V European Conference on Computational Fluid Dynamics
ECCOMAS CFD 2010
J. C. F. Pereira and A. Sequeira (Eds)
Lisbon, Portugal, 14–17 June 2010

TEMPERATURE INLET-WALL BOUNDARY CONDITION IDENTIFICATION OF TRANSIENT INVERSE CONVECTIVE HEAT TRANSFER PROBLEMS WITHIN CHANNELS/PIPES LAMINAR FLOW

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Key words: Inverse Convective Heat Transfer Problem, Parameter Estimation, Stream-Vorticity Formulation, Finite Difference Method, Boundary Condition Identification, Point Heat Source Estimation.

Abstract. *This paper deals with the method for determining simultaneous inlet-wall boundary condition in transient convective heat transfer in a channel/pipe which is based on the solution of the inverse problem referred as the Levenberg-Marquardt method. This method is an iterative regularization algorithm for parameter estimation. The direct problem is formulized based on the stream-vorticity version of the incompressible Navier-Stokes equations together with energy equation. The solution of this inverse problem requires a finite set of temperature measurements taken by noisy/non-noisy sensors located near the boundaries of the domain. Some cases are considered, in which simultaneous wall-inlet boundary condition identification and point heat source strength estimation within a channel/pipe are performed. The results of the present study are compared to those of the exact boundary conditions, and good agreement is achieved.*

1 INTRODUCTION

Today, inverse problems have many applications in different branches of science and engineering. In fact, the use of inverse analysis techniques represents a new research paradigm in numerous engineering applications such as transient heat convection processes. Inverse problems of heat transfer utilize temperature measurements to estimate unknown quantities required in the manner in which physical problems are formulated in mathematical representation modeling. In Inverse Convective Heat Transfer Problems (ICHTPs), temporal/spatial parameters such as inlet and wall boundary conditions have been found by known parameters of temperature field.

ICHTPs in the sense of Hadamard definition [1] are ill-posed and too sensitive to errors in the measured data. According to the ill-posed nature of inverse problems, common numerical methods for solving direct problems are not applicable in solving inverse problems and thus, specific numerical techniques are required to establish stability conditions for solving inverse problems.

Currently, there are techniques that are capable of resolving the inherent ill-posedness in inverse problems by improving the least squares method via adding minimization and regularization techniques. These techniques, based on the solution of whole domain, are divided into two categories: sequential and whole domain methods. In the whole domain method, estimation of unknown quantities in all of domain (spatial or time) is done, but in the sequential method, the whole domain is divided into several subdomains and then unknown quantities are estimated sequentially in each subdomain. To solve inverse problems, two regularization methods, direct regularization methods such as Tikhonov method and iterative regularization methods such as Levenberg-Marquardt method, can be used.

A quite large number of techniques have been proposed for the solution of inverse problems such as iterative regularization techniques. As a result, there are few considerable numbers of the published works concerned with the ICHTPs due to the mathematical complexities which are limited to current twenty years. Most of these ICHTPs works considered problems involving hydrodynamically fully developed laminar flows and also estimation of the unknown quantities with constant values.

Moutsouglou [2] studied the estimation of the heat flux boundary condition of the upper wall in a steady-state channel flow by using the temperature values of the lower wall. The inverse method used in his research was Beck’s function estimation method. In the works published by Liu and ? zisik [3], Raghunath [4] and Bokar and ? zisik [5], the estimation of the unknown inflow temperature distribution and in Huang and ? zisik [6] and Park and Lee [7] papers, estimating the unknown wall heat flux distributions with constant and temperature-dependent thermal conductivity coefficients, respectively, in steady-state flows using conjugate gradient method were performed. Prud’homme and Nguyen [8], and Machado and Orlande [9, 10] estimated the unknown time/spatial-varying wall heat flux distribution within a channel using the conjugate gradient method for Newtonian and non-Newtonian fluid. All the above studies were done for hydrodynamically fully developed laminar flows.

In other works, simultaneous estimation of two unknown quantities was considered. Hsu et al. [12] estimated simultaneously spatial distribution of inflow and wall heat flux boundary conditions using inverse analysis in a steady state laminar pipe flow. Colaço and Orlande [13] estimated simultaneously two space-time varying heat flux boundary conditions in forced convective heat transfer using function estimation version of the adjoint conjugate gradient method. They studied laminar flow within a two-dimensional channel with parallel surfaces and used a finite volume method for numerical simulation of the flow field. The flow considered in these works was developed hydrodynamicly.

Moreover, they solved the same inverse forced heat convection problem in a two-dimensional channel with non-parallel surfaces and in a concentric tube annulus in [14, 15]. They also used the finite volume method for numerical simulation of the flow field in a curvilinear coordinates system.

In this article, to demonstrate the capability of inverse analysis methods in solving practical problems, transient inverse convective heat transfer in a two-dimensional/ axisymmetric domain is solved to determine the unknown quantities such as thermal inlet and wall boundary conditions. The governing equations of the direct problem are the Stream-Vorticity Formulation (SVF) of the transient incompressible Navier-Stokes equations together with transient energy equation. The SVF is used to simulate transient convective heat transfer in a channel/pipe. The direct problem and other partial differential equations in the Cartesian coordinates system are solved numerically using the implicit factorized finite difference method in delta form proposed by Beam-Warming [16]. Central and upwind differencing are used to descretize the diffusion and convection fluxes. In addition, a central finite difference method together with a successive line under relaxation method has been used to solve the stream function equation in an iterative manner. For grid generation, a simple algebraic method is used to have sufficient grids in the boundary layer region. In order to solve governing equations and estimate unknown quantities accurately, 50×35 grid points and time step, $\Delta t = 0.1$ are used and the duration of the simulation time is supposed to be 120 (a nondimensional value).

