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NUMERICAL MODEL OF THE ELECTRICAL TRANSFORMER EPOXY CASTING PROCESS AND ITS HIERARCHICAL VALIDATION

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Abstract. *Main objective of this work is to design and carry out the hierarchical validation and verification (V&V) procedure of the numerical model of the free surface flow in casting applications. Casting processes encompass many techniques that may significantly differ from each other. However, for the purpose of this study one specific process, namely gravitational epoxy casting of an electrical transformer was chosen. Nevertheless, the proposed V&V procedure have rather universal character. Casting process is characterized with a very complex shape of the free surface and coupling between physical properties of the epoxy composition, velocity, temperature and polymerization reaction, making its modelling very demanding task. The Volume of Fluid (VOF) method was used as a framework for development of the mathematical model of the transformer casting process. The polymerization reaction of the epoxy composition was modelled by introduction of the curing degree transport equation and the reaction kinetics determined experimentally. The strong coupling between rheological properties of the epoxy composition, kinetics of the polymerization reaction and temperature was considered in the model as well. The surface tension and wall adhesion effects were modelled with the Continuum Surface Force (CSF) model. The dynamics of the surface wetting was also considered. Computations were performed with all purpose Computational Fluid Dynamics code ANSYS Fluent. The hierarchical verification and validation procedure was proposed to asses credibility of the developped model. Experimental results were compared with the numerical calculations showing reasonably good agreement.*

1 INTRODUCTION

During the last thirty years or so, an incredible development and a dramatic cost decrease in computers has caused them to become one of the most important tools in the work of a modern engineer in the field of modelling fluid and heat flows. Computational Fluid Dynamic (CFD) techniques are efficiently used in conceptual studies of new designs, detailed product development, troubleshooting or redesign, optimisation and control of many industrial processes. Broad use of CFD in industrial applications has resulted in more complex problems being solved. However, each CFD code and simulation needs to be inspected if the obtained results are reliable. This objective is accomplished by means of a credibility analysis (or Validation & Verification - V&V analysis) of the numerical model[1,2]. Credibility analysis consists of two fundamental steps: verification and validation. The main objective of simulation validation is ensuring if the physical process is properly described by the governing equations, *i.e. whether the correct equations are being solved*, while verification addresses the correctness of the solution of the model equations *i.e. whether the equations are being solved correctly.* Practically, validation is accomplished by the comparison of calculated results with those measured in the specially designed experiment. The verification procedure consists of two steps. The first step is the code verification which addresses the possible coding errors in the simulation software. In the second step, the solution verification quantifies the numerical errors which appear during the solution procedure[1,3].

Electrical transformers are important parts of many industrial power supply systems. Only functional and reliable insulation ensures proper operation of these electrical devices. This is particularly important for devices operating in rigorous ambient conditions. Transformers working in such conditions, are often built as a dry-type ones. It means that all electrical circuits of such a device are enclosed in tight casing made of epoxy resin. The majority of the dry-type electrical transformers is produced in the Resin Casting Technology or the Resin Injection Moulding Technology. In these technologies the internal elements of the transformer are placed in a mold which is then filled with an epoxy resin composition. After the introductory curing in the mould until gelation point is reached, the product is further cured outside the mold. The mold casting stage is crucial for the final product quality. Problems such as incomplete filling, air voids and bubbles may arise during this stage. Appearance of any of mentioned problems, disqualifies the product, that is why it is of great importance to be able to predict them. Such possibility is given by the Computational Fluid Dynamics (CFD) simulations.

The resin casting is a very complex and highly coupled process. The following unit phenomena, which make up this process can be distinguished:

- free surface flow (two phase flow),
- viscous flow,
- heat transfer,

- curing reaction,
- surface tension,
- static and dynamic wetting of the solid surfaces (static and dynamic contact angle),
- thermal expansion,
- shrinkage of the epoxy cast due to curing reaction progress,
- stress built up due to curing reaction and temperature gradient.

It would be ideally if all these phenomena were incorporated in the mathematical model. However, since the casting itself is reasonably short process, it is not significantly influenced by the last two phenomena. Moreover, they belong rather to the region of interests of the Solid Mechanics, that is why they were neglected in the analysis.

2 MATHEMATICAL MODEL

The mathematical model is based on the single fluid formulation. In this approach it is assumed that all phases (ie gas and liquid) share one velocity, pressure and temperature field. The free surface shape was tracked with use of the well established Volume of Fluid (VOF) method by Hirt and Nichols was used[4]. In this method the transport equation for the volume fraction of the liquid phase is solved:

$$\frac{\partial(\alpha_l\,\varrho_l)}{\partial t} + \boldsymbol{\nabla}\cdot(\alpha_l\,\varrho_l\,\boldsymbol{w}) \;=\; 0 \tag{1}$$

where ϱ_l is the density of the liquid phase, $\boldsymbol{w}$ is the velocity vector common for both phases. The volume fraction α_g of gas phase is calculated using the formula:

$$\alpha_g \;=\; 1-\alpha_l \tag{2}$$

Both phases share the same velocity $\boldsymbol{w}$ and pressure p fields, hence one set of flow governing equations is considered[5]:

