# AN EXPERIMENTAL AND NUMERICAL PREDICTIONS OF MULTIPLE INJECTIONS INTO HORIZONTAL FLOW TO REDUCE LENGTH REQUIRED FOR MIXING 

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#### Abstract

Computational study of three-dimensional flow and mixing characteristics of multiple tees is done using standard $k-\varepsilon$ turbulence model and preliminary results are examined numerically following side tee mixing experiments. The model has been validated with the available experimental results in a Tee mixer in which a side hot stream mixes with a horizontal cold stream perpendicularly. A reasonable decrease in downward mixing length of horizontal pipe is observed. Other factors such as the tee angle do have a similar impact on mixing in this multiple tee geometry.


## 1 I INTRODUCTION

The term "mixing" is applied to processes used to reduce the degree of non-uniformity or system gradient property such as temperature, concentration, and viscosity. Mixing occurs when a material is moved from one region to another region achieving a required degree of homogeneity along with enhancement of heat and mass transfer, often with a system undergoing chemical reaction. In order to produce a uniform mixture by mixing, two things need to occur. First, there must be a bulk or convective flow so as to avoid any stagnant zone. Secondly, there must be an intensive or high-shear mixing zone, in which the homogeneities are broken down. Laminar and turbulent flow type occur simultaneously in the different part of the mixer with a substantial transitional zone in between them depending upon the fluid properties, primarily viscosity.

A general review of turbulent mixing and mixing in pipeline [1] and other reviews about mechanics of jets of various kinds have been presented. Experimental studies as well as numerical studies of the jet injection of a fluid into a fluid of pipeline have been done discussing the principles of applying mathematical models to explain the increase in mixing using different geometries [2] and simulation of mixing in a pipeline with a 90 tee using CFD [3]. These studies are limited to one parameter or the other. Currently higher machine strength empowers to correlate broader ranges. Correction factors introduced may be eliminated using meshing and refinement techniques revisiting these findings. Experimental and numerical investigations of side tee mixing in pipeline are done to study impingement in higher side velocities which might be a source of corrosion related problems as well as of mixing enhancement in some cases discussing opposed tee. Better mixing length was achieved reporting some numerical solution divergence problems with some studies of protruding tee to discuss jet zones after injection. Many angles were investigated to find out better mixing [4]. Experimental investigations of the mass transfer characteristics of immiscible fluids in the two kinds of stainless steel T-junction microchannels, the opposing-flow and the cross-flow T junction developing empirical correlations to predict the volumetric mass transfer coefficients have been studied [5]. The use of electrical resistance tomography is presented to monitor jet mixing via the addition of a conductivity tracer through coaxial and side entry jets. It is useful for comparative study [6]. A study is done investigating flow and mixing characteristics of many jets and concluding that opposition of jets made vortices which resulted in enhancement of mixing [7]. However, downward length required for mixing is not discussed and no reporting available for reduction in this mixing length due to many injections in numbers. A side injection may also result in some stagnant zones and impingement on opposite wall leading towards temperature fluctuations causing thermal fatigue in the pipe wall resulting in cracks [8] or corrosion of some type. To avoid these problems, lower side velocity may of any help on cost of downward horizontal mixing length enhancement which is also not required. An opposite injection, forming opposite tee, may be used to obstruct impingement but again for similar velocities horizontal pipeline mixing length may increase many times for some cases. The common concern is to reduce the horizontal flow mixing length overcoming all side problems. In this study, multiple tee mixing is investigated keeping in view the above troubles. A reasonable decrease in downward mixing length of horizontal flow is observed which may help to develop and understand mixing better especially designing multiple mixing equipment.

## r EXPERIMENTAL WORK

A pipe tee is a simple device for mixing two fluid streams. A tee is formed by two pipe sections joined traditionally at a right angle to each other. One stream passes straight through the tee while the other enters perpendicularly at one side. This flow arrangement is known as a side-tee. However, other flow arrangements may be used, such as having the two opposing streams entering co-axially and leaving through a pipe, which is perpendicular to the entering direction. This is known as an opposed-tee. A third configuration is a coaxial one, when the (feed) stream (the one to be mixed) enters coaxially with the horizontal stream. Hot water jet was introduced from down side of the pipe and low temperature water was introduced from right side whereas mixed fluid was flown out from left side of pipe. A pipe tee mixing rig was constructed to do experiments as shown in Figure 1. Figure 2 show experimental plots of temperature versus position (in) for $U_{j}=14.7 \mathrm{~m} / \mathrm{s}$ for velocity ratios $U_{j} / U_{m}=23.21,36.48$, and 63.84 for 3.175 mm side-tee and experimental plots of temperature versus position along centerline for $U_{j}=$ $3.94 \mathrm{~m} / \mathrm{s}$ for $U_{j} / U_{m}=6.22,9.77$, and 17.1 for 6.35 mm side-tee. Observations using thermocouples for temperature readings at centerline of pipeline show downward length required for mixing is in the range of 9 to 13 in . These preliminary experimental observations are used to validate side tee numerical model.

