SHAPE STABILITY OF INCOMPRESSIBLE FLUIDS SUBJECT TO NAVIER'S SLIP

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ABSTRACT

In a number of situations it is convenient to assume that an incompressible fluid can slip at the solid boundary. Mathematical analysis of resulting problems reveals some nice properties which are not available in presence of the widely used no-slip. On the other hand, the slip conditions are more sensitive to perturbations of the boundary, that is, a small imperfection or roughness can lead to large change of the solution.

In this talk we consider power-law fluids governed by the equations

$$\left.\begin{aligned} \partial_t \mathbf{v} + \operatorname{div}(\mathbf{v}\otimes\mathbf{v}) - \operatorname{div}(\mathbb{S}(\mathbb{D}(\mathbf{v}))) + \nabla p = \mathbf{f} \\ \operatorname{div}\mathbf{v} = 0 \end{aligned}\right\} \text{ in } (0,T)\times\Omega, \tag{1a}$$

completed by the Navier slip boundary condition

$$\left.\begin{aligned} \mathbf{v}\cdot\mathbf{n} = 0 \\ \mathbb{T}\mathbf{n}\times\mathbf{n} + a\mathbf{v}\times\mathbf{n} = \mathbf{0} \end{aligned}\right\} \text{ on } (0,T)\times\partial\Omega, \tag{1b}$$

and by the initial condition

$$\mathbf{v}(0,\cdot) = \mathbf{v}_0 \qquad \text{in } \Omega. \tag{1c}$$

Here $\mathbf{v}, p, \mathbb{D}(\mathbf{v}), \mathbf{f}, a, \mathbf{n}$ stands for the velocity, the pressure, the symmetric part of the velocity gradient, the body force, the friction coefficient and the unit outward normal vector, respectively, $\mathbb{T} = -p\mathbb{I} + \mathbb{S}$ is the Cauchy stress and Ω is a bounded domain in $\mathbb{R}^3$.

We will show sufficient conditions for the boundary perturbations under which the slip remains preserved. More precisely, given a sequence $\{(\Omega_n, \mathbf{v}_n, p_n)\}$ of domains and weak solutions of (1) such that Ω_n converge to Ω in a suitable sense, we prove that $(\mathbf{v}_n, p_n)$ converge to a pair $(\mathbf{v}, p)$ which is a weak solution of (1) on Ω. Numerical examples that confirm the obtained result will be presented.