- Mass conservation equation

$$\frac{\partial\varrho}{\partial t} + \boldsymbol{\nabla}\cdot(\varrho\,\boldsymbol{w}) \;=\; 0 \tag{3}$$

- Momentum conservation equation

$$\frac{\partial\,(\varrho\,\boldsymbol{w})}{\partial t} + \boldsymbol{\nabla}\cdot(\varrho\,\boldsymbol{w}\boldsymbol{w}) = -\boldsymbol{\nabla}p + \boldsymbol{\nabla}\cdot\left[\mu\left(\boldsymbol{\nabla}\boldsymbol{w} + (\boldsymbol{\nabla}\boldsymbol{w})^T\right)\right] + \varrho\,\boldsymbol{g} + \boldsymbol{F}_\sigma \tag{4}$$

In the above equations $\boldsymbol{F}_\sigma$ is the body force normal to the free surface due to surface tension. It has a non zero value only in those cells where the interface is present. The density ϱ is the volume average value of two phase mixture. In the same manner the average viscosity μ is calculated.

Surface tension was modelled using the continuum surface force (CSF) model proposed by Brackbill et al.[6]. In this model the surface tension is accounted for by the addition of an extra body force term $\boldsymbol{F}_\sigma$ in the momentum equation (4) for the computational cells through which the phase interface passes:

$$\boldsymbol{F}_\sigma = 2\,\sigma\,\kappa_l \frac{\alpha_l\,\varrho_l + \alpha_a\,\varrho_a}{\varrho_l + \varrho_a} \boldsymbol{\nabla}\alpha_l \tag{5}$$

where σ is the surface tension between the liquid and air, ϱ is the average density, κ_l is the curvature of the free surface. Near the walls local curvature depends on the contact angle. In this work it was assumed that the contact angle is a function of the common line (contact line) velocity.

Both phases shared one temperature field, therefore the only one energy equation is solved[5]:

$$\frac{\partial}{\partial t}\left[\varrho\left(u + \frac{w^2}{2}\right)\right] + \boldsymbol{\nabla}\cdot\left[\varrho\,\boldsymbol{w}\left(u + \frac{w^2}{2}\right)\right] = \boldsymbol{\nabla}\left(\lambda_{eff}\boldsymbol{\nabla}T\right) + \dot{q}_{v,r} \tag{6}$$

where u is the specific internal energy of two phase mixture, calculated as a mass weighted average and λ_{eff} is the effective heat conduction coefficient, calculated as a volume average. The last term in the energy equation (6) *i.e.* $\dot{q}_{v,r}$ stands for the volumetric heat source due to exothermic curing reaction:

$$\dot{q}_{v,r} = \alpha_l\,\varrho_l\,\dot{\beta}\,\Delta H \tag{7}$$

where $\dot{\beta}$ is the rate of curing reaction progress and ΔH refers to the enthalpy of that reaction[7]. Both quantities are obtained with Differential Scanning Calorimetry (DSC) measurements.

The curing reaction degree β describes overall progress of the reaction without going into details of the actual chemical reactions that take place within a polymer. If one neglects diffusion of the substrates and products in the epoxy composition, the equation which governs the curing degree distribution within the liquid phase can be written as:

$$\frac{\partial\left(\alpha_l\,\varrho\,\beta\right)}{\partial t} + \boldsymbol{\nabla}\cdot\left(\alpha_l\,\varrho\,\beta\,\boldsymbol{w}\right) = \alpha_l\,\varrho\,\dot{\beta} \tag{8}$$

The source term on the right hand side of the equation refers to the reaction progress $\dot{\beta}$ and it is described by the reaction kinetics model. Here the two stage Kamal curing reaction model was chosen[8]:

$$\dot{\beta} = k_1\,\beta^{m_1}\,(1-\beta)^{n_1} + k_2\,(k_{02} + \beta^{m_2})\,(1-\beta)^{n_2} \tag{9}$$

where n_1, m_1, n_2 and m_2 are orders of the appropriate reactions, k_{02} refers to constant parameter of the model and k_1 and k_2 are reaction rate coefficients, that depend on temperature.

The viscosity of the epoxy resin highly depends on the temperature and the curing degree. This relationship is very well correlated by the Macosko-Cross model[9]:

$$\mu = \mu_0 \exp\left(\frac{E_\mu}{R_g T}\right)\left(\frac{\beta_G}{\beta_G - \beta}\right)^{c_0 + c_1\beta} \tag{10}$$

where E_μ refers to the activation energy, β_G stands for the gelation point (it is theoretical value of the curing degree at which composition may be considered as a solid) and μ_0, c_0 and c_1 are model parameters.

In the isothermal casting experiment (see the next section) a highly non-Newtonian analog liquid was used. Its rheological properties were described well by the Cross model[9]:

$$\frac{\mu - \mu_\infty}{\mu_0 - \mu_\infty} = \frac{1}{1 + (k_\mu \dot{\gamma})^n} \tag{11}$$

where μ_∞ is the infinite-rate viscosity, μ_0 refers to the zero-rate viscosity, k_μ is the consistency parameter, n refers to the rate index and $\dot{\gamma}$ is the shear rate.

