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LAGRANGIAN METHODS FOR DETERMINING THE TURBULENT PRANDTL NUMBER IN DNS OF WALL TURBULENCE

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Abstract. *Turbulent Prandtl number (Pr_t) is one of the most important parameters in the modeling of turbulent transport in boundary layer flows or flow through ducts and pipes. Also it is important in obtaining closure for the system of heat transfer equations when eddy viscosity models (such as k-ε models) are used. Previous studies on the Pr_t have revealed a dependence of its behavior on the molecular Prandtl number of the fluid and also on the distance from the wall. Many software packages, however, ignore these effects on the turbulent Prandtl number and use a constant value irrespective of the molecular Prandtl number or the distance from walls. The present study aims at using a Lagrangian approach in conjunction with direct numerical simulation (DNS) to determine eddy viscosity and eddy conductivity in turbulent channel flow with heat transport. A physically sound approach to determining turbulent Prandtl number is to use Churchill's interpretation of eddy viscosity and eddy conductivity. So, eddy viscosity is determined from the ratio of contributions of turbulent and molecular motion to the transport of momentum, while eddy conductivity is obtained from the ratio of contributions of turbulent and molecular motion to transport of heat. A low molecular Prandtl number, Pr = 0.1, and a high, Pr = 100, fluid are simulated for this study.*

1. INTRODUCTION

One of the major theoretical problems in turbulent transport arises in relating scalar fluctuations to fluctuating velocities. Understanding the behavior of turbulent Prandtl number, which provides a link between turbulent heat and momentum transport, can help in providing a better picture of turbulent transport. Turbulent Prandtl number is defined as the ratio of eddy diffusivity of momentum to the eddy diffusivity of heat, as follows:

$$\mathrm{Pr}_t = \frac{\nu_t}{\alpha_t} = \left(\frac{\overline{u'v'}\dfrac{\partial T}{\partial y}}{\overline{T'v'}\dfrac{\partial U}{\partial y}} \right) \tag{1}$$

Since it is not trivial to determine these four quantities experimentally, namely, the turbulent shear stress, velocity gradient, turbulent heat flux and temperature gradient, accurately at a point in the turbulent flow field, it is difficult to calculate the turbulent Prandtl number precisely. With recent advances in large scale computing, direct numerical simulation (DNS) has been carried out to study heat transfer[1,2,3] in turbulent channel flow and recently at very high molecular Prandtl numbers.[4,5,6]

The current study is aimed at exploring the behavior of turbulent Prandtl number in Poiseuille channel flow for different Prandtl numbers and as a function of the distance from the channel wall. Since the approach is the same for any molecular Prandtl number, the study provides a consistent way of determining the turbulent Prandtl number for a range of molecular Prandtl numbers ranging from small to large.

2. BACKGROUND

2.1 Turbulent Prandtl Number Modeling

Osborne Reynolds proposed a simple model for Pr_t, where he assumed that the eddy viscosity is equal to the eddy conductivity, resulting in a value of one for turbulent Prandtl number. This is the famous Reynolds analogy[7], which works well in the turbulent boundary layers. The widely used commercial software for simulating fluid flow, ANSYS FLUENT, uses a constant value of $Pr_t = 0.85$, irrespective of the wall distance or the molecular Prandtl number.[8] Yakhot et al.[9], and Kays & Crawford,[10] among others, presented intuitive and interesting correlations to predicting turbulent Prandtl number for low molecular Prandtl numbers. Each of these works predicts a variation of turbulent Prandtl number depending on the values of the molecular Prandtl number or the distance from channel wall. Although much work has been done to study the turbulent Prandtl number for lower molecular Prandtl numbers, scarce availability of DNS data for higher molecular *Pr,* has restricted the study of effects of higher molecular *Pr*. Works of Schwertfirm & Manhart[5] and Hasegawa & Kasagi[6] have recently studied higher molecular *Pr* effects on Pr_t but only for regions close to the channel walls.

2.2 Physical Interpretation of the Turbulent Prandtl Number using Churchill's turbulent transport model

Churchill[11] developed a theoretical framework, based on turbulence scaling other than the conventional scaling based on *viscous units* (i.e., scaling based on the friction velocity and the

friction temperature) to describe turbulent scalar transport. Within that framework, the turbulent Prandtl number has a quite intuitive physical meaning. Churchill proposed that the local fraction of shear stress due to fluctuations in velocity is a superior dimensionless variable for the modeling of turbulent flows than the friction velocity. This local fraction of the shear stress is defined as

$$(\overline{u'v'})^{++} = -\frac{\rho\overline{u'v'}}{\tau} \tag{2}$$

The analogue of $(\overline{u'v'})^{++}$ for the transport of heat is

$$(\overline{T'v'})^{++} = \frac{\rho c\overline{T'v'}}{j} \tag{3}$$

Where, $(\overline{T'v'})^{++}$ is defined as the local fraction of heat flux density due to the turbulent fluctuations.

