LOCAL PROJECTION STABILIZATION FOR THE NUMERICAL SIMULATION OF CONVECTION DOMINATED FLOWS

Petr Knobloch[*], **Lutz Tobiska**[†]

[*]Charles University, Faculty of Mathematics and Physics
Sokolovská 83, 186 75 Praha 8, Czech Republic
e-mail: knobloch@karlin.mff.cuni.cz

[†]Otto–von–Guericke–Universität Magdeburg, Institut für Analysis und Numerik
Postfach 4120, 39016 Magdeburg, Germany
e-mail: tobiska@mathematik.uni-magdeburg.de

ABSTRACT

We consider the application of the finite element method to the numerical solution of incompressible flow problems with a particular emphasis on dominant convection. It is well known that standard discretization techniques are not appropriate in this case and some stabilization has to be introduced. We apply the local projection stabilization which was proposed by Becker and Braack [1] and has become rather popular during the last decade. A review on its applications to incompressible flow problems can be found in [2].

Local projection stabilizations preserve the stability properties of the popular residual–based stabilizations [3] but do not require the computation of second order derivatives and can be easily applied to non–steady problems. Moreover, they do not lead to undesirable couplings between various variables. A further advantage of these techniques is that they are symmetric. Therefore, if they are applied to optimization problems, the operations 'discretization' and 'optimization' commute.

We present a generalization of the local projection stabilization applied to the Oseen problem which allows to use local projection spaces defined on overlapping sets. This enables to define the local projection method without the need of a mesh refinement or an enrichment of the finite element space and increases the robustness of the local projection method with respect to the choice of the stabilization parameters. The convective stabilization term is slightly modified, which leads to an optimal estimate of the consistency error even if the stabilization parameters scale correctly with respect to convection, diffusion and mesh width. Moreover, we establish an optimal error estimate with respect to a norm which is stronger than the usual local projection norm.

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