Other two important properties that may significantly vary with temperature, are density and surface tension. Density is assumed to be a linear function of temperature, while in case of the surface tension Macleod equation was used.

Further details on the mathematical model and constitutive equations of the physical properties and its measurements can be find in the work[10]

3 HIERARCHICAL VALIDATION PROCEDURE

In case of the V&V analysis of very complex real-life problems two important issues arise. First of all it is extremely difficult to carry out reliable validation experiment of the complex process. Problems are caused not only by the low measurement quality but what is probably more important by the precise determination of all unambiguity conditions *i.e.* boundary and initial conditions, physical properties of materials or geometry. That is why a hierarchical validation approach is recommended for such complex processes[3]. At the top of the hierarchical validation procedure there is the complete system, which incorporates all model subtleties, *cf.* Figure 1). At the lower level the system is decomposed into a few subsystems. Each subsystem considers real system geometry and three or more physical phenomena. At the benchmark cases level the simplified system geometry is used. Every benchmark usually encompasses two or three coupled physical phenomena. The lowest level concerns unit problems. At this level highly specialized measurements of a single physical phenomenon for which specific equipment is applied are carried out. The number of layers and its specific division is obviously system dependent. Moreover, an optimal system decomposition does not exist and it can vary. The most important disadvantage

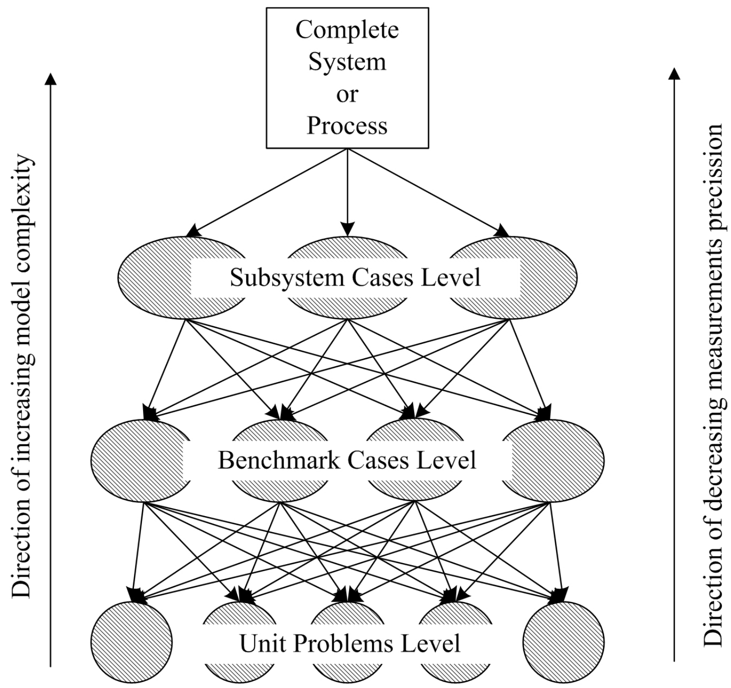

Figure 1: Hierarchical approach to designing of validation procedure.

is that an entity from the higher level can not be considered as a linear combination of entities from the lower level. Furthermore, if materials with unknown physical properties are used at any stage of the procedure then part of the measurement results at the unit problem level have to be used to create constitutive equations for unknown properties and are useless for the validation purpose.

An proposition of the hierarchical validation procedure of a dry transformer casting process is presented in the Figure 2. At the system level the attention focusses on the casting of a transformer in the real installation, where all phenomena mentioned in the first section of this article take place simultaneously. Below there are two subsystems distinguished *i.e.* structural mechanics and fluid mechanics. The structural mechanics subsystem covers building up of stress and strain fields during the transformer epoxy casting, driven by the curing reaction and temperature gradient. The fluid mechanics subsystem considers the free surface flow, curing reaction propagation and thermal effects. At this level the non-isothermal casting experiment was conducted on specially designed experimental stand. In this experiment real-life epoxy composition was used. The next level of the diagram is occupied by the benchmark experiment carried out with analog liquid in the isothermal conditions. The bottom level consists of the set of unit problems (elementary phenomena). Among them falling drop experiment is considered as a core

