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NUMERICAL SIMULATIONS OF THE LORENTZ FORCE FLOWMETER

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Abstract. *Lorentz force velocimetry (LFV) is a contactless technique for the measurement of liquid metal flowrates at high temperature*[1,2]*. It consists in measuring the force acting upon a magnet system, and arising from the interaction between an external magnetic field and the flow of an electrically conducting fluid. In this preliminary study, the existing device is improved in order to make the measurement essentially independent of the fluid electrical conductivity. The present flowmeter consists in two coils placed around a circular pipe. The forces produced by each coil are recorded in time as the liquid metal flows through the pipe. It is highlighted that the cross-correlation of these forces can be used to determine the flowrate of turbulent liquid metal flows. The study is entirely numerical and uses a second-order finite volume method.*

1 INTRODUCTION

A key feature of electromagnetism is that a force is generated when an electrically conducting material moves through a magnetic field. If the material is in a fluid state, this principle can be used to determine its flowrate. At low temperatures, flowrates can be measured through inductive flowmeters[3]. By contrast, measurements in metallurgical flows of liquid metals at high temperatures cannot be carried out using conventional inductive flowmeters, since electrodes cannot be inserted in the flow. The present work is devoted to a non-contact electromagnetic flow measurement technique called Lorentz force velocimetry (LFV)[1,2]. It is based on measuring the force acting upon a magnet system that interacts with the flow of an electrically conducting fluid.

More precisely, the purpose of the present study is to improve the existing device, in order to make the measurement essentially independent of the fluid electrical conductivity. This is highly desirable in metallurgy, where the temperature and composition of the alloy can vary significantly in time and space. In turn, these variations bring uncertainties on the value of the fluid electrical conductivity. The proposed Lorentz force flowmeter (LFF) is based on temporal correlations, taken from force measurements at two locations. The feasibility of this novel version of Lorentz force velocimetry is demonstrated entirely numerically.

2 BASIC PRINCIPLE

A Lorentz force flowmeter (LFF) measures the integrated Lorentz force, resulting from the interaction between a liquid metal in motion and an applied magnetic field. In the present manuscript, the magnet system consists in two current-carrying coils placed around a circular pipe and generates the so-called primary magnetic field $\mathbf{B}$, given by Biot–Savart's law[4],

$$\mathbf{B}(\mathbf{r}) = \sum_{i=1,2} \mathbf{B}_i(\mathbf{r}) = \sum_{i=1,2} \frac{\mu_0 J_i}{4\pi} \oint \frac{\mathbf{dl} \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3}, \tag{1}$$

where J_i is the magnitude of the primary electric current circulating in the i^{th} coil, $\mu_0 = 4\pi 10^{-7}$H/m is the vacuum permeability, $\mathbf{dl}$ is a length element of the coil, $\mathbf{r}'$ is the position of the coil and $\mathbf{r}$ denotes the location where the magnetic field is evaluated. The magnetic field lines are sketched in figure 1 for a pipe of length L and two coils separated by a distance Δ. The currents flowing through the coils have same signs (left figure) or opposite signs (right figure). Since the magnetic field interacts with the flow velocity $\mathbf{u}$, eddy currents, also called secondary currents, are induced in the liquid metal. These in turn create an induced magnetic field $\mathbf{b}$, referred to as the secondary magnetic field. In this work, the magnetic diffusion time is assumed much smaller than the timescale of large eddies. Therefore, the secondary magnetic field becomes negligible with respect to the primary magnetic field, namely $|\mathbf{b}| \ll |\mathbf{B}|$. This is referred to as the quasi-static approximation[5]. In this framework, eddy currents are described by a simplified Ohm's law, for moving electrically conducting fluids, and has the form

$$\mathbf{j} = \sigma(-\boldsymbol{\nabla}\phi + \mathbf{u} \times \mathbf{B}). \tag{2}$$

The fluid electrical conductivity is denoted σ and the electric field is assumed to be the gradient of the electrical potential ϕ. According to the conservation of electric charge, eddy currents are divergence-free. Hence, the electrical potential satisfies the Poisson equation, $\nabla^2\phi = \nabla \cdot (\mathbf{u} \times \mathbf{B})$. In addition, eddy currents are maximum where the angle between $\mathbf{u}$ and $\mathbf{B}$ is large, see figure 1.