Using the above theory, the eddy viscosity to viscosity ratio (ν_t / ν) has to be equal to the ratio of the local fraction of momentum transfer due to turbulent fluctuations, to that due to molecular motions, and, thus, the eddy viscosity obtained a physical meaning that was independent of its diffusive origin. Similarly, he defined the eddy diffusivity over molecular diffusivity ratio to be equal to the local fraction of heat flux density due to the turbulent fluctuations to that of local heat flux density due to molecular motion. In this way, the eddy diffusivity can also be interpreted as a physical quantity that is independent of its heuristic diffusive origin. The Pr_t in terms of the local fraction of fluctuations is expressed as

$$\frac{\mathrm{Pr}_t}{\mathrm{Pr}} = \frac{(\overline{u'v'})^{++}\left(1-(\overline{T'v'})^{++}\right)}{(\overline{T'v'})^{++}\left(1-(\overline{u'v'})^{++}\right)} \tag{4}$$

Equation (4) suggests that Pr_t is a function of molecular Pr, and a function of the distance from the wall, as are the local fractions of transport due to turbulence. The goal of the present paper is to determine the turbulent Prandtl number for different molecular Prandtl numbers and the behavior of the turbulent Prandtl number at different distances from the walls.

3. METHODOLOGY

The psuedospectral DNS algorithm developed by Lyons et al.[12] was used to determine the velocity field. Fluid is considered to incompressible Newtonian with constant physical properties. Periodic boundary conditions are applied in the streamwise, x, and spanwise, z, directions. Reynolds number, Re, for the flow was 10500 based on bulk velocity and hydraulic diameter. The computational box was $(4\pi h, 2h, 2\pi h)$ in the streamwise, wall-normal and spanwise directions, with h = 150 in viscous wall units.

By tracking the trajectories of heat markers in the flow field created by the DNS, the Lagrangian scalar tracking (LST) method was used to generate the mean temperature profiles. The motion of the heat marker was divided into two parts, namely a convection part and a molecular diffusion part. The convection part was calculated from interpolating the fluid velocity at the marker position.[13] For each time step, a three dimensional random walk was imposed on the particle motion to simulate the molecular diffusion effect and this was added to the

convection part of the motion. The values for the random walk were taken from a Gaussian distribution with zero mean and standard deviation, $\sigma = \sqrt{2\Delta t / \mathrm{Pr}}$, for each one of the three space dimensions in viscous wall units. This is based on the Brownian motion theory. Full details of all the Lagrangian runs used here can be found in Le and Papavassiliou[14] and in Mitrovic et al.[15] Additional descriptions and validations for the LST methodology can be found elsewhere[16,17,18].

For determining Pr_t, fluids with Pr of 0.1 and 100 were simulated by releasing 145,161 markers into the flow field. Two sets of simulations were carried out. In the first set of simulation, trajectories of fluid markers (these do not exhibit the Brownian motion at the end of each time step, these are similar to case of $Pr \rightarrow \infty$) were simulated. The fluid markers were released uniformly from the *y*-*z* plane, and their trajectories were stored in every timestep of Δt = 0.125. The channel was then divided in the *y* direction into 600 uniform bins of 0.5 wall unit width each. The number of fluid markers in each of the 600 bins at the initial times t^+ were stored. After a time interval of size Δt = 1, the number of markers that jumped but landed inside the same bin and those that jumped far enough to move out of the bin were counted. If a marker possessed the momentum to go out of a bin, that marker was assumed to have contributed to turbulent momentum transport, while those that stayed within a bin were assumed to have contributed to molecular momentum transport. The ratio of the markers counted in the above way provided the value of the fraction of turbulent to molecular momentum flux. These calculations were repeated at different times in the simulations, in order to calculate 100 more values of this fraction and to get a larger statistical sample of values.

In the second set of simulations, 145,161 heat markers were released from the *x*-*z* plane. The markers moved into the flow field until they were allowed to attain uniform distribution at t^+ = 3000 wall units. The *y* direction was again split into 600 uniform bins of 0.5 wall unit width each, and markers in each of these bins at initial times between t^+ of 3000 and 4000 were monitored. After a time step Δt = 21.5, which is based on the Lagrangian material time scale in the vertical direction for the logarithmic region[19] of turbulent channel flow, the distance traveled by these markers due to convection and that due to molecular diffusion (which is normalized by $Pr^{1/2}$, since the molecular diffusion is calculated based on the standard deviation and depends on $\sqrt{1/\mathrm{Pr}}$), were calculated. If a marker moved out of a bin with convective effects greater than molecular diffusion effects, then the marker was classified as a turbulent marker, while in the case of molecular diffusion dominating the marker motion, it was classified as a molecular marker. The ratio of the turbulent to the molecular markers was assumed to be equal to the ratio of eddy conductivity to thermal conductivity. Using these results the Pr_t was estimated.

4. RESULTS

Simulations were carried out as described in Section 3. The local fraction of shear stress and heat flux density due to turbulence is presented in Figure 1. The local fraction of shear stress due to turbulence increases from zero near the channel wall to value of around 0.65 near the center of the channel. The fraction of normal turbulent heat flux also increases from zero at channel wall to nearly 0.13 and 1 for Pr = 0.1 and Pr = 100, respectively. Thus, for a higher molecular Pr, the contribution of turbulence to normal heat flux is found to be higher, as expected. The figure also shows that the normal turbulent heat flux for a higher molecular Pr increases rapidly to attain a constant value, for Pr = 100 around $y^+ \approx 28$, as compared to a case of lower molecular Pr, for Pr = 0.1, around $y^+ \approx 55$.