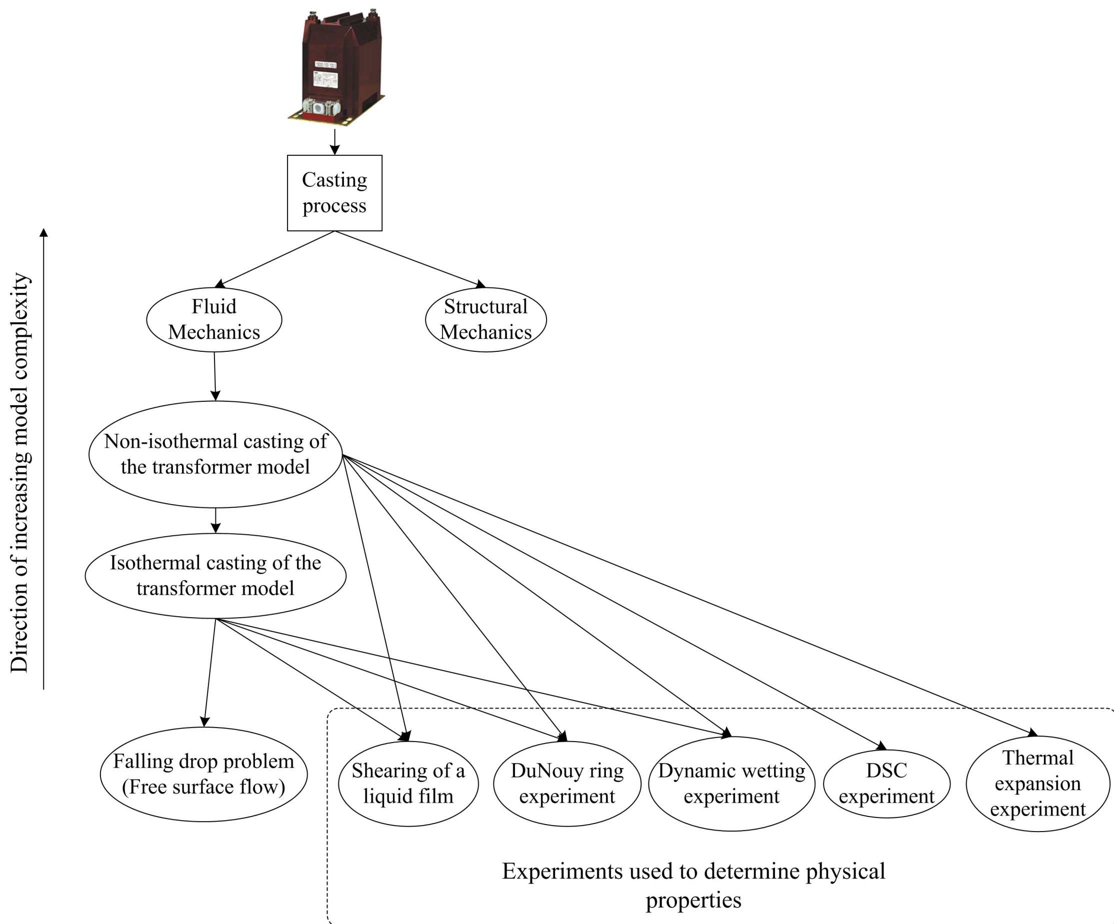

Figure 2: Hierarchical validation procedure of the dry transformer casting process.

one. In this experiment main efforts were put on capturing the shape of the free surface *i.e.* shape of the falling drop and the bulk water surface after this drop hit it. Other unit experiments conducted at this level were used to estimate crucial physical properties of the materials that participated in the experiments at upper levels:

- Differential Scanning Calorimeter (DSC) measurements allowed one to develop curing reaction kinetic model of the epoxy composition,
- continuous shearing of the thin liquid film of analog liquid and epoxy resin using rotational rheometer to establish non-Newtonian rheological behavior of the analog liquid and rheological model of the epoxy composition (especially dependence of its viscosity on temperature and curing degree),
- du Nouy ring experiment to measure surface tension of the analog liquid and epoxy composition,

- dynamic wetting experiment to estimate the static and the dynamic contact angle between analog liquid or epoxy composition and all solid surfaces that participated in the upper level experiments,

- thermal expansion experiment to describe the temperature dependence of the epoxy composition density.

There were no heat transfer unit problem conducted at this level of the V&V structure as it was assumed that the software used to develop the mathematical model was already validated with respect to this phenomenon.

4 RESULTS

4.1 Falling drop experiment

According to the hierarchical validation procedure presented in the previous section the falling drop experiment occupies the unit problem level, *cf.* Figure 2. In this experiment a movement and shape of the free surface were recorded when the liquid drop was falling into a bulk liquid placed in a cylindrical beaker. The drop was formed with use of manual syringe. The beaker was closed from above to minimize external influence. Subtleties like drop formation and free shape of the free surface hit by the drop make this process a good problem for the free surface shape validation.

The only phenomenon that needs to be considered in this experiment is the free surface laminar flow of the incompressible gas liquid system under isothermal conditions. Geometry and materials where chosen in the way to maximize the problem simplicity and hence to minimize uncertainties introduced to the simulation with unambiguity conditions. A demineralized water and ambient air were used to represent liquid and gas phases. The interior of the beaker without external walls was assumed as a computational domain. Mathematical description of this problem in the framework of single fluid formulation comprises of continuity, momentum and liquid volume fraction equations only.

The three fully structural and axisymmetric numerical grids were prepared with the average size of the grid cell equal to 0.0004 m (grid 1), 0.0002 m (grid 2) and 0.0001 m (grid 3).

