Monolithic FEM techniques for nonlinear flow with temperature/concentration, pressure and shear-dependent viscosity

J. Hron*, S. Turek†, H. Damanik†, A. Ouazzi†, P. Pustějovská*

*Mathematical Institute, Charles University
Sokolovská 83, 18675 Prague, Czech Republic
e-mail: hron@karlin.mff.cuni.cz

†Applied Mathematics, TU Dortmund
Vogelpothsweg 87, 44227 Dortmund, Germany

ABSTRACT

We present special numerical simulation methods for non-isothermal incompressible viscous fluids which are based on LBB-stable FEM discretization techniques together with monolithic multigrid solvers. For time discretization, we apply the fully implicit Crank-Nicolson scheme of 2nd order accuracy while we utilize the high order $Q_2\,P_1$ finite element pair for discretization in space which can be applied on general meshes together with local grid refinement strategies including hanging nodes. To treat the nonlinearities in each time step as well as for direct steady approaches, the resulting discrete systems are solved via a Newton method based on divided differences to calculate explicitly the Jacobian matrices. In each nonlinear step, the coupled linear subproblems are solved simultaneously for all quantities by means of a monolithic multigrid method with local multilevel pressure Schur complement smoothers of Vanka type. For validation and evaluation of the presented methodology, we perform the MIT benchmark 2001 of natural convection flow in enclosures to compare our results w.r.t. accuracy and efficiency. Additionally, we simulate problems with temperature and shear dependent viscosity and analyze the effect of an additional dissipation term inside the energy equation. Moreover, we discuss how these FEM multigrid techniques can be extended to monolithic approaches for viscoelastic flow problems and other applications.

REFERENCES

[1] H. Damanik, J. Hron, A. Ouazzi, S. Turek, A monolithic FEM-multigrid solver for non-isothermal incompressible flow on general meshes, *J. Comp. Phys.* **228**, pp. 3869-3881 (2009)