## $r$ MODEL EQUATIONS

The flow of fluids in a pipe is governed by the equations of continuity and motion. The equation of continuity in 3D cylindrical coordinates is:

$$
\begin{equation*}
\frac{\partial \rho}{\partial \mathrm{t}}+\frac{1}{\mathrm{r}} \frac{\partial}{\partial \mathrm{r}}\left(\rho \mathrm{r} \mathrm{u}_{\mathrm{r}}\right)+\frac{1}{\mathrm{r}} \frac{\partial}{\partial \theta}\left(\rho \mathrm{u}_{\theta}\right)+\frac{\partial}{\partial \mathrm{z}}\left(\rho \mathrm{u}_{\mathrm{z}}\right)=0 \tag{3.1}
\end{equation*}
$$

The equations of motion are as follows:


Figure 1: Schematic diagram of experimental setup

> The $\quad \mathrm{r}$ - component, $\begin{aligned} & \left.\frac{\partial u_{r}}{\partial t}+u_{r} \frac{\partial u_{r}}{\partial r}+\frac{u_{\theta}}{r} \frac{\partial u_{r}}{\partial \theta}-\frac{u_{\theta}^{2}}{r}-u_{z} \frac{\partial u_{r}}{\partial z}\right]= \\ & -\frac{1}{\rho} \frac{\partial p}{\partial r}+v\left[\frac{\partial}{\partial r}\left(\frac{1}{r} \frac{\partial}{\partial r}\left(r u_{r}\right)\right)+\frac{1}{r^{2}} \frac{\partial^{2} u_{r}}{\partial \theta^{2}}-\frac{2}{r^{2}} \frac{\partial u_{\theta}}{\partial \theta}+\frac{\partial^{2} u_{r}}{\partial z^{2}}\right]+g_{r}\end{aligned}$

The $\theta$ - component,
$\left[\frac{\partial u_{\theta}}{\partial t}+u_{r} \frac{\partial u_{\theta}}{\partial r}+\frac{u_{\theta}}{r} \frac{\partial u_{\theta}}{\partial \theta}-\frac{u_{r} u_{\theta}}{r}-u_{z} \frac{\partial u_{\theta}}{\partial z}\right]=$
$-\frac{1}{\mathrm{r}} \frac{1}{\rho} \frac{\partial \mathrm{p}}{\partial \theta}+\mathrm{v}\left[\frac{\partial}{\partial \mathrm{r}}\left(\frac{1}{\mathrm{r}} \frac{\partial}{\partial \mathrm{r}}\left(\mathrm{ru}_{\theta}\right)\right)+\frac{1}{\mathrm{r}^{2}} \frac{\partial^{2} \mathrm{u}_{\theta}}{\partial \theta^{2}}-\frac{2}{\mathrm{r}^{2}} \frac{\partial \mathrm{u}_{\mathrm{r}}}{\partial \theta}+\frac{\partial^{2} \mathrm{u}_{\theta}}{\partial \mathrm{z}^{2}}\right]+\mathrm{g}_{\theta}$


Figure 2: Experimental plots of temperature versus position for $U j=14.7 \mathrm{~m} / \mathrm{s}$ for $\mathrm{Uj} / \mathrm{Um}=23.21,36.48$, and 63.84 for 3.175 mm side-tee.


Figure 3: Experimental plots of temperature versus position along centerline for $\mathrm{Uj}=3.94 \mathrm{~m} / \mathrm{s}$ for $\mathrm{Uj} / \mathrm{Um}=$ $6.22,9.77$, and 17.1 for 6.35 mm side-tee.
and the z - component,

$$
\begin{align*}
& {\left[\frac{\partial u_{z}}{\partial t}+u_{r} \frac{\partial u_{z}}{\partial r}+\frac{u_{\theta}}{r} \frac{\partial u_{z}}{\partial \theta}+u_{z} \frac{\partial u_{z}}{\partial \mathrm{z}}\right]=} \\
& -\frac{1}{\rho} \frac{\partial p}{\partial z}+v\left[\frac{\partial}{\partial r}\left(r \frac{\partial u_{z}}{\partial r}\right)+\frac{1}{r^{2}} \frac{\partial^{2} u_{z}}{\partial \theta^{2}}+\frac{\partial^{2} u_{z}}{\partial \mathrm{z}^{2}}\right]+g_{z} \tag{3.4}
\end{align*}
$$

The temperature field of the fluid flowing in pipes can be resolved by solving the energy equation.