The inverse approach is constructed by using an iterative inverse analysis algorithm based on the Levenberg-Marquardt parameter estimation method [17]. The temperature histories are delivered by noisy ($\sigma = 0.05$) or non-noisy ($\sigma = 0.0$) simulated temperature measurements assumed as sensors located on the outlet of the channel/pipe. Three cases are considered, in which the time/space varying inflow/wall boundary condition, time-varying point heat source and simultaneous estimation of two unknown functions, inflow-wall boundary condition identifications are performed in a channel/pipe laminar flow to assess the performance of inverse analysis method to solve inverse convective heat transfer problems. The results of the present study are compared to those of exact heat source, and boundary and inflow conditions, and good agreement is achieved.

2 INVERSE ANALYSIS

The algorithm of inverse analysis is composed of two sections. In the first section, through considering the physical model of the problem as direct problem, a numerical method is employed to solve this direct problem. In direct problem, geometry and computational domain, governing equations, and initial and boundary conditions along with other parameters are assumed to be known and the objective of the section is to achieve the distribution of the physical variables such as temperature, $T(x, y, t)$, within the domain. In the second section, by defining an objective function, applying optimizing and reqularization methods for error minimization and eliminating the fluctuations presented in the measured temperature, an estimation of the unknown parameters and functions is acquired.

Inverse analysis is a technique which provides efficient means for implementing the temperature values measured by sensors in estimation of mathematical models, parameters, functions, and initial and boundary conditions, more effectively. In contrast to a direct problem, an inverse problem includes some unknown quantities which are estimated regarding to temperatures measured in various time and sensor positions. In an inverse problem error is calculated according to following equation [17]:

$$\mathbf{e} = \mathbf{T}^m - \mathbf{T}^c(\mathbf{P}) \tag{1}$$

In order to minimize the error in equation (1), there are various methods. One of the most practical methods is the least squares method [17]:

$$S(\mathbf{P}) = \left[\mathbf{T}^m - \mathbf{T}^c(\mathbf{P})\right]^T \left[\mathbf{T}^m - \mathbf{T}^c(\mathbf{P})\right] \tag{2}$$

In this method to minimize the error, gradient of the equation (2) with respect to unknown quantities must be set equal to zero:

$$\frac{\partial S(\mathbf{P})}{\partial \mathbf{P}} = \frac{\partial S(\mathbf{P})}{\partial P_1} = \frac{\partial S(\mathbf{P})}{\partial P_2} = \ldots = \frac{\partial S(\mathbf{P})}{\partial P_N} = 0 \tag{3}$$

3 DIRECT PROBLEM

Transient incompressible two-dimensional/axisymmetric Navier-Stokes equations are employed to model the forced convective heat transfer. This set of equations is expressed in nondimensionalized form as follow:

$$\frac{\partial \tilde{\mathbf{U}}}{\partial t} + \frac{\partial \tilde{\mathbf{F}}}{\partial x} + \frac{\partial \tilde{\mathbf{G}}}{\partial y} + j\frac{\tilde{\mathbf{H}}}{y} + \mathbf{Sc} = \frac{1}{\text{Re}}\left[\frac{\partial \tilde{\mathbf{R}}}{\partial x} + \frac{\partial \tilde{\mathbf{S}}}{\partial y} + j\frac{\tilde{\mathbf{W}}}{y}\right] \tag{4}$$

where $\tilde{\mathbf{U}}$ vector is:

$$\tilde{\mathbf{U}}^T = \left[0, u, v, T\right] \tag{5}$$

and j is defined as follows:

$$j = \begin{cases} 0 & \text{two dimensional flow} \\ 1 & \text{axisymmetric flow} \end{cases} \tag{6}$$

The following non-dimensional parameters are used to non-dimensionalize the flow variables:

$$t = \frac{t^*}{L_{ref} / u_{ref}}, \qquad x_i = \frac{x_i^*}{L_{ref}}, \qquad u_i = \frac{u_i^*}{u_{ref}}, \qquad p = \frac{p^*}{\rho U_{ref}^2} \qquad T = \frac{T^*}{T_{ref}} \tag{7}$$

Considering the above non-dimensional parameters, non-dimensional numbers are:

$$\text{Re} = \frac{\rho u_{ref} L_{ref}}{\mu}, \qquad \text{Pr} = \frac{c_p \mu}{k} \tag{8}$$

Viscous and inviscid flux vectors in equation (4) are presented in reference [18]. Defining vorticity and stream functions and inserting them in momentum equations, the pressure terms which make the numerical solution of momentum equations difficult are eliminated. Therefore, transient vorticity-function transport equations, along with energy equation for an incompressible two-dimensional/axisymmetric flow in Cartesian coordinates system in their non-dimensional forms are as follows:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} - \frac{j}{y}\frac{\partial \psi}{\partial y} = -\left[1 + j(y-1)\right]\zeta \tag{9}$$

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} + j\mathbf{H} + \mathbf{Sc} = \frac{1}{\text{Re}}\left[\frac{\partial \mathbf{R}}{\partial x} + \frac{\partial \mathbf{S}}{\partial y} + j\frac{\mathbf{W}}{y}\right] \tag{10}$$

where $\mathbf{U}$ vector is:

$$\mathbf{U}^T = \left[\zeta, T\right] \tag{11}$$

Viscid and inviscid vectors in equation (10) are.

$$\mathbf{U} = \begin{pmatrix} \zeta \\ T \end{pmatrix}, \quad \mathbf{F} = \begin{pmatrix} u\zeta \\ uT \end{pmatrix}, \quad \mathbf{G} = \begin{pmatrix} v\zeta \\ vT \end{pmatrix}, \quad \mathbf{H} = \begin{pmatrix} 0 \\ vT \end{pmatrix}, \quad \mathbf{Sc} = \begin{pmatrix} 0 \\ G(x,y,t) \end{pmatrix}$$

$$\mathbf{R} = \begin{pmatrix} \zeta_x \\ \left(\frac{T_x}{\text{Pr}} + u\tau_{xx} + v\tau_{xy}\right) \end{pmatrix}, \quad \mathbf{S} = \begin{pmatrix} \zeta_y \\ \left(\frac{T_y}{\text{Pr}} + u\tau_{xy} + v\tau_{yy}\right) \end{pmatrix}, \quad \mathbf{W} = \begin{pmatrix} \left(\zeta_y - \zeta / y\right) \\ \left(\frac{T_y}{\text{Pr}} + u\tau_{xy} + v\tau_{yy}\right) \end{pmatrix} \tag{12}$$

where function $G(x,y,t)$ is a point heat source and is defined as follows

$$G(x,y,t) = g(t)\,\delta(x - x_s, y - y_s) \tag{13}$$

Thus, through implementing stream and vorticity functions, the number of dependent variables in the governing equations is reduced by one, although the order of derivatives is increased by one. Equation (9) is solved using a finite difference method and the non iterative implicit approximate factorized algorithm in the delta (Δ) form, presented by “Beam and Warming” [16].