In Figure 3 formation of the drop in the experimental conditions is compared with the results of numerical simulations carried out on three successively refined grids. In the experiment drop formed and tore off after time approximately equal to 0.038 s from the beginning of the flow. In numerical simulations it was appropriately: 0.03 s for the first grid, 0.034 s for the second grid and 0.036 s in case of the third grid. Qualitatively all simulation results satisfactory represent shape of the drop in the experiment. Drop shape obtained on the coarser grids (grid 1 and 2) seem to be closer to the experimental results than the finest grid (grid number 3). In case of the finest grid the drop is artificially elongated in the axial direction, see Figure 3d. This is not observed in the experiment and in results obtained with two coarser grids, see Figures 3a, b and c. However, the finest

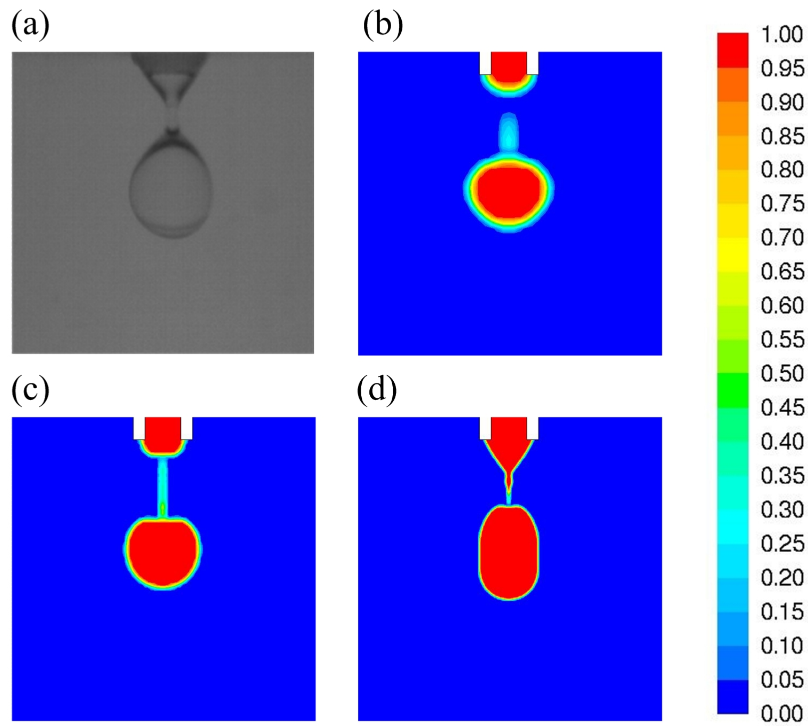

Figure 3: Shape of the drop just after it was formed and tore off the liquid body, time 0.04 seconds, for the grid dependence study: (a) experiment; (b) grid 1; (c) grid 2; (d) grid 3.

grid represents most accurately drop forming and tearing of phenomenon. Observed shape of the bridge between the drop and bulk liquid and instabilities after breaking it are very close to the experimental data. It is very likely that bridge breaking in the case of both coarser grids (grid 1 and 2) is caused by the numerical errors. It explains shorter time at which it happened than it was observed in the experiment.

In Figures 4 position of the drop leading edge in time is reported. Good agreement between measurement data and simulation results was obtained. It can be noticed that as the grid is refined results closer to the experimental curve were obtained.

4.2 Isothermal casting experiment

In the case of the isothermal experiment the work focused on highly accurate capturing of the characteristic geometrical features of the free surface. The experiment was carried out on the casting experimental facility under ambient conditions with analog liquid, see[10]. In Figure **??** the free surface shape of the analog liquid in the isothermal casting experiment after 10 seconds from its beginning is presented. The experimental result is compared with the numerical simulations obtained on the three successively refined grids. Qualitatively, results obtained with the numerical calculations are in good agreement with the experimental data. Features characteristic to the free surface flows, like hydraulic