$$
\begin{align*}
& \rho \hat{C} p\left(\frac{\partial T}{\partial t}+u_{r} \frac{\partial T}{\partial r}+\frac{u_{\theta}}{r} \frac{\partial T}{\partial \theta}+u_{z} \frac{\partial T}{\partial \mathrm{z}}\right)= \\
& k\left[\frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right)+\frac{1}{r^{2}} \frac{\partial^{2} T}{\partial \theta^{2}}+\frac{\partial^{2} T}{\partial \mathrm{z}^{2}}\right]+2 \mu\left\{\left(\frac{\partial u_{r}}{\partial r}\right)^{2}+\left[\frac{1}{r}\left(\frac{\partial u_{\theta}}{\partial \theta}+u_{r}\right)\right]^{2}+\left(\frac{\partial u_{z}}{\partial r}\right)\right\} \\
& +\mu\left\{\left(\frac{\partial u_{\theta}}{\partial z}+\frac{1}{r} \frac{\partial u_{z}}{\partial \theta}\right)^{2}+\left(\frac{\partial u_{z}}{\partial r}+\frac{\partial u_{r}}{\partial z}\right)^{2}+\left[\frac{1}{r} \frac{\partial u_{r}}{\partial \theta}+r \frac{\partial}{\partial r}\left(\frac{u_{\theta}}{r}\right)\right]^{2}\right\} \tag{3.5}
\end{align*}
$$

Initial conditions were set such as for each case separately to get earlier convergence after having experience of simulation studies. The boundary conditions used in this study are ( $i$ ) values of velocities are specified at the entrance of the horizontal pipe and entrance of the side-tee (ii) at all walls, no-slip condition is applied. Temperatures are specified for the horizontal fluid and the side fluid.

## 〔 NUMERICAL MODEL

Mass and momentum conservation equations were solved. The energy equation is enabled during simulations and flow pattern were observed using temperature fields. The general purpose three-dimensional CFD package FLUENT was used to solve the governing equations. A SIMPLE [9] based algorithm is used to solve the pressurevelocity equations and then the equation of energy is solved sequentially. A threedimensional numerical model was constructed of jet diameter 6.35 mm and of horizontal pipe diameter of 25.4 mm using pre-processor Gambit. An unstructured tetrahedral grid was chosen.

It has already been investigated that during simulation no significant effect is observed due to upstream and down stream lengths in simulations. The standard $\mathrm{k}-\mathrm{e}$ turbulence model is used to account for turbulence. Other turbulence models were tried including the Realizable k-e model and the Reynolds stress model. The standard $\mathrm{k}-\mathrm{e}$ model was found to be adequate and produced numerical results that agreed well with experimental values. This is consistent with other work that was published in the literature including simulation of mixing in pipelines with t-jet and also in the simulation of mixing in a jet agitated tank [3, 4, 10, and 11].


Figure 4: Grid display of multiple-tee showing the four side jet and the horizontal pipe with an inlet and an outlet of the horizontal pipe.

The mesh size was small enough in order to properly resolve the fields that were solved for and the effect of mesh size was negligible at sizes smaller than that. The gradient refinement of $0.0005 \mathrm{~K} / \mathrm{m}$ gave the best approximation for the highest temperature at side fluid entrance than others. Boundary conditions are different for each case. A very good agreement between experiments and simulation results of side injection into horizontal pipe flow, validating the numerical model is established using this geometry. When a plane was constructed to observe the mixing numerically, it was
seen that the multiple tees on same plane of horizontal pipe may be considered as many single tee joined on a single cross sectional plane of horizontal pipe.

## - RESULTS AND DISCUSSIONS

The simulation of multiple-tee geometry has been carried out. Figure 4 shows the grid displaying the four side-tees with a horizontal pipe inlet and outlet. A 25.4 mm diameter horizontal pipe and side-jet diameter of 6.35 mm are considered. Cases with different velocities were simulated as shown in Table 1. Temperature is taken for horizontal- and side-streams as 283 K and 323 K respectively with a difference of 40 degree.

Three cases are studied for side velocities of $3.94 \mathrm{~m} / \mathrm{s}$ and $1.313 \mathrm{~m} / \mathrm{s}$ with horizontal $0.23 \mathrm{~m} / \mathrm{s}$ and $0.92 \mathrm{~m} / \mathrm{s}$ as shown in Table 1. Instead of measuring the temperature downstream of a row of heated jets injected into a cold stream cross-sectional planes are used to measure the completeness of mixing. The mixing length is a measure of how well the temperature reached to average temperature (equilibrium) at any specific axial plane. All results presented here were obtained with temperature-independent properties. Temperature difference was set to be small to avoid physical effects of fluids due to temperature. Velocities of opposing jet were controlled because at higher opposing velocities solution convergence was difficult numerically.