3.1 Initial and boundary conditions for numerical solution of direct problem

Regarding wall and entrance boundary conditions, initial values for flow variables including distributions of stream and vorticity functions and temperature are considered for the solution of governing equations.

In this study, boundary conditions, inflow, outflow and wall boundary conditions, are applied explicitly. In viscous flow, boundary condition applied on body surface is no-slip condition, accordingly velocity magnitudes *u* and *v* on the surface are zero, which dictates the following stream function boundary condition [19]:

$$\psi = \text{Constant} \tag{14}$$

Based on Thom’s method [20], vorticity function boundary condition applied on a wall where unit vector is in opposite direction to y axis direction and has no movement is applied as follows:

$$\zeta_{M,j} = \frac{-1}{\left[1+\kappa(y-1)\right]}\left.\frac{\partial^2\psi}{\partial y^2}\right|_{M,j} = \frac{-1}{\left[1+\kappa(y-1)\right]}\frac{2\left(\psi_{M-1,j}-\psi_{M,j}\right)}{\left(\Delta y\right)^2} \tag{15}$$

Two cases are possible for temperature boundary condition:

- Body surface temperature is known.
- The surface is adiabatic. In this case, temperature values are obtained from the following equation:

$$\frac{\partial T}{\partial \text{n}} = 0 \tag{16}$$

For inflow boundary condition, variables such as velocity vector and temperature must be known. Through using them, other variables such as stream and vorticity functions can be computed. Assuming a constant velocity, *u*, for inflow, stream function boundary condition is calculated by applying the following relation:

$$\psi = \int_0^y u_o\,dy = u_o\,y \tag{17}$$

Using the following relation, vorticity function at entrance boundary is found:

$$\zeta_{1,j} = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \left(-\frac{\partial^2\psi}{\partial x^2} + \frac{\partial u}{\partial y}\right)_{1,j} = \frac{-3\psi_{1,j}+4\psi_{2,j}-\psi_{3,j}}{\left(\Delta x\right)^2} - \frac{u_{1,j+1}-u_{1,j-1}}{2\Delta y} \tag{18}$$

In outflow boundary, assuming a fully developed regime, one can use linear extrapolation to obtain stream function and temperature variable. Vorticity function is also reachable through discritization of the vorticity equation at this boundary.

4 MESH GENERATION

Considering that governing equations of the problem are expressed in a finite difference form, a structured mesh generated by a simple algebraic method [18] is used to solve numerically partial differential equations and then, to compute the velocity field and temperature of the fluid.

Considering (x_l, y_l) and (x_u, y_u) as lower and upper boundary coordinates, geometrical domain of the flow field can be descritized using the following equations:

$$x = x_l + (x_u - x_l)\bar{\alpha} \qquad\qquad y = y_l + (y_u - y_l)\bar{\alpha} \tag{19}$$

where $\bar{a}$ is defined as:

$$\bar{\alpha} = 1 + \bar{\beta}\left[\frac{1-\alpha^{(1-j/j_{\max})}}{1+\alpha^{(1-j/j_{\max})}}\right] \qquad\qquad \alpha = \frac{\bar{\beta}+1}{\bar{\beta}-1} \tag{20}$$

and $\bar{b}$ varies between 1.001 to 1.05.

5 INVERSE PROBLEM

Levenberg-Marquadrant parameter-estimation method is utilized to solve the inverse forced convective problems. Since this study investigates forced convective heat transfer cases, temperature field does not influence the governing equations, vorticity and stream function equations. In fact vorticity and stream functions are not affected by temperature field, thus they are not related to the unknown parameter in the entrance or wall thermal boundary conditions. The only parameter which is sensitive to unknown parameters is temperature. As a result, the only equation investigated in the inverse analysis is energy equation.

5.1 Levenberg-Marquardt method

This technique is an iterative algorithm designed to solve least squares parameter-estimation problems. The algorithm is a modified version of the least squares method suitable for determining linear and nonlinear problems. At the start point, close to initial guess, Levenberg-Marquardt method acts like steepest decent method. However, as it approaches the solution, it acts similar to Newton-Gauss method. In this algorithm to solve an inverse problem, one should assume the function of the unknown inflow or wall boundary condition in the following form [17]:

$$g(t) = \sum_{n=1}^{N} P_n C_n(t), \qquad\qquad F(s,t) = \sum_{n=1}^{N} P_n D_n(x,t) \tag{21}$$

where $C_n(s,t)$ and $D_n(s,t)$ can be known functions like polynomials or B-splines, etc. By the definition, an inverse heat conduction problem with $F(\mathrm{s},t)$ as the unknown function is converted into an inverse heat conduction problem with unknown parameter $\mathbf{P}$. Based on the minimization of the least squares norm, equation (2), the solution is achieved by the following equation:

$$\mathbf{P}^{k+1} = \mathbf{P}^k + \left[\left(\mathbf{J}^k\right)^T \mathbf{J}^k + \Upsilon^k \Omega^k\right]^{-1} \left(\mathbf{J}^k\right)^T \left[\mathbf{T}^m - \mathbf{T}^c(\mathbf{P}^k)\right] \tag{22}$$

where matrix Ω^k is defined as:

$$\Omega^k = \mathrm{Diag}(\mathbf{P}) = \mathrm{Diag}\left[\left(\mathbf{J}^k\right)^T \mathbf{J}^k\right] \tag{23}$$

As mentioned above during iterations to compute the unknown parameter, matrix $\left(\mathbf{J}^k\right)^T \mathbf{J}^k$ should not be singular i.e. determinant should not equal zero. Otherwise, iteration procedure can not continue.

