_j} (-\rho \bar{v}_j c'_\alpha) + R_{s\alpha} - \dot{m} c_\alpha, \quad \alpha = 1,5; \quad (4)$$

$$\frac{\partial}{\partial x_j} \left(\frac{c}{3k} \frac{\partial U_R}{\partial x_j} \right) - k c U_R + 4 k_s \sigma T_s^4 + 4 k_g \sigma T^4 = 0, \quad (5)$$

$$k = k_g + k_s;$$

$$\sum_{i=1}^4 \rho_i c_{pi} \varphi_i \frac{\partial T_s}{\partial t} = q_3 R_3 - q_2 R_2 + k_s (c U_R - 4 \sigma T_s^4) + \alpha_v (T - T_s); \quad (6)$$

$$\rho_1 \frac{\partial \varphi_1}{\partial t} = -R_1, \rho_2 \frac{\partial \varphi_2}{\partial t} = -R_2, \rho_3 \frac{\partial \varphi_3}{\partial t} = \alpha_c R_1 - \frac{M_c}{M_1} R_3, \rho_4 \frac{\partial \varphi_4}{\partial t} = 0; \quad (7)$$

$$\sum_{\alpha=1}^5 c_\alpha = 1, \quad p_e = \rho R T \sum_{\alpha=1}^5 \frac{c_\alpha}{M_\alpha}, \quad \vec{v} = (v_1, v_2, v_3), \quad \vec{g} = (0, 0, g),$$

$$\dot{m} = (1 - \alpha_c)R_1 + R_2 + \frac{M_c}{M_1}R_3 + R_{54} + R_{55},$$

$$R_{51} = -R_3 - \frac{M_1}{2M_2}R_5, R_{52} = v_g(1 - \alpha_c)R_1 - R_5,$$

$$R_{53} = 0, R_{54} = \alpha_4 R_1, R_{55} = \frac{\alpha_5 v_3}{v_3 + v_{3*}} R_3.$$

Reaction rates of these various contributions (pyrolysis, evaporation, combustion of coke and volatile combustible products of pyrolysis) are approximated by Arrhenius laws whose parameters (pre-exponential constant k_i and activation energy E_i) are evaluated using data for mathematical models [11,13].

$$R_1 = k_1 \rho_1 \varphi_1 \exp\left(-\frac{E_1}{RT_s}\right), R_2 = k_2 \rho_2 \varphi_2 T_s^{-0.5} \exp\left(-\frac{E_2}{RT_s}\right),$$

$$R_3 = k_3 \rho \varphi_3 s_\sigma c_1 \exp\left(-\frac{E_3}{RT_s}\right), R_5 = k_5 M_2 \left(\frac{c_1 M}{M_1}\right)^{0.25} \frac{c_2 M}{M_2} T_s^{-2.25} \exp\left(-\frac{E_5}{RT_s}\right).$$

where the following notations have been introduced: $\frac{d}{dt}$ is the symbol of the total (substantial) derivative; α_v is the coefficient of phase exchange; ρ - density of gas – dispersed phase, t is time; v_i - the velocity components; T, T_s - temperatures of gas and solid phases, U_R - density of radiation energy, k - coefficient of radiation attenuation, P - pressure; c_p – constant pressure specific heat of the gas phase, $c_{pi}, \rho_i, \varphi_i$ – specific heat, density and volume of fraction of condensed phase (1 – dry organic substance, 2 – moisture, 3 – condensed pyrolysis products, 4 – mineral part of forest fuel), R_i – the mass rates of chemical reactions, q_i – thermal effects of chemical reactions; k_g, k_s - radiation absorption coefficients for gas and condensed phases; T_e - the ambient temperature; c_α - mass concentrations of α - component of gas - dispersed medium, index $\alpha=1,2,\dots,5$, where 1 corresponds to the density of oxygen, 2 - to carbon monoxide CO , 3 - to carbon dioxide and inert components of air, 4 - to particles of black, 5 - to particles of smoke; R – universal gas constant; M_α, M_C , and M molecular mass of α -components of the gas phase, carbon and air mixture; g is the gravity acceleration; c_d is an empirical coefficient of the resistance of the vegetation, s is the specific surface of the forest fuel in the given forest stratum, g – mass fraction of gas combustible products of pyrolysis, α_4 and α_5 – empirical constants. To define source terms which characterize inflow (outflow of mass) in a volume unit of the gas-dispersed mixture \dot{m} , outflow of oxygen R_{51} , changing carbon monoxide R_{52} , generation of black R_{54} and smoke particles R_{55} . Coefficients of multiphase (gas and solid phase) heat and mass exchange are defined $\alpha_v = \alpha S - \gamma C_p \dot{m}, S = 4\varphi_s / d_s$. Here $\alpha = Nu\lambda / d_s$ – coefficient of heat exchange for sample of forest combustible material (for example needle), Nu – Nusselt number for cylinder, λ – coefficient of heat conductivity for pine needle; γ – parameter, which characterize relation between molecular masses of ambient and inflow gases. It is supposed that the optical properties of a medium are independent of radiation wavelength (the assumption that the medium is “grey”), and the so-called diffusion approximation for radiation flux density were used for a mathematical description of radiation transport during forest fires.