Figure 4 shows temperature (K), velocity ( $\mathrm{m} / \mathrm{s}$ ) contours, and velocity vectors for the case two with the side velocity $3.94 \mathrm{~m} / \mathrm{s}$ into four side-tees with $0.92 \mathrm{~m} / \mathrm{s}$ horizontal pipe fluid velocity. Mixing is achieved earlier i.e. dividing the side fluid into four part and injecting it into same horizontal fluid enhanced mixing and reduced the pipe length required for mixing. At lower horizontal velocities back mixing is apparent when the side velocities are higher. Therefore, at higher side velocities significance of horizontal velocity becomes less than side velocities. It was found that for a side velocity of 3.94 $\mathrm{m} / \mathrm{s}$ and horizontal velocity of $0.23 \mathrm{~m} / \mathrm{s}$, velocity ratio 17.10 , length required for $95 \%$ mixing is 11 D . A comparison of the length required for $95 \%$ mixing for these different geometries is given in Table 3. It can also be seen from Table 2 that keeping everything same except using multiple-tees, as in case 1, instead of single side-tee $95 \%$ mixing is achieved faster. So multiple-tee arrangement may be used where large amount of fluids are to be mixed and higher velocities are to be avoided. An interesting observation is found that when the mass flow rate was divided into four equal jets of same side entry diameter (case three: each jet velocity is $1.313 \mathrm{~m} / \mathrm{s}$ ), the length required for $95 \%$ mixing was reduced to almost one third that of $90^{\circ}$ side-tee as shown in Table 3.

| Case | Horizontal Fluid |  | Side Fluid Velocity, m/s |  |  |  | Temperature of Side Fluid, K |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Velocity, $\mathrm{m} / \mathrm{s}$ | Temperature, K | Side <br> 01 | Side $02$ | Side <br> 03 | $\begin{array}{\|l\|} \hline \text { Side } \\ 04 \end{array}$ |  |
| 1 | 0.23 | 283 | 3.94 | 3.94 | 3.94 | 3.94 | 323 |
| 2 | 0.92 | 283 | 3.94 | 3.94 | 3.94 | 3.94 | 323 |
| 3 | 0.23 | 283 | 1.313 | 1.313 | 1.313 | 1.313 | 323 |

Table 1: Velocities of horizontal and side fluids for multiple-tees

| Case | Side <br> Velocity $\mathrm{m} / \mathrm{s}$ | Horizontal velocity $\mathrm{m} / \mathrm{s}$ | Side to horizontal velocity ratio |  |  |  | Mixing Length in horizontal pipe diameters |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | 01 | 02 | 03 | 04 |  |
| 1 | 3.94 | 0.23 | 17.1 | 17.1 | 17.1 | 17.1 | 4.5D |
| 2 | 3.94 | 0.92 | 4.3 | 4.3 | 4.3 | 4.3 | 4.0D |
| 3 | 1.313 | 0.23 | 5.7 | 5.7 | 5.7 | 5.7 | 3.5D |

Table 2: Side to horizontal velocity ratios with mixing length in diameter of horizontal pipe.

| Case | Horizontal <br> velocity, <br> $\mathrm{m} / \mathrm{s}$ | Side <br> velocity, <br> $\mathrm{m} / \mathrm{s}$ | Length Required for <br> $95 \%$ mixing in <br> horizontal pipe <br> diameter |
| :--- | :--- | :--- | :--- |
| Right-angle, side 6.35 mm tee with <br> 25.4mm horizontal | 0.23 | 3.94 | 11D |
| Four multiple 6.35 mm tees with <br> 25.4 mm horizontal | 0.23 | 1.313 <br> each | 3.5 D |

Table 3: Comparison of length required


Temperature $(\mathrm{K})$ contours in a central z-plane.


Temperature $(\mathrm{K})$ contours at 2 D . The range is from 310.9 to $316.4 \mathrm{~K}(\Delta \mathrm{~T}=5.5)$.


Temperature (K) contours showing 95\% mixing attained at 4D for temperature range from 312.8 to 316.0 K drawn for temperature scaled as above


Velocity vectors scaled by a factor of 2 at the entrance of the jets.

Figure 5: Temperature ( K ) and Velocity ( $\mathrm{m} / \mathrm{s}$ ) contours and velocity vectors for case two

## 1 CONCLUSION

CFD simulations are a useful tool to assist in finding the optimum design. Multiple tees reduced the mixing length required for $95 \%$ mixing three times than for single tee. Dead and stagnant zones are reduced by using multiple tees. Impingement of jet is tackled by the opposing entering jet stream avoiding the pipe wall. Numerous parameters e.g. diameter ratio, velocity ratio, angle injection, gravity effects may be studied to investigate effect on mixing which either boost mixing for one case or knock mixing for other case. Corrosion, cracking and other major problems may be reduced or controlled using these geometries for different types of fluids and adjusting their flow rates.

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