Damping parameter $?^k$ takes positive value. The objective of defining Υ^k and Ω^k is damping the fluctuations, avoiding the instabilities caused by ill-posed nature of inverse problems and assuming small values for $\left(\mathbf{J}^k\right)^T \mathbf{J}^k + \Upsilon^k \Omega^k$ when the value of $\left(\mathbf{J}^k\right)^T \mathbf{J}^k$ becomes considerably small [17].

To apply equation (22), one needs to have an equation in order to calculate sensitivity coefficient matrix $\mathbf{J}$ as well as initial and boundary conditions by employing the governing equations of direct problem and also the corresponding initial and boundary conditions. Differentiating energy equation (10) with respect to unknown parameters, the following equation is achieved as sensitivity equation:

$$\frac{\partial \mathbf{U}_J}{\partial t}+\frac{\partial \mathbf{F}_J}{\partial x}+\frac{\partial \mathbf{G}_J}{\partial y}+j\frac{\mathbf{H}_J}{y}+\mathbf{Sc}_J=\frac{1}{\mathrm{Re}}\left[\frac{\partial \mathbf{R}_J}{\partial x}+\frac{\partial \mathbf{S}_J}{\partial y}+j\frac{\mathbf{W}_J}{y}\right] \tag{24}$$

In the above equation, each of the vectors has one component. Dependent parameter of equation (24) is $\mathbf{J}$ parameter. The only nonzero boundary condition for the above equation is $F(s,t)$, which is inflow or wall boundary condition as imposed on the corresponding boundary:

$$\mathbf{J}(x,y,t)=\frac{\partial F(s,t)}{\partial \mathbf{P}} \tag{25}$$

5.2 Iterative algorithm convergence criterion

To implement equation (22) and obtaining desirable results, one needs a criterion to stop the iteration procedure of Levenberg-Marquardt method. This criterion is a necessary condition to avoid the amplification of the measurement errors on the computed solution, to obtain a result from iteration procedure (to converge inverse solution). If a criterion is employed to stop iteration procedure, the inverse problem together with the iteration algorithm as regularization method will become well-posed, hence the following criterion is applied:

$$S(\mathbf{P}^{k+1})<\varepsilon \tag{26}$$

where ε is the tolerance value to avoid unstable solutions and to converge the iteration:

$$\varepsilon = M\,I\,\sigma^2 \tag{27}$$

6 INVERSE ANALYSIS RESULTS

In order to demonstrate the capability of inverse analysis method in estimating the unknown inflow and wall boundary conditions in heat transfer problems, the results of the transient inverse forced convective heat transfer in a duct/pipe are investigated. This study is conducted using the Levenberg-Marquardt method for unknown parameter estimation. Computational field is descritized to 50×35 grids. For numerical solution of direct problem and sensitivity equation in the inverse problem, mesh is generated by a simple algebraic technique. Figure 1 depicts the computational field and mesh points, used in numerical solution of partial differential equation for duct/pipe geometry.

As stated before, inverse heat transfer problems in contrast to direct ones include one or more unknowns, estimated by temperatures measured in sensor locations. A normal distribution of zero mean values of fluctuating errors is added to the exact computational temperature field from $t=0$ to $t=120$. Then, these computed values are used as the simulated temperature measurements in the sensor locations:

$$T^m(x,y,t)=T^c(x,y,t)+\omega\times\sigma\times T^c_{\max} \tag{28}$$

where using the normal distribution, ω is a random number between (-2.576, 2.576) with a confidence of 0.99. Sensors are assumed to be located adjacent to geometrical boundaries. In this study, the computed temperature values without error ($\sigma=0$) as well as with $\sigma=0.05$ have been used for estimation of the unknown parameters. Time step, $\Delta t=0.1$, is employed to solve partial differential equations. Root Mean Square (RMS) error of estimated functions, $g(t)$ and $F(x,t)$, are also calculated using:

$$e_{RMS}=\sqrt{\frac{1}{I}\sum_{i=1}^{I}\left[g_{est}(t_i)-g_{exact}(t_i)\right]^2} \qquad e_{\mathrm{RMS}}=\sqrt{\frac{1}{I\times I_b}\sum_{j=1}^{I_b}\sum_{i=1}^{I}\left[F_{\mathrm{est}}(x_j,t_i)-F_{\mathrm{exact}}(x_j,t_i)\right]^2} \tag{29}$$

Before proceeding to explain the examples of inverse forced convective heat transfer in a duct/pipe, it is necessary to verify the data extracted from numerical simulation of a flow in a duct/pipe with initial and boundary conditions (direct problem). Hence, an example has been conducted for numerical simulation of a flow in a duct with $\mathrm{Re} = 500$ and $\mathrm{Pr} = 1$ (figure 2) to verify and validate numerical simulation of the direct problem whose results are presented in figures 3 and 4. In this example, maximum velocity at the centerline has been 1.47 which differs from the empirical value by 2 percents. This solution confirms the appropriate accuracy of the numerical solution of the direct problem for application in numerical solution of inverse convection problems.

In this study, estimation of point heat source strength and inflow/wall boundary condition, and simultaneous estimation of two unknown functions, inflow-wall boundary condition based on three cases is conducted to test the performance of the iterative inverse analysis. The strength of the heat source is a linear function of time, but the boundary conditions are a linear/quadratic/triangular function of time or space. The initial guesses of the parameters and functions are taken equal to zero. It should be mentioned that, in inverse and direct convective heat transfer problems, temperature at exit boundary is calculated using approximate condition $\partial T / \partial x = 0$.

