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A LAGRANGIAN SMOOTHED PARTICLE HYDRODYNAMICS – SPH – METHOD FOR MODELLING WAVES-COASTAL STRUCTURE INTERACTION

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Abstract. *Wave-structure interaction generates very complex phenomena involving nonlinear processes, like wave propagation and transformation, run-up, wave breaking, and overtopping. Additionally, complex coastal structures are constructed, with impermeable or porous structures, composed by blocs or arc crown wall structures, etc. Consequently, in practical engineering projects, there are a large number of cases for which there is no appropriated empirical formula. For those cases, physical modeling is currently employed due to the accuracy of this approach and the possibility to model large areas. However, its accurate simulation on physical models strongly depends on the model scale used and needs an understanding of model and scale effects for the correct representation of the phenomenon. For studies of interaction between waves and complex structures as coastal structures, numerical modeling presents a very attractive complement to physical modeling. However, only some numerical models allow simulating wave breaking and wave overtopping correctly. The SPH (Smoothed Particle Hydrodynamics) method is a relatively new method that computes trajectories of fluid particles which interact according to the Navier-Stokes equations. The recent advances on SPH models show that Lagrangian method is a very promising alternative approach to simulate wave-structure interaction due to its completely mesh-free technique.*

This paper presents an application of a free surface modeling by means of the SPHysics numerical model based on Lagrangian approach. A typical impermeable coastal structure of the Portuguese Atlantic coast is considered in the present study and features of free surface elevation and overtopping are analyzed for two different geometric configurations, differing in the crest level. These two cases represent a range of overtopping conditions varying from small discharges, more difficult to model numerically, to a considerable amount of overtopping.

1 INTRODUCTION

Seawalls are structures that allow the protection of coastal areas from the wave attack. In the project of those structures, a wave-structure interaction study should be made to define the viability and efficiency of the structure, namely the overtopping discharge and the forces applied on the structure. Wave-structure interaction generates very complex phenomena involving nonlinear processes, like wave propagation and transformation, run-up, wave breaking and overtopping. Coastal structures could have different structural characteristics: could be impermeable or porous structures, composed by artificial blocs, be an arc crown wall structures, etc.

Numerical models, more or less complexes depending on the approach and on the physical assumptions, allow simulating the propagation of waves and the nearshore transformation. The models based on the nonlinear Boussinesq equations, such as COULWAVE [1], give good predictions comparing with field data and laboratory physical modeling. However, it does not model the breaking and highly nonlinear processes that occur when waves impinge the coastal structures, such as breaking and overtopping. Numerical models based on Euler or Navier-Stokes equations, such as CANAL [2] based on Boundary Element Method, or FLUINCO [3], based on a mixed Euler-Lagrange formulation of the free surface, allow modeling wave-structure interaction and calculating velocity and pressure field. However, those numerical models do not simulate wave breaking.

Only few numerical models allow simulating the very complex phenomena of wave breaking and overtopping. Those models are generally based on fluid dynamic equations, i.e. the Navier-stokes equations, and developed using an Eulerian approach. Numerical simulation of free surface flows is treated using the Volume of Fluid (VOF) approach [4], such as the Reynolds Average Navier-Stokes (RANS) model COBRAS-UC [5] or the commercial program FLUENT [6, 7]. However, the accuracy of wave breaking and overtopping simulations strongly depend of the mesh and a fine grid is necessary to ensure modeling those phenomena without diffusion of the fraction volume index (particularly for thin layers of fluid).

A new approach appears in 1994, when Monaghan [8] shows the first application of Lagrangian method for modeling free surface flows. The recent advances on Smoothed Particle Hydrodynamics (SPH) models, since 1994, show that Lagrangian method is a very promising alternative approach to simulate wave breaking and overtopping due to its completely mesh-free technique.

Several numerical models are constructed using the SPH method. One of these is the SPHysics model [9, 10, 11], inspired by the formulation of Monaghan [12].

SPHysics model is here applied for modeling wave propagation with breaking and overtopping of an impermeable seawall, a common coastal defense structure employed at the Portuguese coast. Numerical results of free-surface deformation at several positions along the flume and of the mean overtopping discharge are compared with experimental data from a model scale tests made at the National Civil Engineering Laboratory (LNEC) in the framework of the Composite Modelling of the Interactions between Beaches and Structures (CoMIBBs) project – HYDRALAB III European project [13]. Two different geometric configurations, differing in the crest level, are considered. These two cases represent a range of overtopping conditions varying from small discharges, more difficult to model numerically, to a considerable amount of overtopping discharge.

2 SPH NUMERICAL MODEL

SPH method consists to integrate the hydrodynamics equations of motion on each particle in the Lagrangian formalism. The partial differential equations of continuum fluid dynamics are transformed into SPH forms, i.e. particle forms, by integral equations using integral interpolants [12, 14, 15]. The fundamental principle is to approximate any function A(r) by:

$$A(r) = \int_{\Omega} A(r')W(r-r',h)dr' \quad (1)$$

where r is the vector position, *W* is the weighting function, *h* is called the smoothing length. The interpolation function, i.e. weighting function or kernel, allows determining the interaction among neighboring particles included in the influence domain, controlled by the smoothing length *h*, typically higher than the initial particle spacing.

Figure 1 shows a typical compact support of a kernel function. The kernels should be verified several conditions of positivity, compact support, Delta function behavior. Different kernels were developed and can be found in the literature [15].