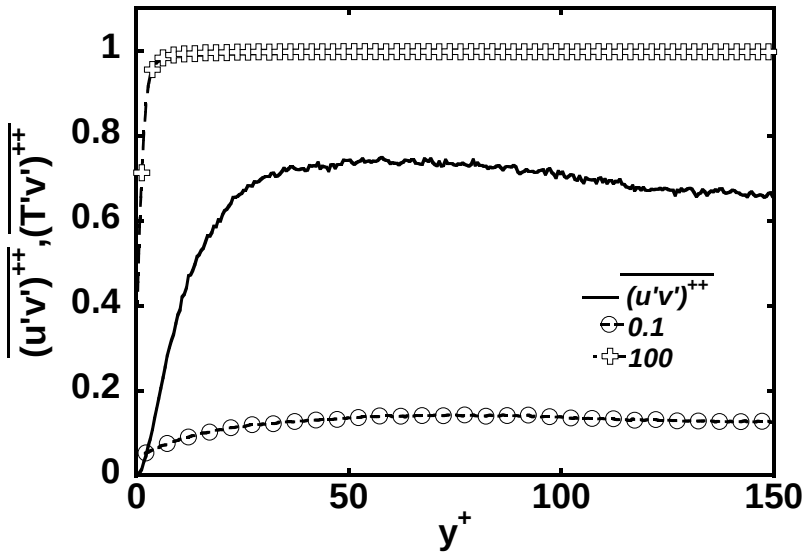

Figure 1: Local fraction of shear stress due to turbulence and normal turbulent heat flux as a function of normal distance from the channel wall.

The Pr_t calculated using the above values of turbulent fraction of shear stress and normal heat flux is shown in figure 2 for molecular Pr of 0.1 and 100. The plot also has data for Pr_t from prior DNS studies by Kawamura et al.[20,21] and Schwertfirm & Manhart[5]. The results obtained herein qualitatively agree with the previously available data. This Lagrangian approach coupled with the new scaling concept of Churchill provides a new way of looking at the turbulent Prandtl number. Also this study confirms that this universal approach to determining the Pr_t is feasible.

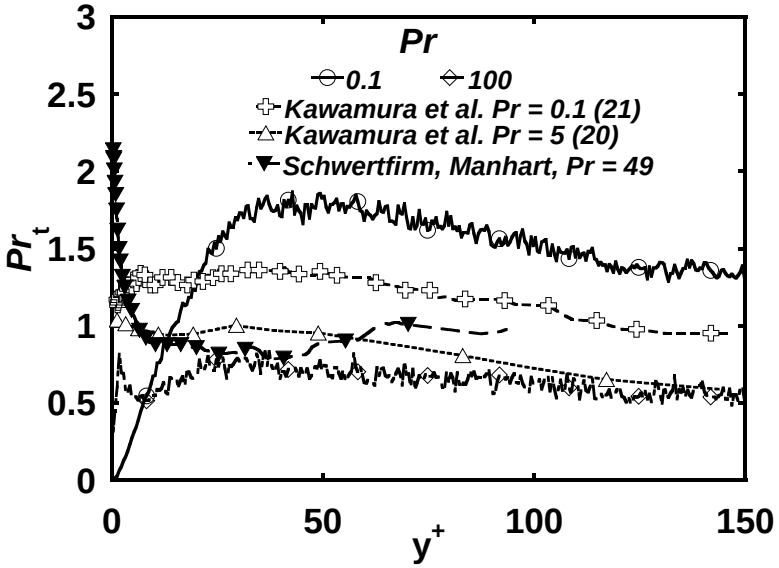

Figure 2: Turbulent Prandtl number as a function of the normal distance from the channel walls for different fluid Prandtl numbers for Poiseuille channel flow.

5. CONCLUSIONS

The turbulent Prandtl number has been determined using a Lagrangian approach and incorporating Churchill's theory of scaling shear stress and heat flux. This Lagrangian method provides a logical and consistent approach to modeling and calculating the turbulent Prandtl number. The local fraction of momentum and heat flux density due to turbulence has been estimated. The heat flux density due to turbulence shows an increase with increasing *Pr*. Since eddy sizes increase as one moves away from the wall, there is an increased contribution of turbulence to shear stress and normal heat flux when moving away from the wall and towards the center of the channel. The Pr_t shows a decrease as the molecular *Pr* increases. The Pr_t is around 1.5 for $Pr = 0.1$, while it decreases to a value around 0.7 for a molecular $Pr = 100$ in the outer region of the channel, where an almost constant value is attained. Comparisons of data obtained for the turbulent *Pr* with prior computational results indicate qualitative agreement. Extension of the DNS/LST methodology to other cases of *Pr* could lead to a correlation between the Pr_t and the parameters that affect it, using a unique and consistent approach.

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