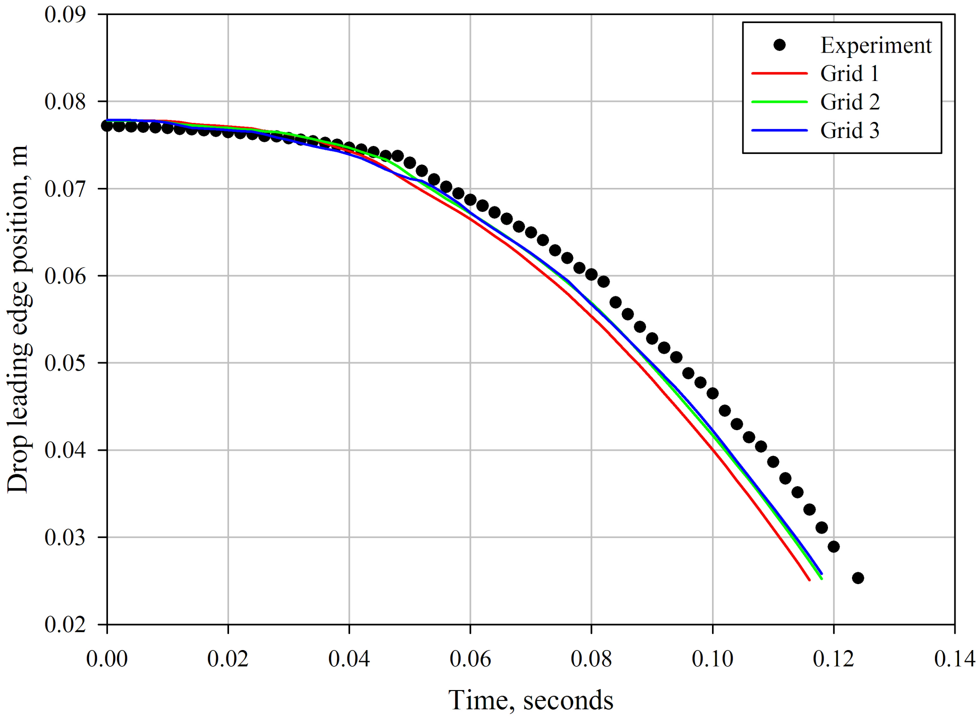

Figure 4: Position of the drop leading edge for the grid dependence study.

jump around the liquid jet impinging the top coil surface and the surface of the bottom arm of the core are captured very well.

Figure 6 demonstrate changes of the liquid stream width at the top of the transformer coil with time. Experimental data are compared with solutions obtained on three refined grids. It can be seen that the numerical solution converges as the grid is refined. Unfortunately, this converged solution differs significantly from the experimental observations. Discrepancy between numerical solutions and experimental data is increasing as the liquid width in lower part of the coil is considered. Moreover, as it can be considered as an acceptable for the initial stage of the experiment, it is slowly increasing with time. The liquid stream width was growing slowly during the whole experiment, while in the simulations it reached pseudo-steady state value quite quickly.

4.3 Non-isothermal casting experiment

The non-isothermal occupies the highest level of the validation hierarchical structure, *cf.* Figure 2. The experiment was carried out on the same experimental facility as the isothermal one. In the experiment two-component epoxy composition was utilized as a filling medium. Before the experiment about 17 litres of the resin components were mixed in a hermetic vessel connected to a vacuum pump. The composition was deaerated and

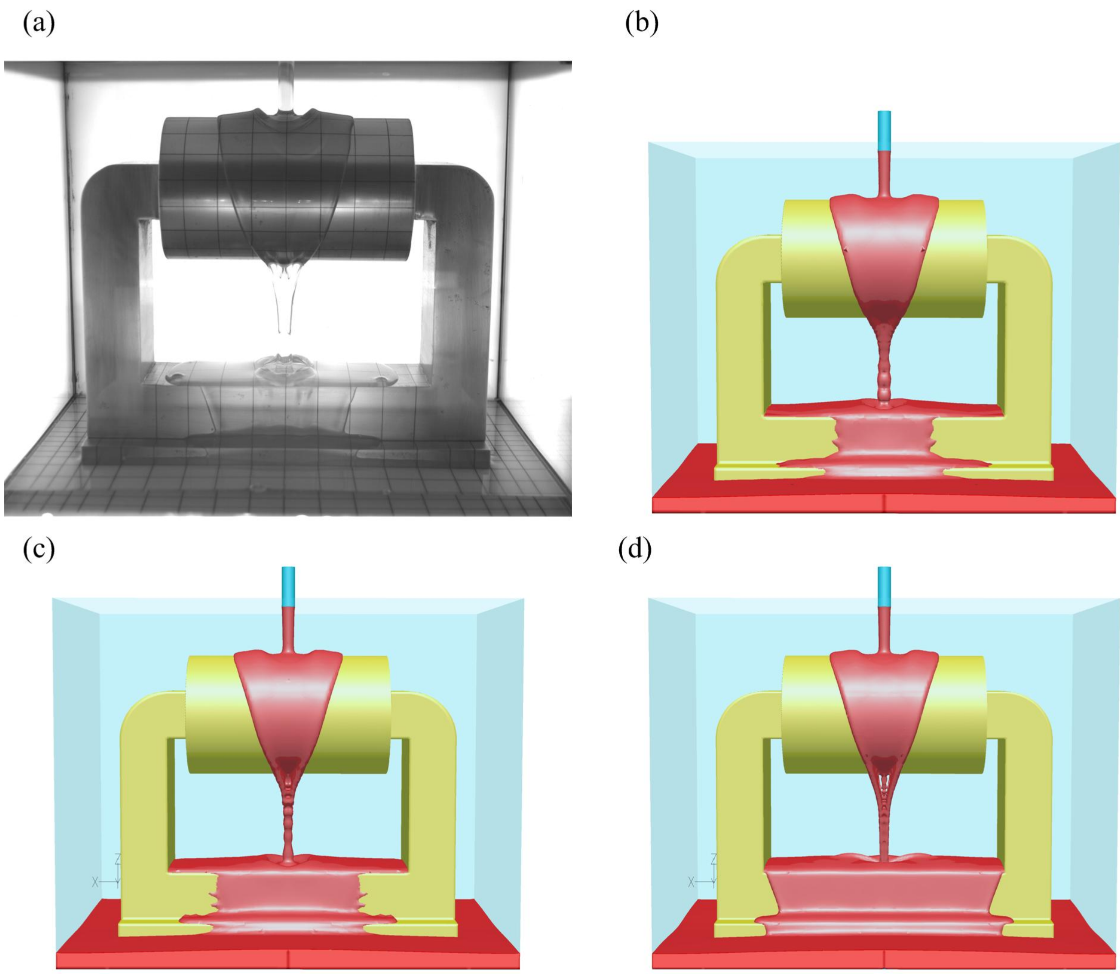

Figure 5: Free surface shape of the analog liquid at the time 10.0 seconds after the casting process start: (a) isothermal experiment, (b) grid 1, (c) grid 2, (d) grid 3.