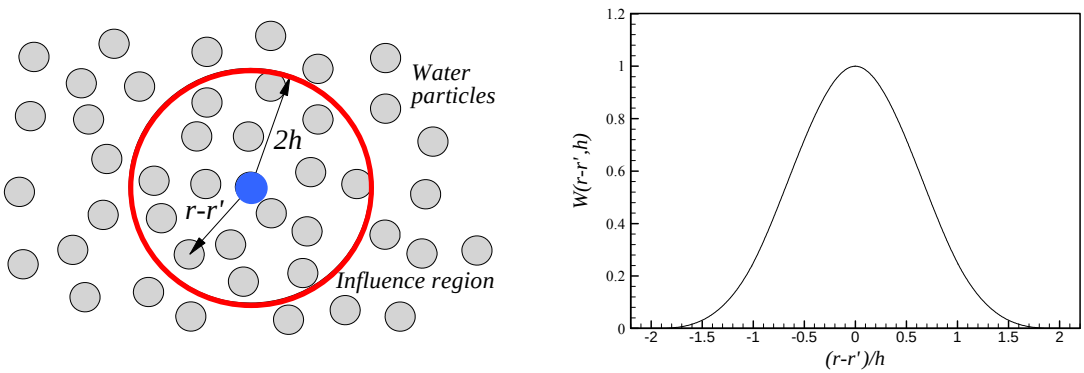

Figure 1: Typical kernel function and compact support of the kernel.

The relation given in Eq.1 is written as an approximation of the function *A* at a particle *a*, in discrete notation:

$$A(r) = \sum_{b} m_b \frac{A_b}{\rho_b} W_{ab} \quad (2)$$

where the summation is over all the particles within the region of compact support of the kernel function. The mass and density are noted *mb* and *rb* respectively and *Wab*=*W*(*ra-rb*, *h*) is the kernel.

Two types of SPH model were developed: strict incompressible and weakly incompressible SPH model. The major differences between the weakly compressible SPH [12, 16, 17] and the incompressible SPH [18, 19, 20] lie in that the former calculated the pressures explicitly using an equation of state, while the latter employs a strict incompressible formulation for what the pressure is obtained implicitly by solving a pressure Poisson equation derived from the mass and momentum equations.

3 SPHYSICS NUMERICAL MODEL

SPHysics code is an open-source Smoothed Particle Hydrodynamics program developed jointly by researchers of several Universities [9, 10, 11]. The model is inspired by the formulation of Monaghan [12]. The fluid in the standard SPH formalism is treated as weakly compressible. The model presents a modular form and a variety of features are available to choose different options, like: 2D and 3D model, time scheme (Predictor-Corrector or Verlet algorithm), constant or variable time step, various kernels, viscosity models (artificial, laminar and Sub-Particle Scale turbulence model), density filter (Shepard or MLS), and solid boundary conditions (dynamic boundaries, repulsive forces). Detail of numerical implementation and references are available at the website of SPHysics [21].

For the present numerical simulations, the quadratic kernel [22] is used to determine the interaction between the particles. The fluid is treated as weakly compressible which allows the use of an equation of state to determine fluid pressure. The relationship between the pressure and the density was assumed to follow the equation of state provided by Batchelor [23]. The compressibility is adjusted to slow the speed of sound so that the time step in the model, based on the sound velocity, is reasonable. Integration in time is performed by the Predictor-Corrector model using a variable time step. The repulsive boundary condition, developed by Monaghan [24], is used and allows preventing a water particle crossing a solid boundary. Variable time step is used to ensure the CFL condition.

It was shown, in previous study [25], that Sub-Particle Scale – SPS – turbulence model [18] provided better results compared to artificial viscosity model [12] since SPS model avoids the strong dissipative effects of artificial viscosity model. So the SPS turbulence model is used in the present simulations.

Particles are usually moved using the XSPH variant due to Monaghan [26], with ε_{XSPH}=0.5 (values ranged between 0 and 1). The method is a correction for the particle velocity, which is recalculated taking into account the velocity of that particle and the average velocity of neighbouring particles. However, it was shown in [25] that instabilities appear during wave propagation due to the XSPH correction, particles cross the solid boundary, fluid flow exhibits unphysical behaviours and the program crashes. Consequently, in the present simulations, the XSPH correction is not used and ε_{XSPH}=0.

4 CASE STUDY OF A COASTAL STRUCTURE

The numerical results are compared with experimental data collected at the National Civil Engineering Laboratory (LNEC), Portugal, in the framework of the *Composite Modeling of the Interactions between Beaches and Structures (CoMIBBs)* project, a joint research activity of the HYDRALAB III European project [27]. Several tests had been made with different geometrical scales, using for that two different wave flumes.

The experimental work used in this paper consists of wave propagation, with breaking and wave overtopping of an impermeable seawall, a common coastal defense structure employed at the Portuguese coast [13]. The test used was performed in the large wave flume of LNEC with 3m width, 73m length and 2m height (Figure 2), that has an operating water depth of 2m and is equipped with a piston-type wave-maker and an active wave absorption system, which allows the absorption of reflected waves.

Figure 2: Overview of wave flume and model structure.

The structure comprises a seawall with a 2:3 slope fronted by a 1:20 beach foreshore. physical model was built and operated according to Froude's similarity law, using a geometrical scale of 1:10 and it was built to reproduce the prototype cross-section shown in Figure 3. The bottom profile is composed by a horizontal bottom with 35.74m length and a bottom with a slope of 1:20 during 18.675m. The impermeable structure has a crest located at 1.684m from the bottom, i.e. Rc=0.534m.