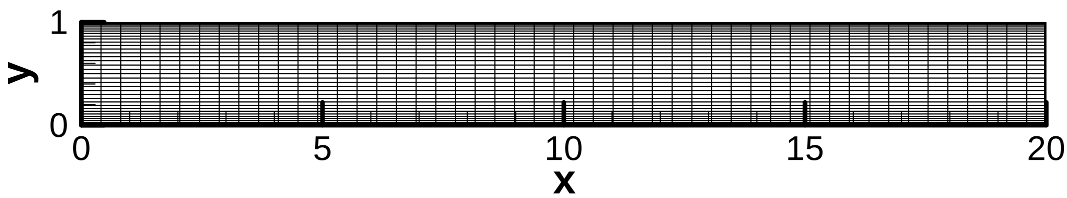

Figure 1: Grid 50×35 to solve governing equations of the problem in computational domain.

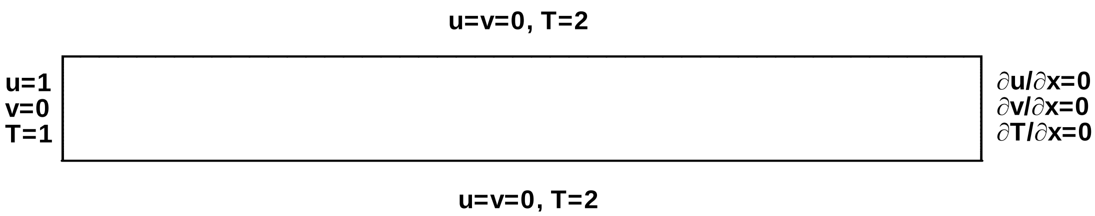

Figure 2: Boundary conditions to verify solution f the governing equations of the direct problem.

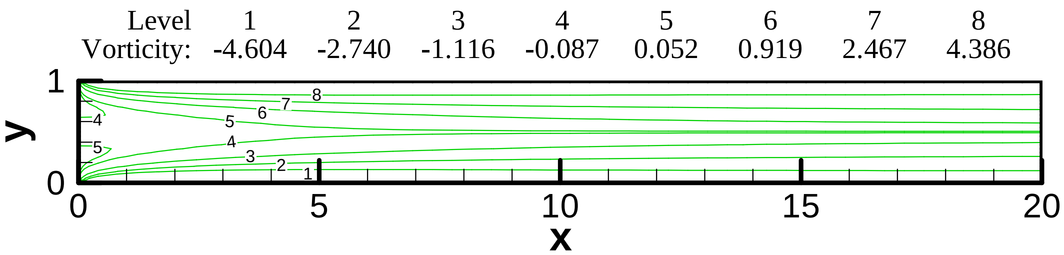

Figure 3: Contours of vorticity function, example of the verification.

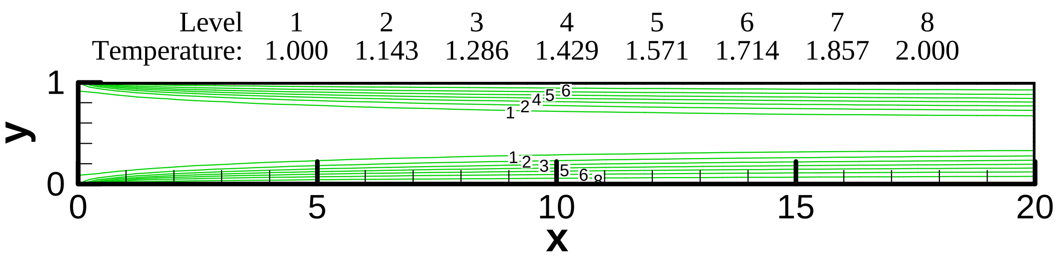

Figure 4: Contours of temperature, example of the verification.

6.1 Case study 1: estimation of point heat source

In the first case, inverse convective heat transfer with an unknown point heat source and the boundary conditions depicted in figure 2 is investigated. It is assumed that the strength of the heat source positioned at x = 5 and y = 0.5 is a linear time varying function which the aim of the inverse analysis is to find its two coefficients:

$$g(t) = P_1 + P_2\, t = 9 + 3t \tag{30}$$

The measured values of a sensor is located at the interface of lower and exit boundaries at $x = 19.95$ and $y = 0.05$. Figure 3 shows temperature contours of the estimated flow field. The sensitivity coefficients of the parameters and the estimation of the heat source strength are shown in figures 4 and 5, respectively, and the results are also summarized in table 1.

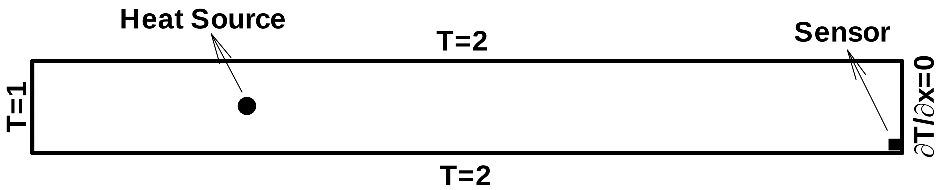

Figure 2: The boundary conditions and heat source position, case 1.

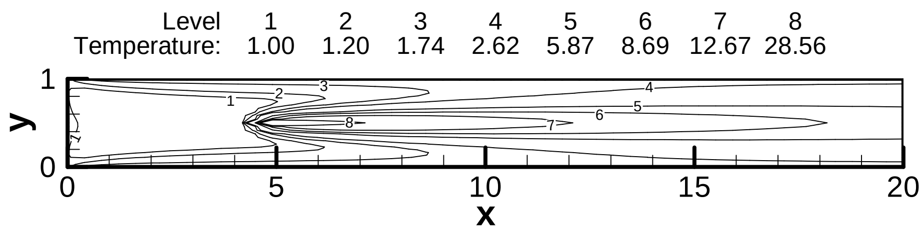

Figure 3: The temperature contours, case 1, $t = 120$ and. $\sigma = 0.05$

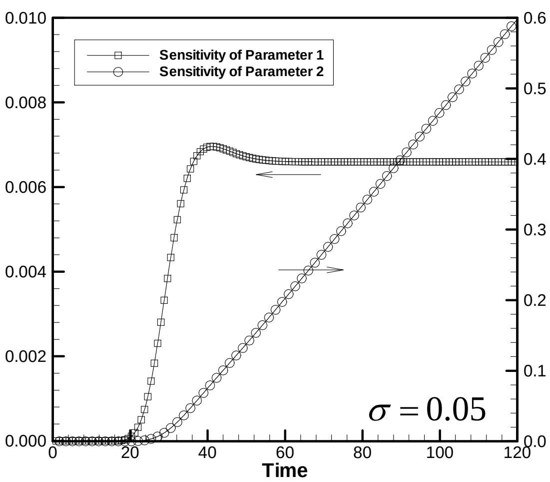

Figure 4: Non-dimensional sensitivity coefficients of the sensor versus the parameters.