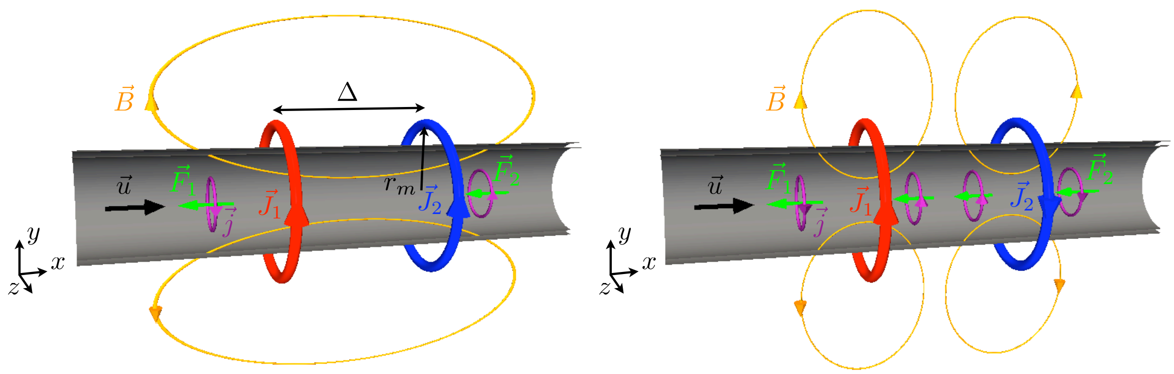

Figure 1: Principle of Lorentz force velocimetry. Two current-carrying coils produce the primary magnetic field $\mathbf{B}$, which interacts with the flow $\mathbf{u}$ of an electrically conducting fluid and induces eddy currents $\mathbf{j}$. The coil currents have either same (left) or opposite (right) signs. The force acting upon the coils is equal in magnitude (but opposite in direction) to the sum of the Lorentz forces F_1 and F_2 acting on the flow.

The interaction between the primary magnetic field $\mathbf{B}_i$, generated by the i^{th} coil, and the eddy current $\mathbf{j}$, induced by all coils, yields a Lorentz force,

$$\mathbf{F_L}_i = \mathbf{j} \times \mathbf{B}_i, \tag{3}$$

which globally brakes the flow. Furthermore, the secondary magnetic field magnetic field $\mathbf{b}$ interacts with the primary current and induces a reaction force, acting on the i^{th} coil. By virtue of the reciprocity principle[2], the following integrated Lorentz force,

$$F_i = \frac{1}{V}\int \mathbf{F_L}_i dV = -F_i^r, \tag{4}$$

is equal in magnitude but opposite in direction to the reaction force F_i^r acting upon the i^{th} coil. In addition, F_i is proportional to the mean velocity of the flow and the electrical conductivity of the fluid. Previous works have shown that F_i, or equivalently F_i^r, can be used to determine the mean flowrate assuming the fluid conductivity is known[1,2]. However, the electrical conductivity of the fluid is often unknown or fluctuates in time. It is therefore desirable to develop Lorentz force flowmeters which operate independently of the electrical conductivity of the fluid.

To remove this dependency, a variant of flowmeter is investigated. It focuses on the time evolutions of the Lorentz forces, which are cross-correlated. By definition, the location of the maximum correlation gives the time shift for which the two forces are most similar. Therefore, information about the mean flowrate can be obtained. The main advantage of this technique is that the resulting flowrate is indenpendent on the fluid properties. In the present application, the challenge stands in using the coils as sensors.

The quality of the measurement is assessed in two ways. First, its reliability is analysed by quantifying the amplitude of the correlation peak. Second, the measured time shift is compared to its exact value, in order to determine the calibration factor. The investigation is performed for different coil radii r_m, separations Δ and signs of the coil-carrying currents.