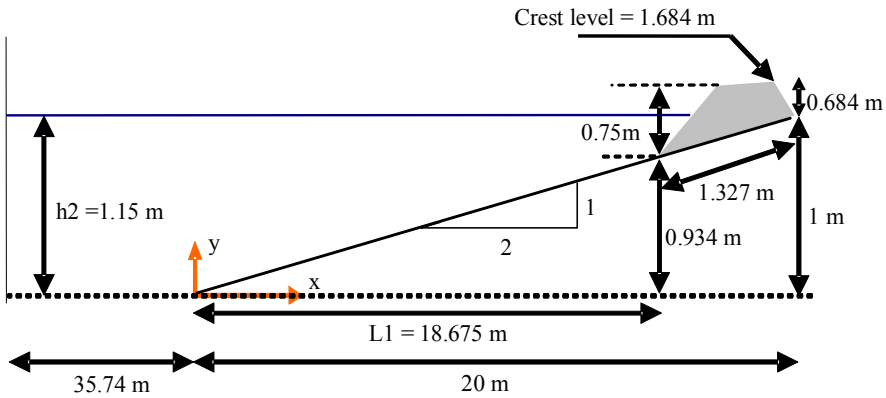

Figure 3: Schematic representation of the prototype cross-section and the coordinate system.

The flume is equipped with several wave gauges, sensor pressures at the structure and one overtopping device designed to measure the volume of overtopping (Figure 4). To determine the free-surface elevation, the flume was equipped with six resistive-type wave gauges. A fixed array of two gauges, located in front of the wave-maker, was needed for the dynamic wave absorption system. A moveable array of four gauges was used to characterize the free-surface elevation along the flume. A resistive type wave gauge was located 3mm above the face of the model structure to determine run-up levels (Figure 4). A special attention was paid to the breaking area where video cameras were located, allowing the analysis of the wave breaking characteristics. A tank with a triangular weir on one of its sides was located at the back of the structure to collect the water overtopping the structure (Figure 4). The water was conducted to the tank by means of a chute, 50cm wide. A water-level gauge was used inside the tank to measure the variation in water level within a test run.

Figure 4: Equipment used and overview of wave overtopping.

The incident regular wave used here has a period, T=3.79s, a wave height, H=0.40m, and water depth is d=1.15m. The wave length is 12.04m for this water depth. Tests lasted for 5 minutes and for each regular wave condition and the test was repeated at least six times, each with the moveable array in a different position, in order to have the surface elevation measured at twenty four different locations along the flume.

Figure 6 presents the mean overtopping discharges obtained for the test conditions considered in this study. The figure also shows the means, μ, standard deviations, σ, and coefficients of variation, σ/μ, of the discharges. For this case, breaking occurs around x=8.5m (x=0m at the toe of the foreshore). After that, due to interaction between the incident and the reflected waves, a maximum value of the surface elevation is reached for x around 11m.

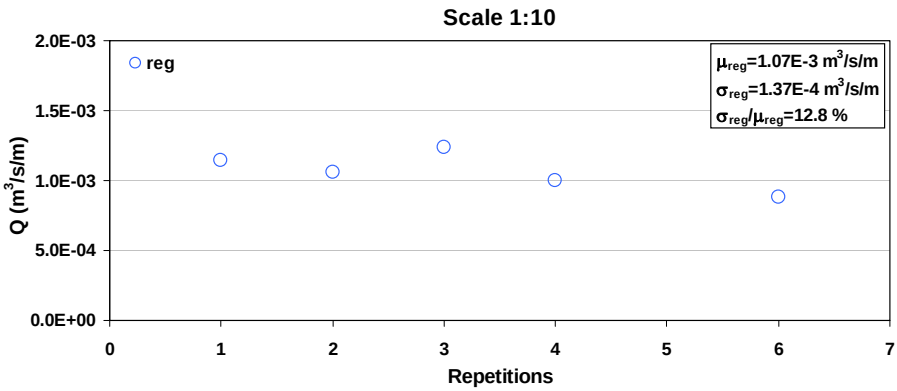

Figure 6: Mean overtopping discharge, Q, obtained in physical model for the test conditions considered in this study.

5 RESULTS AND DISCUSSION

The computational domain reproduced the full dimensions of the ramp and the structure at scale 1:10. Wave generation is performed in SPHysics by a flexible wavemaker in order to impose a wave velocity profile. However, the wavemaker does not include dynamic absorption. Solid boundaries are defined by solid particles regularly spacing.

Initially, water particles are placed in the flume using a Cartesian distribution, i.e. particles are regularly distributed. This is a condition of SPH method when smoothing length of the kernel is constant. Velocity is zero and pressure is hydrostatic. Figure 7 presents a view of the initial distribution of solid and fluid particles in the full computational domain and near the structure.