The system of equations (1)–(7) contains terms associated with turbulent diffusion, thermal conduction, and convection, and needs to be closed. The components of the tensor of turbulent stresses $\overline{\rho v'_i v'_j}$, as well as the turbulent fluxes of heat and mass $\overline{\rho v'_j c_p T'}$, $\overline{\rho v'_j c'_\alpha}$ are written in terms of the gradients of the average flow properties using the formulas

$$\begin{aligned} -\overline{\rho v_i v_j} &= \mu_t \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) - \frac{2}{3} K \delta_{ij}, \\ -\overline{\rho v_j c_p T'} &= \lambda_t \frac{\partial T}{\partial x_j}, \quad -\overline{\rho v_j c'_\alpha} = \rho D_t \frac{\partial c_\alpha}{\partial x_j}, \\ \lambda_t &= \mu_t c_p / Pr_t, \quad \rho D_t = \mu_t / Sc_t, \quad \mu_t = c_\mu \rho K^2 / \varepsilon, \end{aligned}$$

where μ_t , λ_t , D_t are the coefficients of turbulent viscosity, thermal conductivity, and diffusion, respectively; Pr_t , Sc_t are the turbulent Prandtl and Schmidt numbers, which were assumed to be equal to 1. In dimensional form, the coefficient of dynamic turbulent viscosity is determined using local equilibrium model of turbulence [11]. The length of the mixing path is determined using the formula $l = x_3 k_T / (1 + 2.5 x_3 \sqrt{c_d s / h})$ taking into account the fact that the coefficient of resistance c_d in the space between the ground cover and the forest canopy base is equal to zero, while the constants $k_T = 0.4$ and $h = h_2 - h_1$ (h_2 , h_1 – height of the tree crowns and the height of the crown base).

It should be noted that this system of equations (1)–(7) describes processes of transfer within the entire region of the forest massif, which includes the space between the underlying surface and the base of the forest canopy, the forest canopy and the space above it, while the appropriate components of the surface fire data are calculated using the averaged setting of problem:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j} (\rho v_j) = \dot{m} - (\dot{m}^- - \dot{m}^+) / h_0, \quad j = 1, 2, \quad i = 1, 2, 3; \quad (8)$$

$$\rho \frac{dv_i}{dt} = -\frac{\partial p}{\partial x_j} + \frac{\partial}{\partial x_j} (-\overline{\rho v'_i v'_j}) - \rho s c_d v_i |\vec{v}| - \rho g_i - \dot{m} v_i + (\tau_i^- - \tau_i^+) / h_0; \quad (9)$$

$$\rho c_p \frac{dT}{dt} = \frac{\partial}{\partial x_j} (-\overline{\rho c_p v'_j T'}) + q_5 R_5 - \alpha_v (T - T_s) + (q_T^- - q_T^+) / h_0; \quad (10)$$

$$\rho \frac{dc_\alpha}{dt} = \frac{\partial}{\partial x_j} (-\overline{\rho v'_j c'_\alpha}) + R_{5\alpha} - \dot{m} c_\alpha + (J_\alpha^- - J_\alpha^+) / h_0, \quad \alpha = 1, \dots, 5; \quad (11)$$

$$\frac{\partial}{\partial x_j} \left(\frac{c}{3k} \frac{\partial U_R}{\partial x_j} \right) - k(c U_R - 4\sigma T_s^4) + (q_R^- - q_R^+) / h_0 = 0; \quad (12)$$

$$\sum_{i=1}^4 \rho_i c_{pi} \varphi_i \frac{\partial T_s}{\partial t} = q_3 R_3 - q_2 R_2 + k(c U_R - 4\sigma T_s^4) + \alpha_v (T - T_s); \quad (13)$$

$$\rho_1 \frac{\partial \varphi_1}{\partial t} = -R_1, \quad \rho_2 \frac{\partial \varphi_2}{\partial t} = -R_2, \quad \rho_3 \frac{\partial \varphi_3}{\partial t} = \alpha_c R_1 - \frac{M_c}{M_1} R_3, \quad \rho_4 \frac{\partial \varphi_4}{\partial t} = 0; \quad (14)$$

