STABILITY AND CONVERGENCE ANALYSIS OF AN X-FEM FORMULATION FOR INCOMPRESSIBLE FLOW

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ABSTRACT

Although the eXtended Finite Element Method (X-FEM) was initially developed for solving fracture problems, it is nowadays widely used in many other applications, such as material inclusions, phase change or contact problems. In particular, for problems involving several materials, discontinuities of the solution across interfaces can be represented using X-FEM, by simply enriching the interpolation.

In this work the stability and convergence of X-FEM for incompressible flow is studied. A first analysis on X-FEM for these problems can be found in [1]. The analysis is extended here considering different types of elements. Convergence and stability are studied using numerical tests for different enrichment strategies, including discontinuous enrichment for pressure field.

The effect of the interface representation on the convergence rate is also analyzed. An usual practice is considering a linear representation of the interface in each element. As noted in [1], this poor representation of interfaces affects the convergence rate for quadratic or higher order elements. In this work, two strategies for a proper representation of interfaces, keeping optimal converge rates, are considered (see figure below). First, a uniform linear submesh is used to represent the interface as a piecewise linear function in each element [2]. Nevertheless, here it is shown that, in order to ensure optimal convergence, a refinement of the linear submesh as the element's size decreases is needed. Submeshing can be avoided if a higher order representation of the interface is used. Optimal convergence rates are recovered when using an approximation of the interface of the same order than the one used for interpolating the solution.

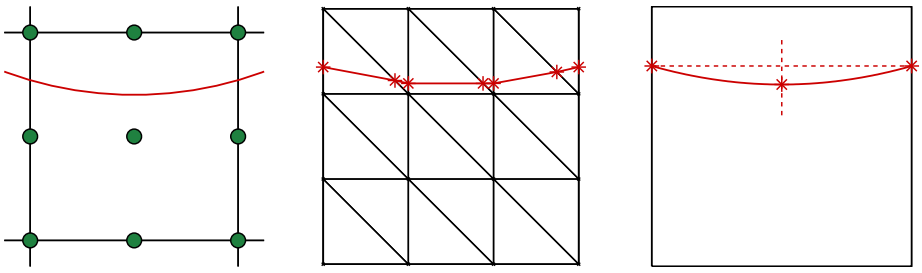

Figure 1: (a) Element cut by the interface, with piecewise linear (b) or quadratic (c) representation.

REFERENCES

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