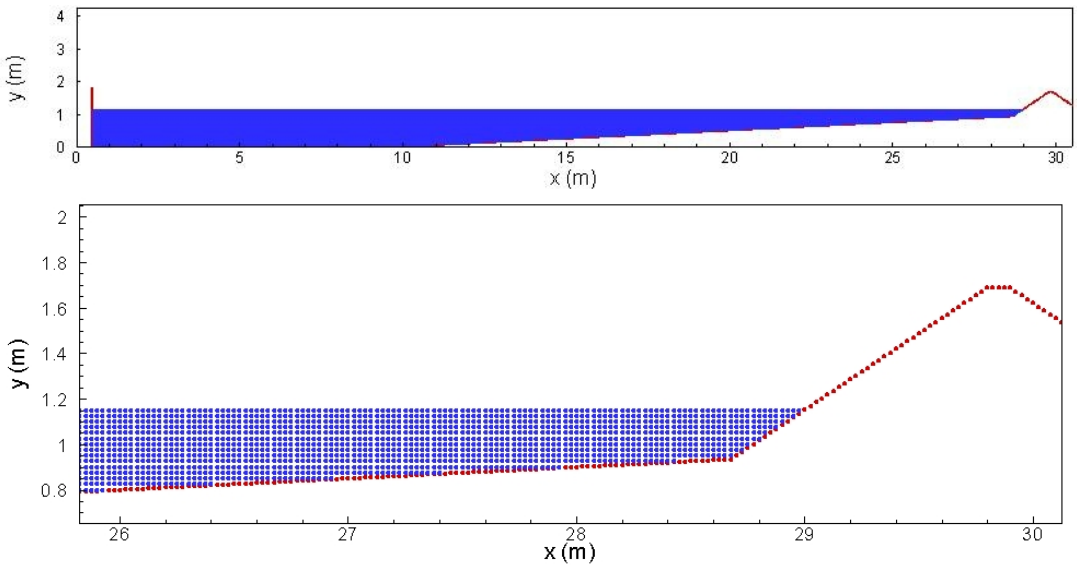

Figure 7: Full computational domain and detail of the initial position of particles near the structure for Rc=0.534m.

Theoretically, it is possible to model large domain with SPH method. However, a fine numerical resolution is very time consuming and it is impossible with the actual serial version of the code to model large areas. Because of that, the computational domain is smaller than the physical flume: the horizontal bottom is 10m length (slightly smaller than the wave length for the local deep) instead of 35.7m. However, it was verified that the wave characteristics obtained by SPHysics at the beginning of the 1:20 slope using this short domain are similar to the wave characteristics obtained at the same section using a wave propagation model, CANAL [2] using a domain with a horizontal bottom of 35.7m. Figure 8 shows the free surface elevation at X=0m, the toe of the foreshore, obtained using SPHysics and CANAL codes.

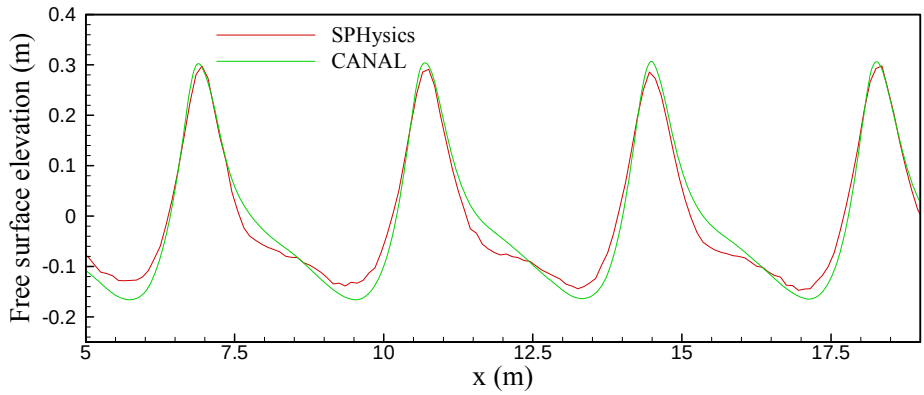

Figure 8: Free surface elevation at X=0m, toe of the foreshore, obtained by SPHysics and CANAL.

Here, the convergence study performed to define the best discretization in terms of particle dimension, i.e. particle volume, is presented. Table 1 shows the characteristics of the nine numerical configurations studied: particle dimension, do, particle volume and total number of particles for each discretization.

Firstly, the free surface elevation obtained for the nine configurations is compared in two sections of the flume, X=0m and X=10.0m, Figure 9.

do (m)	Volume (m^3/m)	Number of particles
0.025	$6.25.10^{-4}$	40517
0.01949	$3.80.10^{-4}$	66032
0.01667	$2.78.10^{-4}$	89883
0.01533	$2.35.10^{-4}$	106020
0.01420	$2.01.10^{-4}$	123405
0.01353	$1.83.10^{-4}$	135767
0.01292	$1.67.10^{-4}$	148629
0.01210	$1.46.10^{-4}$	166183
0.01150	$1.32.10^{-4}$	187164

Table 1 – Particle characteristics and total number of particles.

At X=0m, the free surface elevation between 10s and 21.5s, corresponding to just wave propagation, presents the same behaviour. Wave characteristics, as wave height and period, are very similar, except for the larger particle dimension, 0.025 and 0.01949m, where the obtained error is around 10% and 4%, respectively.

However, at X=10.0m and between 15s and 26.5s, the free surface elevation shows some differences. In this case, interaction between the incident and the wave reflected by the structure is more sensible to the discretization, particularly in the estimation of the wave crest amplitude. The error increases with the increase of the wave crest amplitude as can be seen in Figure 9. For 17.6s, which correspond to the instant where incident wave interact with the first wave reflected by the structure, convergence is obtained only for the finer discretizations and the errors obtained for the larger particle dimensions, do=0.025m, 0.01949m and 0.01533m, are around 22%, 13% and 9%, respectively. For the interaction between the incident and the second reflected wave, the error decrease to around 15% and 4% for the discretization do=0.025m and 0.01949m, respectively. The wave crest amplitude decreases from 0.416m to 0.283m from the first to the second incident-reflected wave interaction.