3 GOVERNING EQUATIONS AND NUMERICAL METHOD

In the proposed device, cross-correlations are performed between the Lorentz forces due to each coil. Since the Lorentz forces depend on the electric potential and the velocity field, these quantities are needed at each time step. In this manuscript, we assume that the flow is unaffected by the Lorentz force. For the parameters considered here, the maximum streamwise Lorentz force is indeed less than 5% of the driving force. It is thus expected to have a weak influence on the flow. With this assumption, the incompressible flow dynamics are governed by the Navier–Stokes equations, as in classical hydrodynamics. The equations of motion are thus given by

$$\begin{aligned} \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\boldsymbol{\nabla})\mathbf{u} &= -\boldsymbol{\nabla} p + \nu\nabla^2\mathbf{u}, \\ \boldsymbol{\nabla}\cdot\mathbf{u} &= 0, \end{aligned} \tag{5}$$

p being the kinematic pressure (i.e. divided by the fluid density) and ν the fluid kinematic viscosity. These equations are discretised spatially using an unstructured finite volume method based on a collocated formulation. The method is analogue to that used in previous studies[2,6], and it is thus not detailed here. Briefly, the velocity and pressure fields at time $n+1$ are computed using a fractional-step method. First, Eq. 5 is solved for the velocity field at an intermediate time step $\mathbf{u}^{(\star)}$. Then, the pressure field $p^{(n+1)}$ at time $n+1$ is computed through the following Poisson system,

$$\nabla^2 p^{(n+1)} = \boldsymbol{\nabla}\cdot\mathbf{u}^{(\star)}, \tag{6}$$

so as to ensure that $\mathbf{u}^{(n+1)}$ is divergence-free. The velocity field $\mathbf{u}^{(n+1)}$ is finally obtained as $\mathbf{u}^{(n+1)} = \mathbf{u}^{(\star)} - \Delta t\boldsymbol{\nabla} p^{(n+1)}$, Δt being the time step. The electrical potential satisfies the following Poisson equation

$$\nabla^2\phi = \boldsymbol{\nabla}\cdot(\mathbf{u}\times\mathbf{B}), \tag{7}$$

and it is computed explicitely using $\mathbf{u}^{(n+1)}$.

The computational domain consists in a circular pipe, characterised by no-slip and insulating walls located at $r = R$, which yields

$$\begin{aligned} \mathbf{u}|_{r=R} &= 0, \\ (\mathbf{j}\cdot\mathbf{n})_{r=R} &= 0, \\ \left.\frac{\partial\phi}{\partial n}\right|_{r=R} &= 0, \end{aligned} \tag{8}$$

where $\mathbf{n}$ is the unit vector normal to the wall. Moreover, periodic boundary conditions are applied at the inlet and outlet. In order to solve the Poisson system for the potential, the following boundary conditions are applied,

$$\begin{aligned} \phi|_{r=0} &= 0, \\ \left.\frac{\partial\phi}{\partial r}\right|_{r=R} &= 0. \end{aligned} \tag{9}$$

4 SIMULATION SETTINGS

The input parameters for the flowmeter are the coil radius r_m (both coils being assumed to be equal in size), the coil separation Δ and the currents J_1 and J_2 flowing through each coil. Both same-sign currents ($J_1 = J_2$) and opposite-sign currents $J_1 = -J_2$ are considered, see figure 1. The primary magnetic field is computed through Biot–Savart's law, see Eq. 1, by discretising each coil in 1,000 segments. The coil radii and separations equal twice the pipe radius, namely $r_m = \Delta = 2R$.

The pipe is ten times longer than its radius, i.e. $L = 10R$, and it is discretised with 65 points in the streamwise direction. The mesh resolution in the radial and azimuthal directions vary with the azimuthal angle since the mesh is unstructured. However, the pipe wall is discretised with 64 points in the azimuthal direction and the numbers of points along the pipe diameter is approximately 50.