placed in the feeding tank. Temperature of the epoxy composition during the experiment was close to the ambient temperature. It was monitored with a coated thermocouple placed near the inlet to the mould its average value was 293.45 K. The mould itself initially had the ambient temperature equal to 297.1 K. The internal elements of the transformer were heated up to 393 K before they were placed inside the mould. In this experiment the main efforts were carried out to measure the temperature at selected points inside the mould. Hence, a number of coated NiCr-NiAl (type K) thermoelements were placed inside the mould. All thermoelements were connected to the data logger with internal compensation for the cold junctions. The data logger was connected to the PC computer and thermocouples recordings were in suit stored on its hard drive[11].

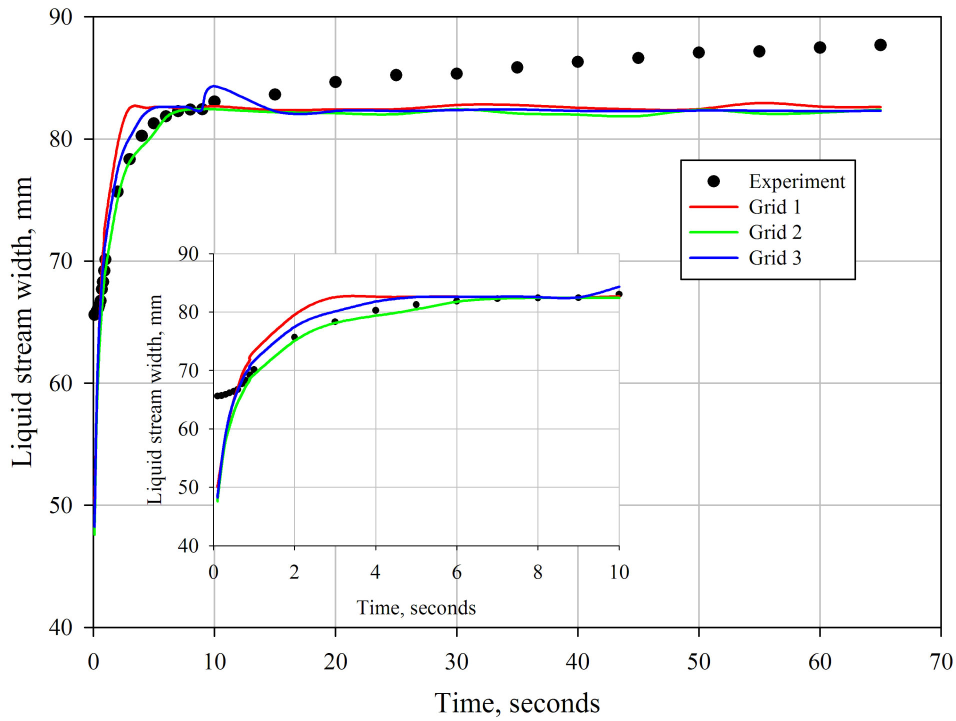

Figure 6: Variation of the liquid stream width at the top of the transformer coil with time for different numerical grids.

Figure 7 presents temperature recorded by the chosen thermoelement located in the middle of the mould height near its back wall. The measurement results are compared with the numerical calculations carried out on three subsequently refined grids. It can be noticed that the temperature peaks are reproduced by the simulations, however they are not so sharp as the ones observed in the experiment. Amplitudes of these peaks decrease for the thermocouples located higher, because the epoxy composition slowly cools down. However, this drop observed in experiment was considerably lower than the one obtained in simulations.

5 CONCLUSIONS

In this thesis an extensive Validation and Verification procedure of the numerical model of the free surface flow in casting applications was proposed. Casting processes encompass many techniques that may significantly differ from each other. However, for the purpose of this study one specific process, namely gravitational epoxy casting of an electrical transformer was chosen. For this process a hierarchical Verification and Validation procedure was designed and carried out. Even though one specific process was considered, the proposed V&V procedure has universal character and can be successfully used for validation of other free surface codes or models.