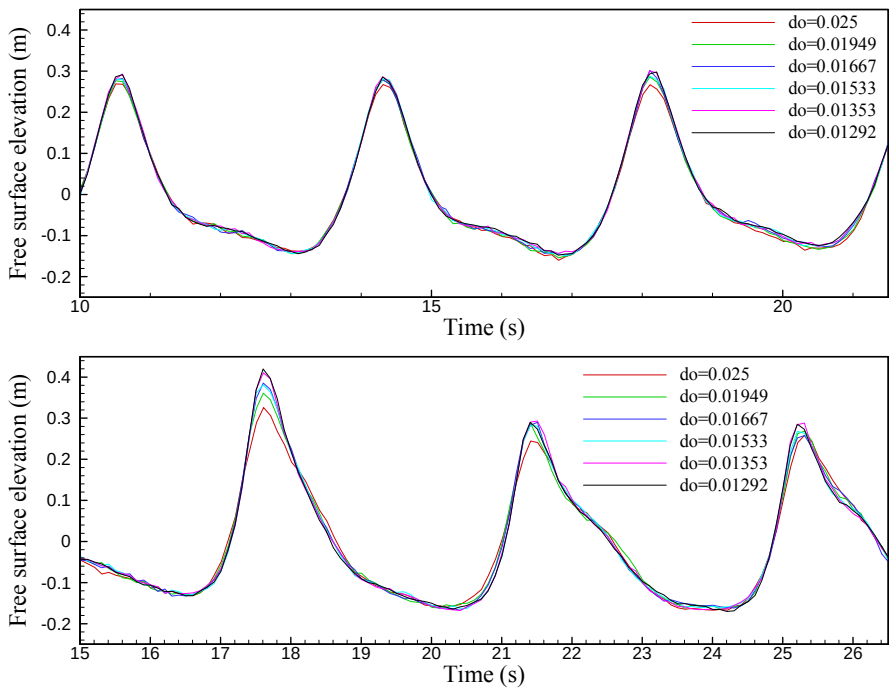

Figure 9: Free surface elevation at X=0m (top) and X=10.0m (bottom) for several discretizations, do.

Secondly, the overtopping is analyzed, since even for the cases where the free surface elevation convergence with the particle volume is verified, the same is not necessarily true for the overtopping discharge convergence. This parameter strongly depends on the accurate simulation of the wave propagation, incident-reflected wave interaction and wave breaking. Consequently, it is very sensitive to the numerical discretization and even more sensitive to the characteristics of the free surface elevation.

To study the convergence of the overtopping discharge, two configurations of the present coastal structure are used: one with the same dimension of crest level as in the physical model, Rc=0.534m, and a second with a smaller crest level, Rc=0.116m. In other words, a convergence study is carried out for a configuration with small and with large overtopping discharge in order to access the sensibility of the numerical model to the discretization.

Figures 10 and 11 show the overtopping volume versus the discretization along the time for the two cases with different structure crest levels , Rc=0.534m and Rc=0.116m, and Figures 12 and 13 present the mean overtopping discharge for these two cases. As the wavemaker was not design for do the dynamic absorption, overtopping discharge is analyzed before the wave re-reflected by the wavemaker reaches the structure, i.e. up to 40s. During this time seven waves reach the structure and overtopping can occur.

As can be seen in Figure 10, for the structure with a smaller crest level (Rc=0.116m), the overtopping volume reaches around 0.50m^3/m after 40s. Convergence of overtopping volume with discretization is obtained with relatively large particle dimension, i.e. do=0.01667m. Figure 12 shows that convergence of the mean overtopping discharge is also reached when particle dimension is 0.01667m. For this configuration overtopping intensity is large for each wave. Mean and total overtopping are not strongly dependent on the discretization.

On the other hand, for Rc=0.534m, convergence is not so evident (Figure 11). Overtopping volume reaches 0.03m^3/m after 40s, i.e. a volume 16 times smaller than that obtained for the previous configuration. As can be seen in Figure 13, convergence is obtained only for particle dimensions smaller than 0.01353m. So, as can be expected, numerical model is more sensitive for smaller overtopping discharge. Due to the small overtopping volume, it is necessary to decrease the particle volume, i.e. the particle dimension, to access accuracy results and independence of results with discretization.

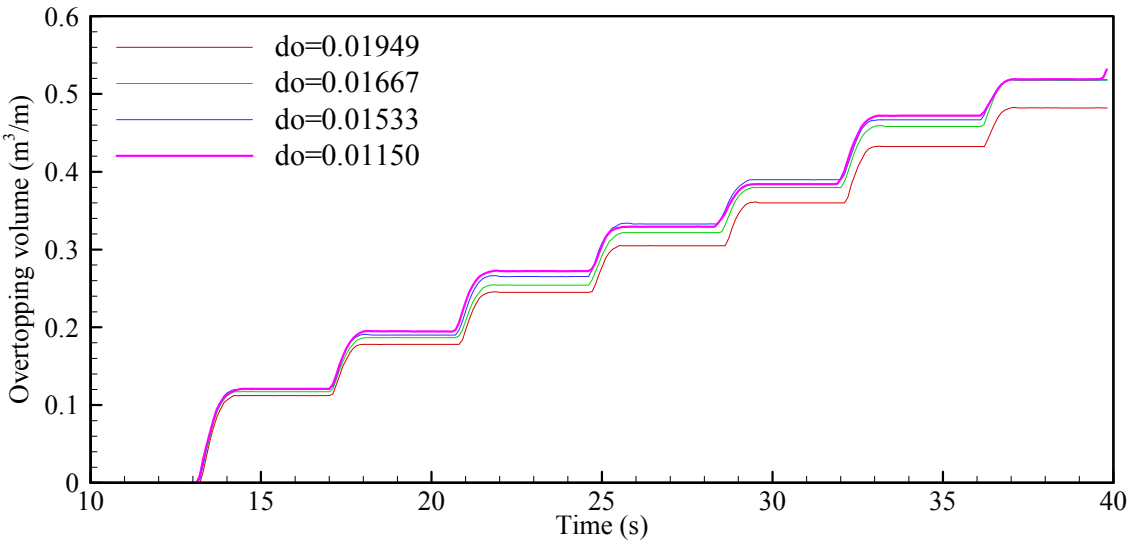

Figure 10: Overtopping volume versus time for Rc=0.116m for several discretizations.