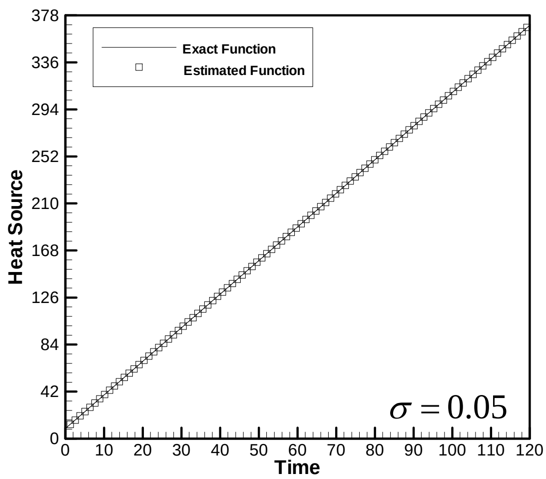

Figure 5: Variation of the estimated strength of the point heat source versus to time, case 1.

$\sigma = 0$				$\sigma = 0.05$			
P_1	P_2	RMS Error	No. of Iteration	P_1	P_2	RMS Error	No. of Iteration
9.000	3.000	0.000	3	8.922	3.002	0.034	5

Table 1: Estimation of the point heat source strength.

6.2 Case study 2: estimation of inflow/wall boundary condition

In the second case, inverse convective heat transfer with an unknown inflow/wall boundary condition is studied. It is assumed that the fitted functions for inflow and wall boundary conditions are quadratic and triangular space varying functions, respectively. The coefficients of these functions are estimated by the measured values of a sensor located at the interface of lower and exit boundaries at $x = 19.95$ and $y = 0.05$. Estimations of these two functions are illustrated in two following examples and their results are shown in table 2.

In example 1 the objective is to find the second order polynomial which represents estimation of inflow temperature as a function of location:

$$F(y) = P_1 + P_2\, y + P_2\, y^2 = 2 + 6y - 6y^2 \qquad (31)$$

The other boundary conditions are depicted in figure 6. Figure 7 illustrates estimated temperature field at *t*=120 with $\sigma = 0.05$. Figure 8 shows the estimated inflow temperature function for $\sigma = 0.05$ and the values of the exact inflow temperatures.

In example 2 the objective is to find the triangular function which represents estimation of wall temperature as a function of location:

$$F(y) = \begin{cases} P_1 \\ P_1 + P_2\,(x-5) \\ P_1 + 2.5P_2 - P_3\,(x-7.5) \\ P_4 \end{cases} = \begin{cases} 3 & \text{for} \quad x < 5 \\ 3 + 2(x-5) & \text{for} \quad 5 \le x < 7.5 \\ 8 - 2.4(x-7.5) & \text{for} \quad 7.5 \le x < 10 \\ 2 & \text{for} \quad x \ge 10 \end{cases} \qquad (32)$$

The other boundary conditions are shown in figure 9. Figure 10 demonstrates estimated temperature field at *t*=120 with $\sigma = 0.05$. Figure 11 shows comparison of the variations of the estimated wall temperature for $\sigma = 0.05$ with those of exact values. These results reveal good agreements of inverse analyses.

Function	$\sigma = 0$						$\sigma = 0.05$					
	P_1	P_2	P_3	P_4	RMS Error	No. of Iteration	P_1	P_2	P_3	P_4	RMS Error	No. of Iteration
Inflow Boundary Condition	2.000	6.000	−6.000	-	0.000	3	1.989	6.119	−6.185	-	0.032	4
Wall Boundary Condition	3.001	1.996	2.392	2.000	0.002	6	3.003	1.997	2.402	2.000	0.003	4

Table 2: Estimation of the inflow and wall boundary conditions, case 2.

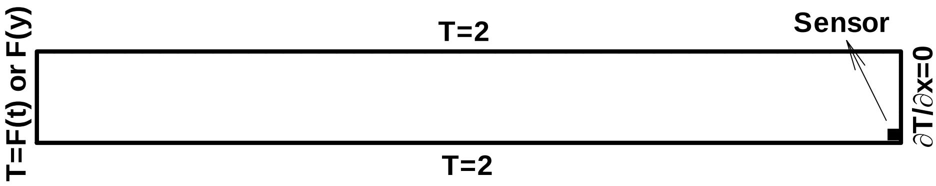

Figure 6: The unknown inflow boundary condition and other boundary conditions, case 2.