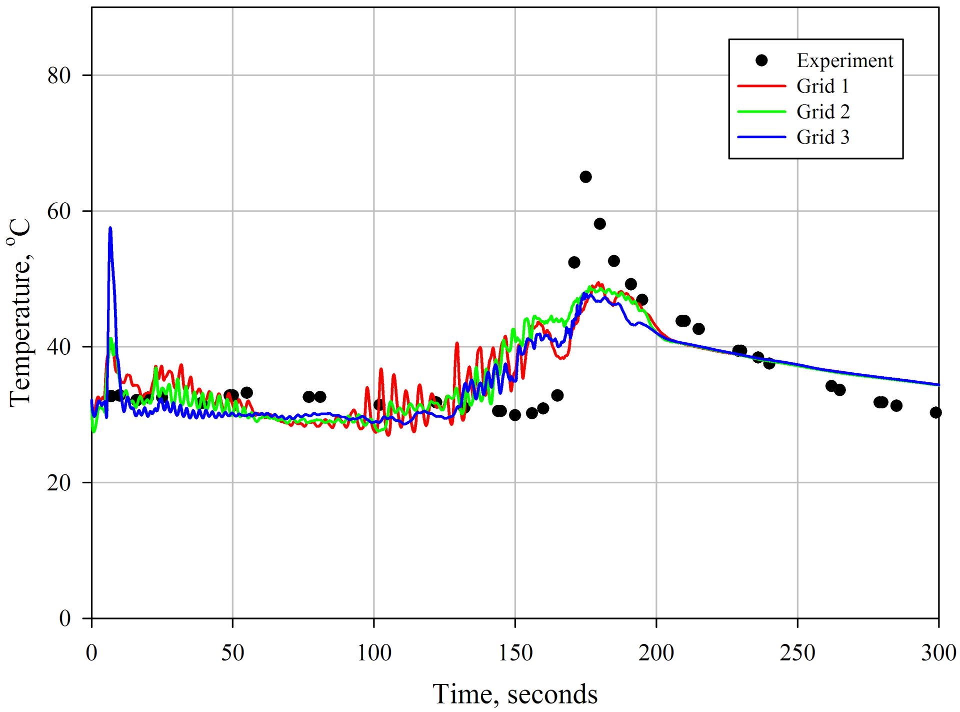

Figure 7: Comparison of the thermocouple T3 recording with solutions obtained on different grids.

Aside from the experimental work the mathematical and numerical models of each experimental setup were developed. The Volume of Fluid (VOF) method together with single-fluid formulation were used as a framework for these models. The wide variety of the measurement data gathered in the performed experiments were successfully utilized to asses correctness of developed numerical models. In the conducted experiments local quantities (*i.e.* temperature in selected points) as well as integral quantities (*i.e.* liquid stream width) or global quantities (*i.e.* position of the drop leading edge) were utilized for validation. Hence, from that point of view worked up procedure can be thought as an exhaustive and efficient source of data for verification and validation of the mathematical models of the free surface flows.

Qualitatively simulation results showed very good agreement with the results of the experiments. However, closer quantitative investigation of the obtained results revealed some deficiencies of the considered numerical model. In some cases the wrong movements and shape of the common line were observed. Because one temperature field common for both phases was assumed it was smeared across the free surface. It was inferred from the fact that temperature peaks obtained in the non-isothermal casting experiment were not so sharp in the simulation results. Although these inaccuracies are observed in the obtained solutions, the presented numerical and mathematical models can be efficiently applied for simulating casting processes.

REFERENCES

[1] I. Babuska and J. T. Oden, Verification and validation in computational engineering and science: basic concepts, *Comput. Meth. Appl. Mech. Eng.*, **193**, 4057–4066 (2004).

[2] J. Banaszek, Credibility analysis of computer simulation of complex heat transfer problems, In proceedings of the *Numerical Heat Transfer 2005*, Eurotherm Seminar 82, A.J. Nowak, R.A. Biaecki and G. Wcel Eds., Gliwice-Cracow, Poland, 141–162 (2005)

[3] W. L. Oberkampf and T. G. Turcano, Verification and validation in computational fluid dynamics, *Progr. Aero. Sci.*, **38**, 209–272 (2002).

[4] C. W. Hirt and B. D. Nichols, Volume of fluid (VOF) method for the dynamics of free boundaries, *J. Comput. Phys.*, **39**, 201–225 (1981).

[5] N. I. Kolev, Multiphase flow dynamics, 2nd Edition, *Springer-Verlag*, **Vol. I** (2005).

[6] J.U. Brackbill, D.B. Kothe and C. Zemach, A Continuum method for modelling surface tension, *J. Comput. Phys.*, **100**, 335–354 (1992).

[7] J. M. Barton, The application of differential scanning calorimetry (DSC) to the study of epoxy resin curing reactions, *Adv. Polymer Sci.*, **72**, 111–154 (1985).

[8] M.R. Kamal and S. Sourour, Kinetics and thermal characterization of thermoset cure, *Polymer Eng. Sci.*, **13**, 59–64 (1973).

[9] J.M. Castro and C.W. Macosko, Kinetics and rheology of typical polyurethane reaction injection molding system, In proceedings of the *Annual Technical Conference*, Society of Plastics Engineering, **26**, 434–438 (1980)

[10] Z. P. Buliski, Numerical Modelling and Credibility Analysis of Free Surface Flows in Selected Industrial Processes, PhD Thesis, *Silesian University of Technology, Gliwice, Poland*, (2010).

[11] Z. P. Buliski and A. J. Nowak, Numerical analysis and experimental validation of the free surface flow and heat trabsfer in electrical transformers moulding processes, *Int. J. Numer. Meth. Heat. Fluid Flow.*, **18**, 356–377 (2008).