A HYBRID LEVEL–SET–CAHN–HILLIARD MODEL FOR TWO–PHASE FLOW

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ABSTRACT

We present an efficient numerical method for simulation of two-phase flow at micro scales such as pore structures. In such a setting, flow physics is strongly influenced by wetting phenomena in regions where the fluid-fluid interface meets solids. The proposed method combines the computationally simple level set approach with a Cahn–Hilliard-type mechanism for contact line motion [1]. The flow dynamics are governed by the incompressible Navier–Stokes equations, which are augmented by a surface tension forcing term that is proportional to the curvature of the fluid–fluid interface. This implementation accounts for effects of gravity, surface tension, different fluid densities and viscosities.

Our method relies on a smoothed color function for identifying the two fluids. Based on this function, a conservative level set model as in [2] and a phase field model based on the Cahn–Hilliard equations [3] are selectively applied. Away from solids (rigid boundaries), a level set implementation can accurately and efficiently represent the transport of the interface with local fluid speed. A conservative reinitialization procedure ensures that the shape of the discretized smoothed color function is retained under interface motion. The basic level set model however fails in prescribing contact line physics in regions where the interface between the fluids interacts with a solid boundary. The Cahn–Hilliard equation, on the other hand, contains several energy terms and enables the control of contact line behavior based on suitable boundary conditions [3]. In order to avoid the use of this computationally more expensive description everywhere, our hybrid method uses the additional modeling feature only in a region around contact lines. A somewhat wider transition region is used to smoothly set the Cahn–Hilliard specific terms to zero in order not to introduce consistency problems.

The flow equations are discretized with finite elements. We use quadratic elements for velocity and linear elements for pressure as well as for the level set/phase field variables. The combined system is advanced in time with a time-lag scheme that first advances the interface position using an extrapolated velocity, and then updates the Navier–Stokes part based on a BDF-2 time-integration scheme. The model is tested on a few benchmark problems for capillary-dominated flow, as two-phase flow in a cylindrical and a conical channel with large surface tension forces. The results show good performance in comparison with reference results generated using a full phase field simulation at high resolutions.

REFERENCES

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