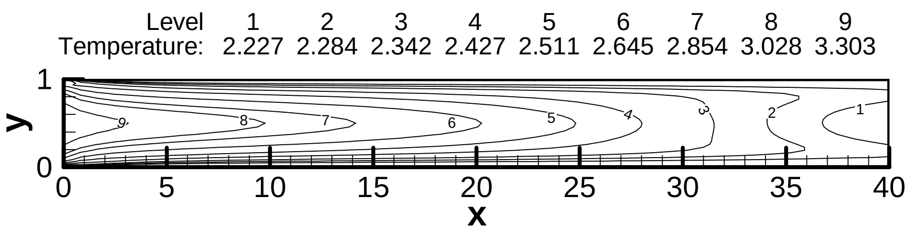

Figure 7: Temperature contours, case 2: inflow boundary condition, $t = 120$ and. $\sigma = 0.05$

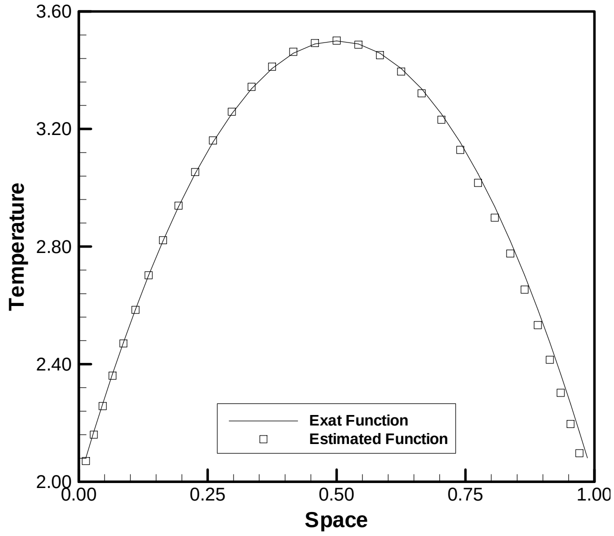

Figure 8: Variation of the estimated inflow boundary condition, case 2.

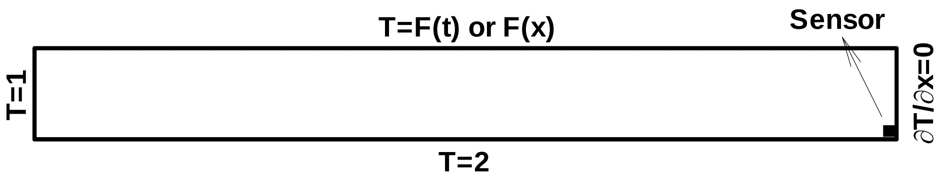

Figure 9: The unknown wall boundary condition and other boundary conditions, case 2.

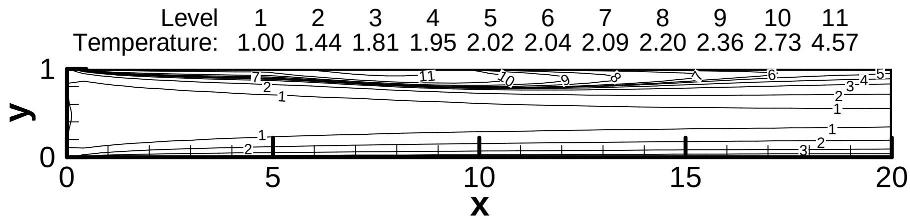

Figure 10: Temperature contours, case 2: wall boundary condition, $t = 120$ and. $\sigma = 0.05$

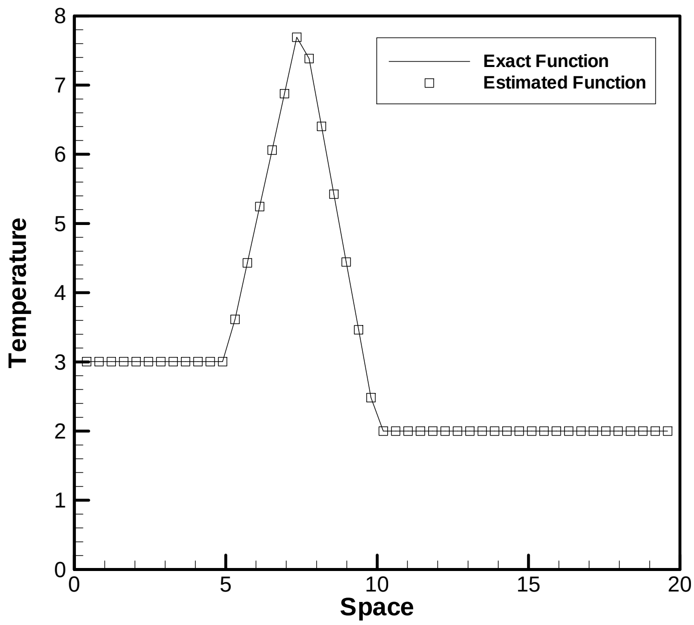

Figure 11: Variation of the estimated wall boundary condition, case 2.

6.3 Case study 3: simultaneous estimation of inflow and wall boundary conditions

This study focuses on an inverse convection heat transfer problem with inflow and wall boundary conditions as unknowns. Boundary conditions of the problem are shown in figure 12. Inflow and wall thermal boundary conditions are the space varying functions of equations (31) and (32), respectively. The aim of this inverse problem is simultaneous estimation of inflow and wall boundary conditions using Levenberg-Marquardt parameter estimation method.

These functions are calculated using measured values by three sensors located at the exit boundary, $x = 19.95$ and $y = 0.05, 0.50, 0.95$. Numerical solution results are represented in table 3.

Figures 13 shows estimated temperature field at *t*=120 with $\sigma = 0.05$ and the inflow and wall temperature variations with $\sigma = 0.05$ 2 are compared with those of the exact temperatures in figures 14 and 15. These figures indicate that inverse analysis estimate these boundary conditions with good accuracy.

7 DISCUSSION AND CONCLUSION

This study investigates the inverse analysis of convective heat transfer in a duct/pipe in Cartesian coordinates to demonstrate the capability of the inverse analysis method to estimate unknown functions in heat transfer problems. Finite difference as the numerical method is employed to solve numerically the direct problem and other partial differential equations resulted from inverse analysis along with structured grids and time/space varying boundary conditions. Then, some cases including a duct/pipe flow with unknown heat source, inflow or wall boundary conditions and simultaneous inflow and wall thermal boundary conditions, considered as thermal systems were investigated by Levenberg-Marquardt parameter-estimation method. Unknown functions under study were estimated with excellent accuracy.