The flow is initialised with the following turbulent-like velocity profile on which random perturbations are superposed[2],

$$\begin{aligned} u_x &= U_c\beta(\alpha)\ln\left[1+\alpha\left(1-\frac{r^2}{R^2}\right)\right] \\ u_r &= u_\theta = 0, \end{aligned} \tag{10}$$

where

$$\beta(\alpha) = \frac{\alpha}{(1+\alpha)\ln(1+\alpha)-\alpha}, \tag{11}$$

and $\alpha = 1000$. Furthermore, the flow is driven by a constant pressure gradient such that the bulk Reynolds number fluctuates around $Re_b = 2U_bR/\nu \approx 3600$, $U_b = 1/V\int udV = 2/R^2\int_0^R\langle u_x\rangle rdr$ being the bulk velocity, V the pipe volume, and $\langle u_x\rangle$ the mean streamwise velocity profile in the radial direction. The mean value of the bulk velocity is fixed through

$$U_b^2 \;=\; \frac{4R}{\rho f}\left(\frac{\partial p}{\partial x}\right), \tag{12}$$

where f is the friction factor[7].

5 RESULTS

The integrated forces produced by each coil and acting upon the flow in the streamwise direction, $\widetilde{F}_{1x}$ and $\widetilde{F}_{2x}$, are non-dimensionalised by removing their mean and dividing by their root-mean-square. Their evolutions in time are shown by figure 2, for same-sign currents (left) and opposite-sign currents (right) flowing through the coils. For clarity, the time history is limited to three crossing of the mean flow through the pipe. Since the flow is turbulent, the forces exhibit several fluctuations and they are statistically shifted in time.

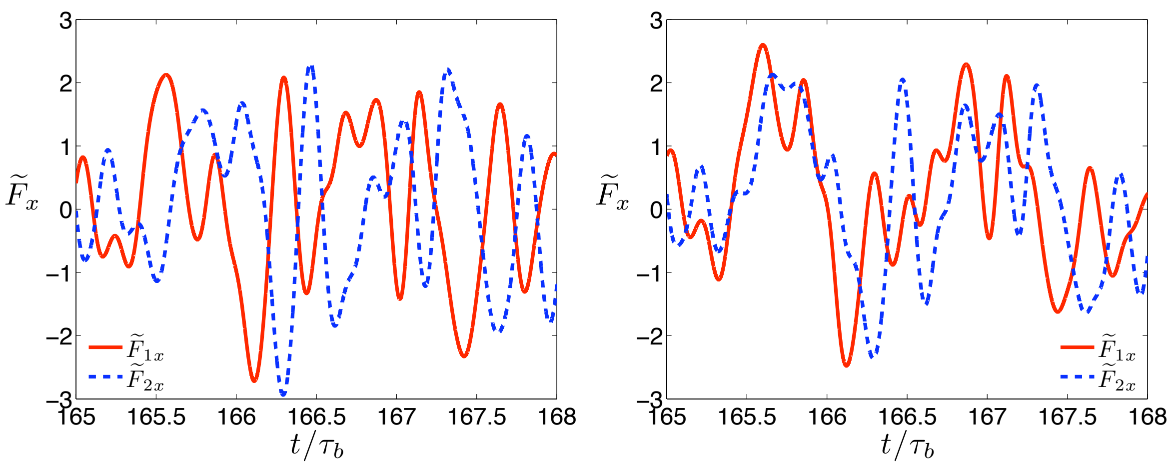

Figure 2: Time evolutions of the integrated forces produced by each coil when a turbulent flow moves through the flowmeter: same-sign currents (left) and opposite-sign currents (right). Times are non-dimensionalised by the crossing time of the mean flow through the pipe, $\tau_b = L/U_b$, where U_b is the bulk velocity.