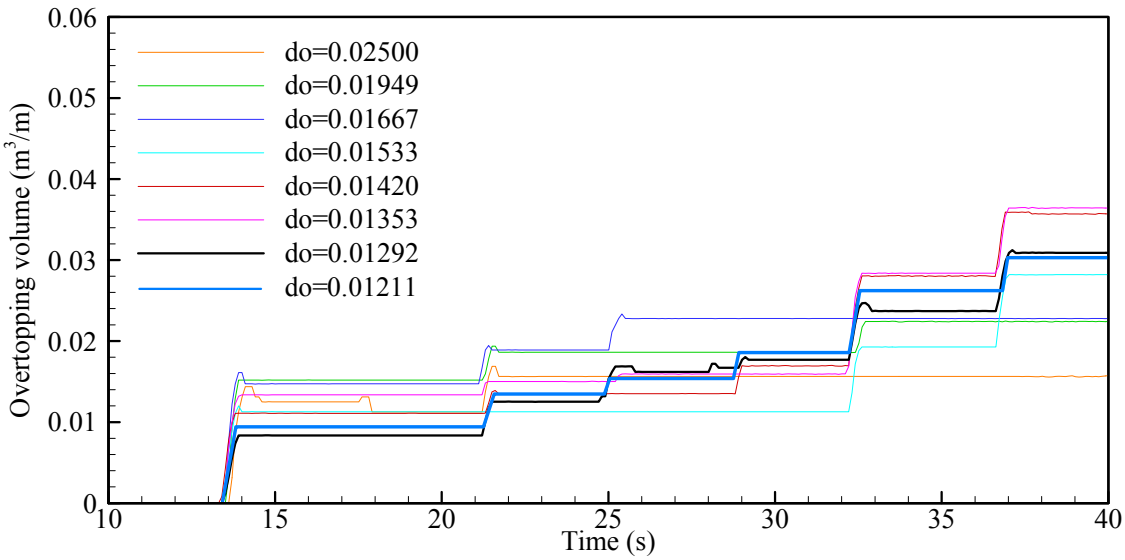

Figure 11: Overtopping volume versus time for Rc=0.534m for several discretizations.

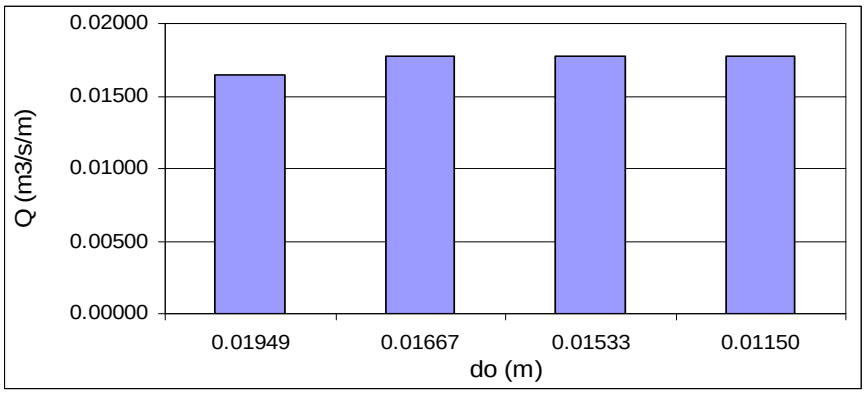

Figure 12: Mean overtopping discharge for Rc=0.116m for several discretizations.

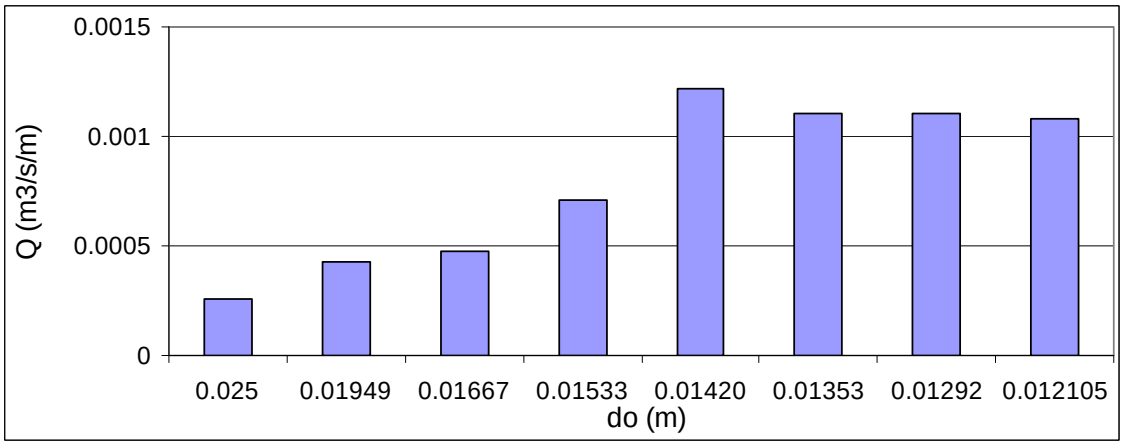

Figure 13: Mean overtopping discharge for Rc=0.534m for several discretizations.

Based on the convergence study, it was decided to run SPHysics for the case study using the discretization with particle dimension equal to 0.012105m.