It is propose to study the propagation of surface fire in two dimensional gorizontal plane, because of the gorizontal sizes of forest massif more than height of surface layer h_0 . The system of equations of general mathematical model of forest fire [11] was integrated between the limits from height of the roughness level - 0 to h_0 . In system of equations (8)-(14) are introduced the next designations:

$$\dot{m} = \rho v_3, \tau_i = -\overline{\rho v'_i v'_i}, J_\alpha = -\overline{\rho v'_3 c'_\alpha}, J_T = -\rho c_p \overline{v'_3 T'}.$$

Upper indexes “+” and “-” designate values of functions at $x_3=h_0$ and $x_3=0$ correspondingly. It is assumed that heat and mass exchange of fire front and boundary layer of atmosphere are governed by Newton law and written using the formulas:

$$\begin{aligned} (q_T^- - q_T^+) / h_0 &= -\alpha(T - T_e) / h_0, \\ (J_\alpha^- - J_\alpha^+) / h_0 &= -\alpha(c - c_{ae}) / h_0 c_p. \end{aligned}$$

Besides, suppose that

$$\int_0^{h_0} \phi dx_3 = \bar{\phi} h_0, \quad (15)$$

Above the litter at the interface of surface fuel bed and space between the underlying surface and the base of the forest canopy the conjugate conditions can be written as follows:

$$\begin{aligned} (\rho v_3)_+ &= (\rho v_3)_-, (v_i)_+ = (v_i)_-, \Phi_+ = \Phi_-, \\ -(\overline{\rho v'_i v'_j})_+ &= -(\overline{\rho v'_i v'_j})_-, -(\overline{\rho v'_i \Phi'})_+ = -(\overline{\rho v'_i \Phi'})_-, \end{aligned} \quad (16)$$

where $\bar{\Phi} \equiv c_p T, c_\alpha$; “+” and “-” are, respectively, the values of functions at the upper and low boundary of domain of interface. This approach substantially simplifies the technology of solving problems of predicting the state of the medium in the fire zone numerically. The thermodynamic, thermophysical and structural characteristics correspond to the forest fuels in the canopy of a different (for example pine forest [11], [13]) type of forest. The conditions of symmetry are used because of the patterns of flow and distributions of all scalar functions are symmetrical relatively to the axes Ox_2 .

3 CALCULATION METHOD AND RESULTS

The boundary-value problems (1)–(7) and (8)-(16) were solved together numerically using the method of splitting according to physical processes [13]. In the first stage, the hydrodynamic pattern of flow and distribution of scalar functions was calculated. The system of ordinary differential equations of chemical kinetics obtained as a result of splitting [13] was then integrated. A discrete analog was obtained by means of the control volume method using the SIMPLE like algorithm [13], [19]. The accuracy of the program was checked by the method of inserted analytical solutions. The time step was selected automatically. Fields of temperature, velocity, component mass fractions, and volume fractions of phases were obtained numerically.

The source of ignition in surface layer is defined as a function of time using experimental data [11] and turned off after the forest fire initiation (t_0 is the time of the fire source formation, characteristic time of setting the maximum temperature in the source [13]). At the moment of ignition the gas combustible products of pyrolysis in low base of crown burns away, and the concentration of oxygen is rapidly reduced [15]. The temperatures of both phases reach a maximum value at the point of ignition. The ignition processes is of a gas - phase nature, i.e. initially heating of solid and gaseous phases occurs, moisture is evaporated. Then decomposition process into condensed and volatile pyrolysis products starts, the later being ignited in the forest canopy. Note also that the transfer of energy from the fire source takes place due to radiation; the value of radiation heat flux density is small compared to that of the convective heat flux. As a result of heating of forest fuel elements, moisture evaporates, and pyrolysis occurs accompanied by the release of gaseous products, which then ignite.

