Convergence analysis of a decoupled scheme for poro-elasticity

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ABSTRACT

The modeling of coupled mechanics and flow in porous media is of great importance in a diverse range of engineering fields. Besides the application in soil mechanics, developments in poroelasticity modeling are also contributing to important achievements in civil, petroleum and even biomedical engineering. In this work we consider the numerical solution of a coupled fluid flow and geomechanics in Biot's consolidation model for poroelasticity. The method combines mixed finite elements for Darcy flow and continuous Galerkin finite elements for elasticity. Under the assumption that the total stress change in time is relatively small, we decouple the pressure equation from the equation for the displacement. We summarize a priori convergence estimates for fully coupled schemes ([1], [2]) and for iteratively coupled schemes ([3]), and provide a convergence result for the decoupled scheme. In the convergence analysis a post-processing operator for the flow variable is used. This, together with a duality argument, allows us to prove higher order of convergence for the deformation variable in the L2-norm. We perform numerical experiments for verifying our theory and modeling engineering applications. We give special attention to the well-known Mandel's problem. This problem has been used as a benchmark problem for the reason that it admits an analytical solution in two dimensions on a finite domain. It is therefore very useful to verify the accuracy of discretization schemes and also provides a nice example of the dynamics involved in solid-fluid interactions.

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