Figure 14 shows the free surface elevation at three gauges located at X=9.5m, 10.5m and 11.0m obtained with the numerical model and from the experimental test. There is a good agreement between SPHysics results and physical model data for the three gauges,

both in terms of wave amplitude and wave period. The small discrepancies observed are probably due to differences in the phase between fundamental frequency and harmonics.

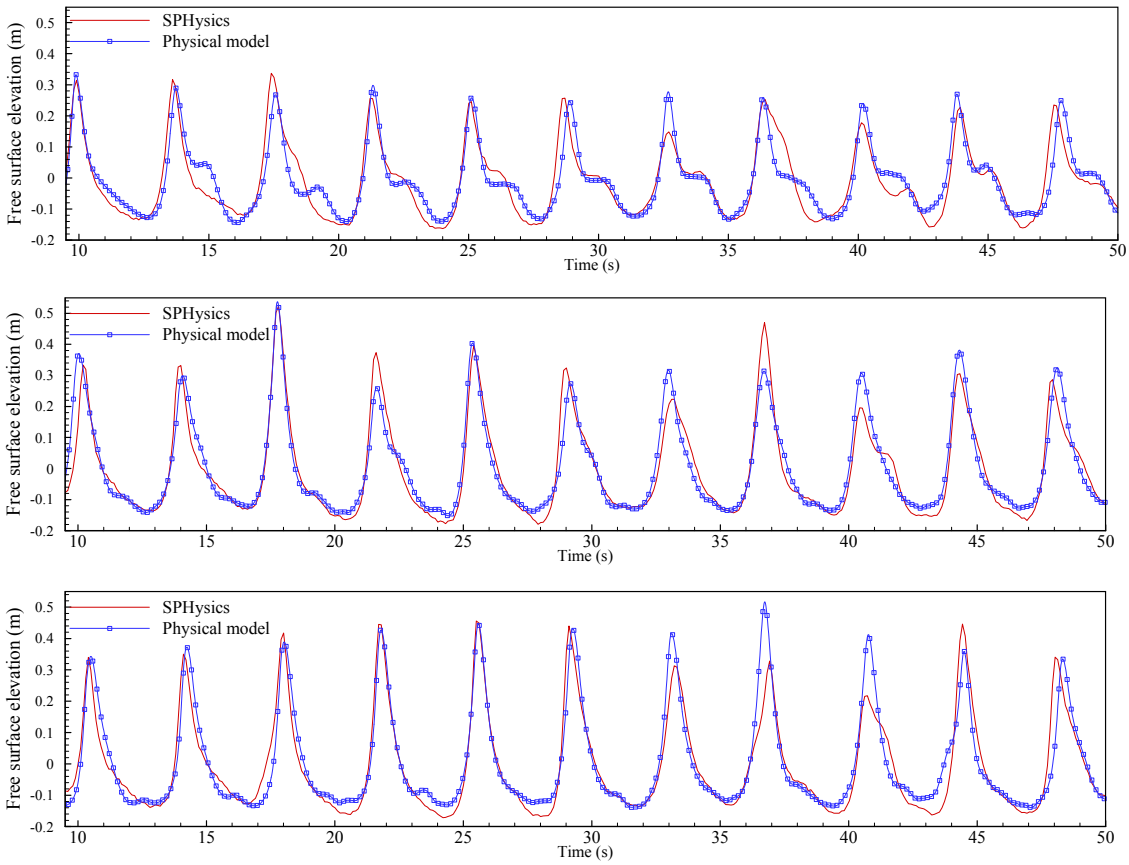

Figure 14: Free surface elevation obtained with SPHysics and experimental data for X=9.5m, 10.5m and 11.0m.

Figure 15 presents a comparison of mean overtopping discharge obtained by the SPHysics and by the physical model. Results obtained using two other numerical codes, COBRAS-UC [5] and AMAZON [29], with a totally different approach for modeling free surface and overtopping, are also present in the Figure 15 [28]. SPHysics results agree well with the range of experimental data, such as results obtained by COBRAS-UC and AMAZON.

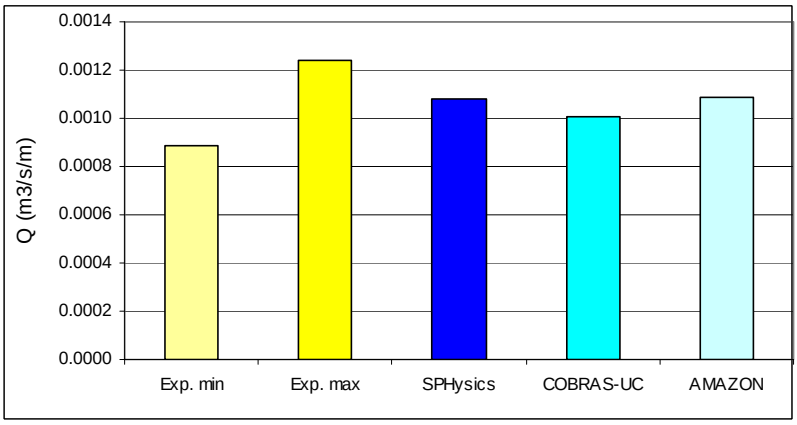

Figure 15: Mean overtopping discharge obtained with physical model, SPHysics, COBRAS-UC and AMAZON.

Finally, figure 16 present, for several instants, the particle position and illustrate the different phenomena modeled in the present simulations: wave propagation, incident-reflected wave interaction, wave breaking, overtopping of the coastal structure, strong splash that can occur between incident and reflected wave.