The effect of the wind on the zone of forest fire initiation is shown in Figures 2-4 present the space distribution of field of temperature for gas phase for different instants of time when a wind velocity $V_e = 7$ m/s. We can note that the isosurfaces are deformed by the action of wind. The isosurfaces of the temperature of gas phase 1,2,3 и 4 correspond to the temperatures $\bar{T} = 1.2, 2, 3$ and 4. In the vicinity of the source of heat and mass release, heated air masses and products of pyrolysis and combustion float up. The wind field in the forest canopy interacts with the gas-jet obstacle that forms from the surface forest fire source and from the ignited forest canopy base. Recirculating flow forms beyond the zone of heat and mass release, while on the windward side the movement of the air flowing past the ignition region accelerates. Under the influence of the wind the tilt angle of the flame is increased. As a result this part of the forest canopy, which is shifted in the direction of the wind from the center of the surface forest fire source, is subjected to a more intensive warming up. The isosurfaces of the gas phase are deformed in the direction of the wind.

Figures 3 and 4 present the distribution of the velocity and isosurfaces of the temperature at the different instants of time when a wind velocity $V_e = 7$ m/s.

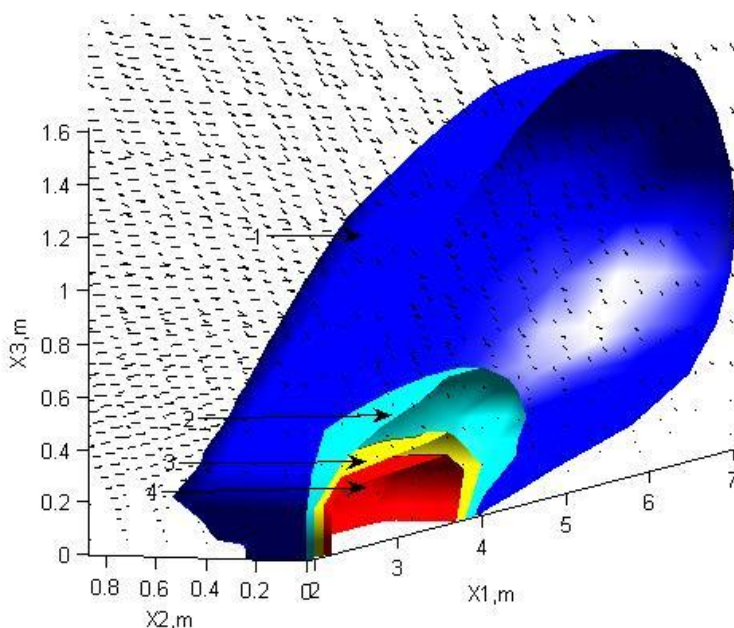


Figure 2.

The vectorial field of velocity and temperature at $t=3.3$ s.

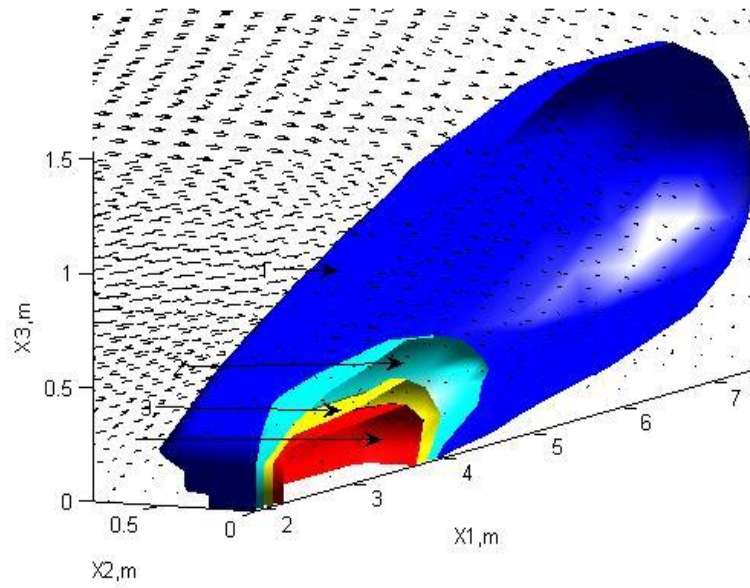


Figure 3.
The vectorial field of velocity and temperature at $t=3.8$ s.