The cross-correlations of the forces produced by each coil are illustrated by figure 3. Two characteristics of the correlations are of particular interest: the time shift T_p associated to their main peak, and the peak magnitude C_M, which indicates the reliability of the measurement. The time T_p is compared to the time needed for the mean flow to travel from the upstream to the downstream coil, namely

$$T_b \;=\; \frac{\Delta}{U_b}. \tag{13}$$

The ratio T_p/T_b is defined as the calibration factor of the measurement. In turn, the flowrate measured by the Lorentz force flowmeter (LFF) is given by

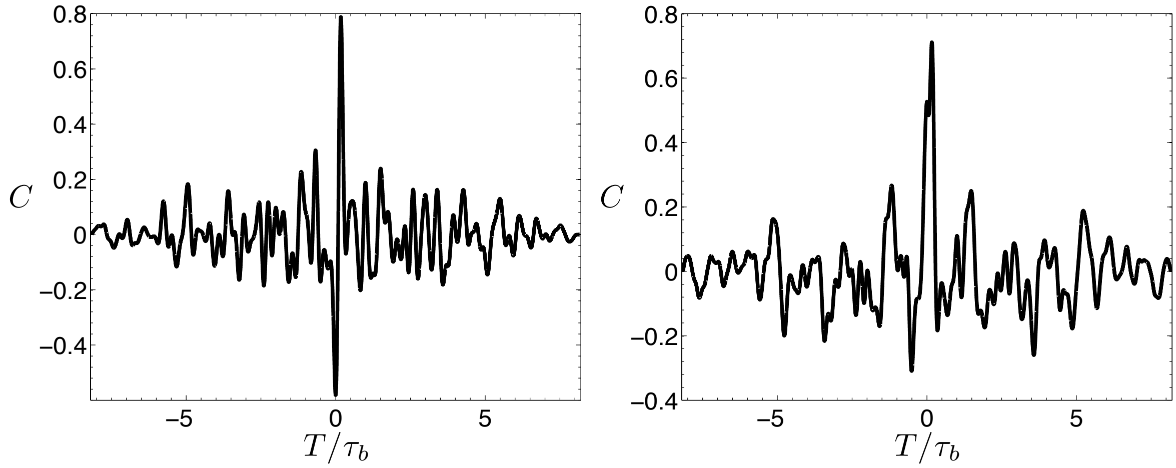

Figure 3: Cross-correlations of the integrated forces produced by each coil when a turbulent flow moves through the flowmeter: same-sign currents (left) and opposite-sign currents (right). The time shifts T are non-dimensionalised by the crossing time of the mean flow through the pipe, $\tau_b = L/U_b$, where U_b is the bulk velocity.

$$U_{LFF} = \frac{T_b}{T_p} U_b, \tag{14}$$

where U_b is the exact flowrate. The results are presented in table 1. In both cases, the main peak of the correlation is well distinguishable from the secondary oscillations. Moreover, it is slightly larger and sharper with same-sign currents than with currents of opposite sign. Remarkably, the calibration factor T_p/T_b is almost independent of the current signs.

Sign of (J_1, J_2)	C_M	T_p/T_b	U_{LFF}/U_b
Same	0.79	0.87	1.15
Opposite	0.71	0.84	1.20

Table 1: Results of the cross-correlations between the forces produced by each coil when a turbulent flow moves through the flowmeter. C_M is the magnitude of the correlation peak and T_p/T_b is the ratio between the time shift associated to the peak and the time-of-flight of the mean flow between the coils. The ratio of the measured to the exact flowrate is U_{LFF}/U_b.

6 CONCLUSIONS

This preliminary study demonstrated numerically the feasibility of a two-coil flowmeter, based on time-correlations of the Lorentz forces produced by each coil. The proposed technique was validated with coil radii and separation equal to twice the pipe radius. Particular attention was given to the magnitude of the correlation peak and its associated time. Currents of same- and opposite-sign, flowing through the coils, were investigated. In both cases, the flowrate of a three-dimensional turbulent flow can be successfully measured. More precisely, the determination of the correlation peak is reliable and the calibration factor is almost insensitive to the sign of the currents.

Future works will aim at analysing systematically the reliability and calibration factor of the measurement for a wide range of coil parameters. The region of maximum sensitivity of the flowmeter should also be identified. Finally, the effect of the magnetic field on the flow will be quantified through dynamic simulations, in which the momentum balance accounts for the Lorentz force as an explicit source term.

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