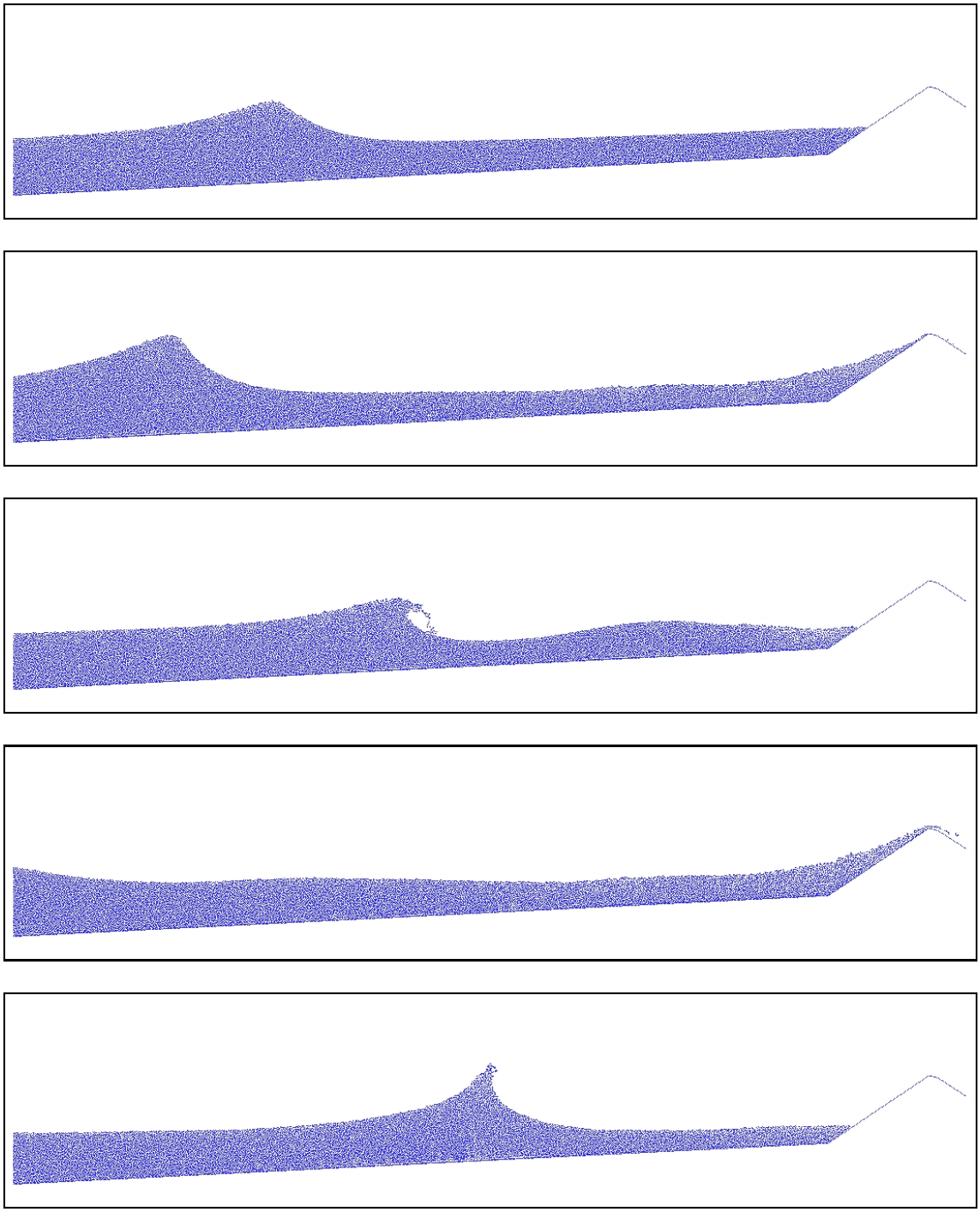

Figure 16: Snapshots of free surface near the sea wall structure.

6 CONCLUSIONS

Smoothed particles hydrodynamics methods, i.e. SPH methods, are an attractive option to model wave-structure interaction, particularly phenomena as wave breaking and wave overtopping that occur in a study of waves reaching a coastal structure, such as seawalls.

The SPHysics code is used and developed at the National Civil Engineering Laboratory (LNEC), Portugal, since 2008 for coastal applications. However, in order to correctly use the SPHysics code, several studies were realized to access its sensibility with some parameters and to analyze convergence with discretization.

The paper presents a convergence study with discretization, i.e. with the dimension of particles, of the free surface elevation and the overtopping discharge for two different seawall crest levels, corresponding to a large and a small overtopping discharge.

It was shown that wave propagation is the phenomenon less sensitive to the discretization. Interaction between incident and reflected waves is more sensitive to the particle dimension. In what concerns to the overtopping discharge, accurate results are only achieved when particle volume is adapted to the intensity of the mean overtopping discharge, i.e. to the overtopping volume per wave. So, in order to obtain accurate results, the particle dimension should be decrease when smaller overtopping discharge volume per wave is simulated. However, increasing the number of particles the CPU time increases drastically.

The numerical results obtained with the particle dimension resulting as the best compromise between accuracy and run time from the convergence study are also compared with experimental data collected at LNEC, consisting of wave propagation, with breaking and wave overtopping of an impermeable seawall. Good agreement is obtained between SPHysics results and physical model data, both in terms of wave amplitude and wave period. Small discrepancies are probably due to differences in the phase between fundamental frequency and harmonics. Finally, SPHysics results of mean overtopping are in good accordance with the experimental data, such as results obtained by COBRAS-UC and AMAZON codes using other approaches.

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