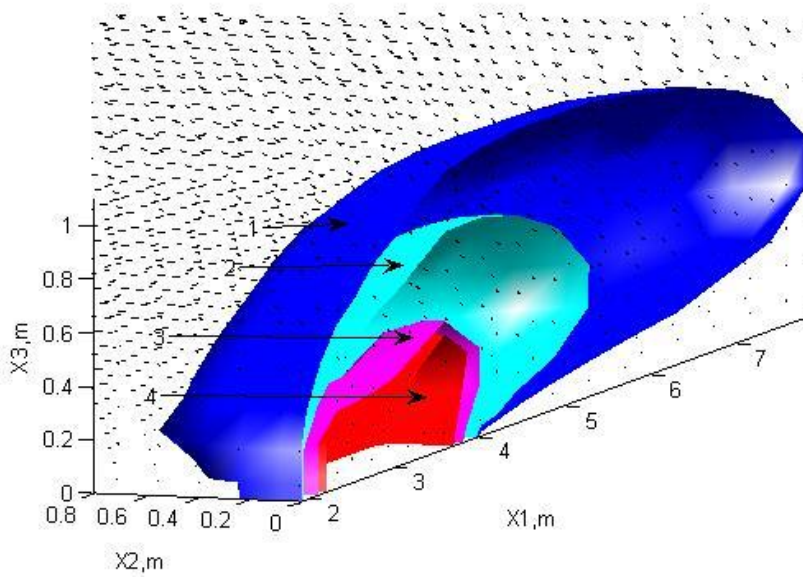


Figure 4.
The vectorial field of velocity and temperature at $t=4.8$ s.

The effect of the wind on the forest fire spread is shown in Figures 5a,b, present the distribution of temperature \bar{T} ($\bar{T} = T/T_e, T_e = 300 K$) (1- 5., 2 - 4.5, 3 - 4, 4 - 3.5) for wind velocity $V_e = 10$ m/s: and a) $t=3$ sec., b) $t=5$ sec. We can note that the isotherms is moved in the forest canopy and deformed by the action of wind. It is concluded that the forest fire begins to spread and the fire front is extent.

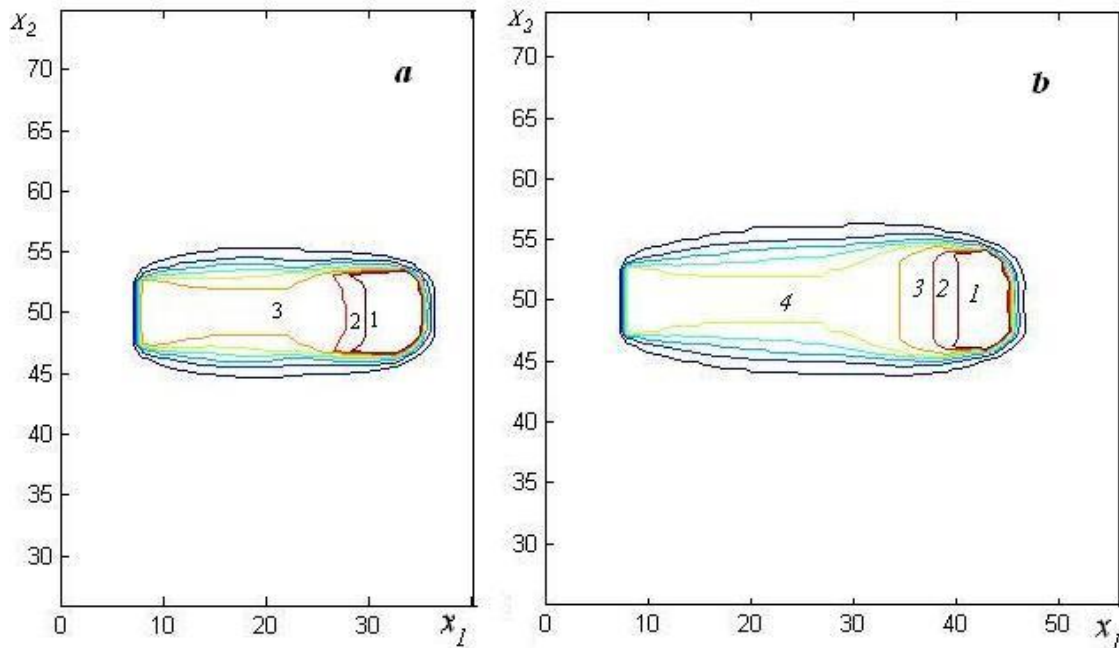


Figure 5.
Fields of isotherms of the forest fire for plane $x_1 x_2$
for different moments of time:
a) $t=3$ sec., b) $t=5$ sec.

4 CONCLUSION

Mathematical model and the result of the calculation give an opportunity to evaluate critical condition of the forest fire initiation and spread which allows applying the given model for preventing fires. The model can overestimate the rate of the crown forest fires spread. The results obtained agree with the laws of physics and experimental data [11, 13, 15, 16]. This work represents the attempt for application of three dimensional models for description of crown forest fires initiation and spread.

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