Function	$\sigma = 0$						$\sigma = 0.05$					
	P_1	P_2	P_3	P_4	RMS Error	No. of Iteration	P_1	P_2	P_3	P_4	RMS Error	No. of Iteration
Inflow Boundary Condition	2.000	6.000	−6.000	-	0.000	7	1.993	6.035	−6.038	-	0.004	5
Wall Boundary Condition	3.001	1.994	2.387	2.000	0.004		2.987	2.243	3.088	2.002	0.233	

Table 3: Estimation of the inflow and wall boundary conditions, case 3.

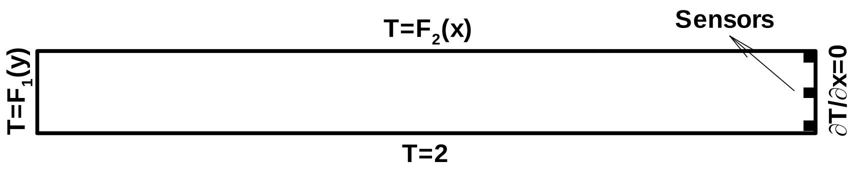

Figure 12: The unknown inflow and wall boundary conditions and other boundary conditions, case 3.

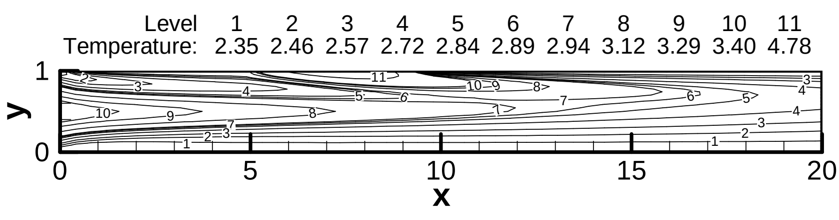

Figure 13: Temperature contours, case 3, $t = 120$ and. $\sigma = 0.05$

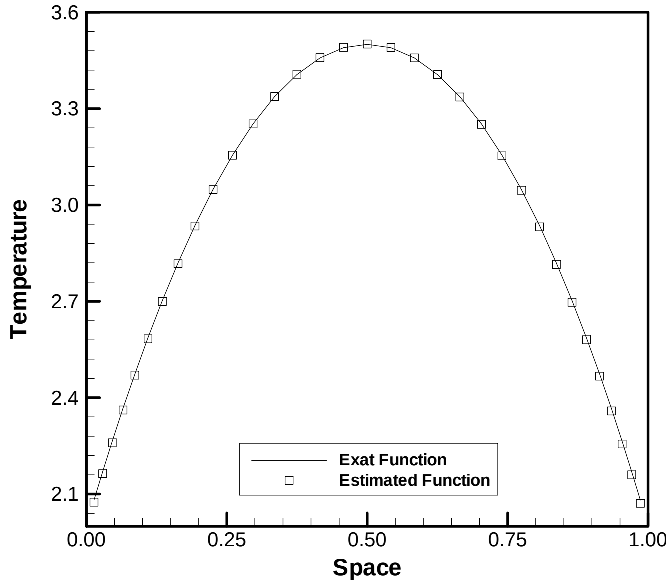

Figure 14: Variation of the estimated inflow boundary condition, case 3.

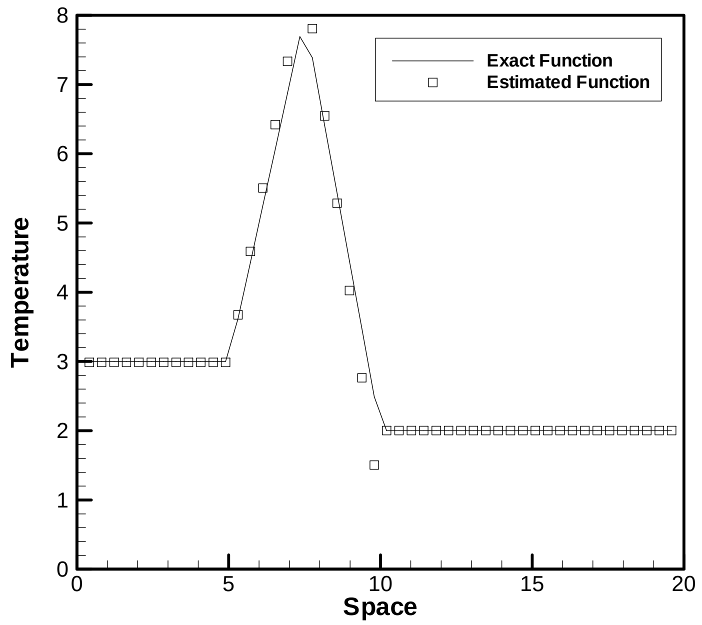

Figure 15: Variation of the estimated wall boundary condition, case 3.

Nomenclature

c_p	specific heat at constant pressure	**Subscripts**	
$D(s,t)$	polynomial coefficients in boundary condition function	b	boundary points
$\mathbf{E}$ ، $\mathbf{F}$	inviscid flux vectors	i	geometrical coordinates
$\mathbf{e}$	error	l	values on lower boundary
$F(s,t)$	temperature boundary condition	r	values on upper boundary
I	number of transient measurements	ref	reference variables
$\mathbf{J}$	sensitivity coefficient matrix	**Superscripts**	
k	thermal conductivity	c	calculated or estimated temperature
$\mathbf{n}$	normal vector	k	iteration number

P	vector of unknown parameters
P	unkown parameter
Pr	Prandtl number
R ‘ **S**	viscous flux vectors
Re	Reynolds number
S(**P**)	objective function
T	temperature

Greek symbols

$\bar{a}$	stretching function used in grid generation
$\bar{b}$	stretching coefficient used in grid generation
e	tolerence
‘	Damping parameter in Levenberg-Marquardt method
G	boundary of computational domain
W	computational domain
m	Dynamic viscosity
r	density
y	stream function
s	constant used for calculating noise in measured data
z	vorticity function

m	measured temperature
T	Transpose
*	dimensional variables
:	flux vectors in primitive variable-based Navier-Stokes equations

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