A HAMILTONIAN PARTICLE METHOD FOR HYDROSTATIC FLOW IN ISENTROPIC COORDINATES

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ABSTRACT

Although our climate is ultimately driven by (nonuniform) solar heating, many aspects of the flow can be understood qualitatively from forcing-free and frictionless dynamics. In the limit of zero forcing and dissipation, our weather system falls under the realm of *Hamiltonian fluid dynamics* and the flow conserves potential vorticity, energy and phase space structure.

We have found a conservative numerical scheme for an adiabatic and hydrostatic atmosphere. The steps are as follows. First, we derive a mixed Eulerian-Lagrangian Hamiltonian description of the continuum atmosphere based on a so-called *parcel formulation*, [1]. Key point here is that we use isentropic coordinates. The next step is to introduce a finite element discretization in the entropic direction. What remains is a stack of shallow water-type flows that are coupled through an elliptic equation in the entropy coordinate only. Next, we use a *Hamiltonian Particle-Mesh method*, [3], to discretise the horizontal dynamics. This method is based on a discrete version of the parcel formulation and uses a symplectic time integrator. Potential vorticity as well as Hamiltonian phase space structure are exactly conserved. Energy is conserved asymptotically in the time step.

Several challenging nonlinear solutions will be tested.

A successful application of our method can be found in [2], where a simplifying layer-model approximation is used in the entropic direction.

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