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THE IMMERSED BOUNDARY METHOD APLLIED TO SIMPLIFIED DRILLING PROBLEMS WITH NON-NEWTONIAN FLUIDS

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Abstract. *Flows found in well drilling processes are much more complex than the flows often studied, as Taylor-Couette and Taylor-Couette-Poiseuille flows, because there are other additional characteristics, for instance: eccentric movement determined by the interaction of internal and external flows and fluids with changeable viscosity due the stress rate (non Newtonian fluid). In this context, a project was initiated in order to develop a computational tool to analyze flows associated with the drilling technology in deep waters using the immersed boundary method (IBM) with physic virtual model o represent the fixed and moving channels. This paper presents the first three-dimensional results of application of IBM with PVM methodology to two simplified drilling problems with non-Newtonian fluids: Taylor-Couette flow and eccentric Taylor-Couette flow. Two non-Newtonian models were implemented, the power-law model and the Carreau-Yasuda model. The finite volume method is applied with a staggered Eulerian grid and second order temporal-spatial schemes were used. The governing equations were solved with a fractional time-step method. Initially, the validation was performed for laminar Hagen-Poiseuille flow by comparing with analytical solution.*

1 INTRODUCTION

Flows found in well drilling processes are much more complex than the flows often studied, as Taylor-Couette and Taylor-Couette-Poiseuille flows, because there are other additional characteristics, for instance: eccentric movement determined by the interaction of internal and external flows and fluids with changeable viscosity due the stress rate (non-Newtonian fluid). In this context, a project was initiated in order to develop a computational tool to analyse flows associated with the drilling technology in deep waters using the immersed boundary method (IBM)[1] with physic virtual model (PVM)[2] to represent the solid structures that defines to drilling system.

In referred project, A first step has been completed, which was developed a numerical code for three-dimensional flow analysis of Newtonian, isothermal and single phase using the methodology of large eddy simulation SGE[3] for the treatment of flow transition to turbulence and turbulent, coupled with the methodology immersed boundary MFI, which allows to model and treat numerically the presence of bodies immersed in static and in motion. Initially, was validated the numerical code base[4], in Cartesian coordinates and without IBM; soon, were made applications for simplified flows associated with drilling engineering considering static and moving interface[5]. In these work, Padilla et al. presents the first results of three-dimensional internal flows, using IBM with PVM, which are an application to simplified drilling problems: Taylor-Couette flow, Taylor-Couette spiral flow and eccentric Taylor-Couette flow.

The present work include new implementations, preliminary tests and qualitative validation performed with the purpose of increasing the functionality of computational tool under development, characterized by application of conventional numerical methods combined with relatively new techniques. Continuing the developments in this second stage, new implementations were realized in order to enable the analysis of non-Newtonian flows.

2 MATHEMATICAL FORMULATION AND METHODOLOGY

Precedent developments precedents[4,5] represent the dynamics of Newtonian flows, isothermal and incompressible, through of conservation of mass and balance of momentum equations. In this approach, the Navier-Stokes equations become modified, in order to represent the Newtonian and non-Newtonian flows, then the generalized Newtonian fluid equations is given by:

$$\nabla.\vec{u} = 0 \tag{1}$$

$$\rho\frac{\partial \vec{u}}{\partial t} + \rho\nabla.(\vec{u}\vec{u}) = -\nabla p + \nabla.\left[\eta(\nabla\vec{u} + \nabla\vec{u}^{T})\right] + \vec{f} \tag{2}$$

u and p represent the velocity and pressure fields, ρ and η is the fluid density and the called viscosity function and f is the Eulerian force field. The additional term f of eq. (2), of the immersed boundary method, represent the solid interface present in the flow, which is equivalent to force exercised for the fluid on the interface. Evidently, this interface is virtual and must represent the physical boundary condition for solid surface static or moving. According IBM, the flow field is solved in an Eulerian domain and the virtual interface is solved in a Lagrangean domain, where the coupling of domains is realized with the force field f. The Eulerian force field is obtained through of the distribution of Lagrangean force, which is evaluated using the PVM[2].

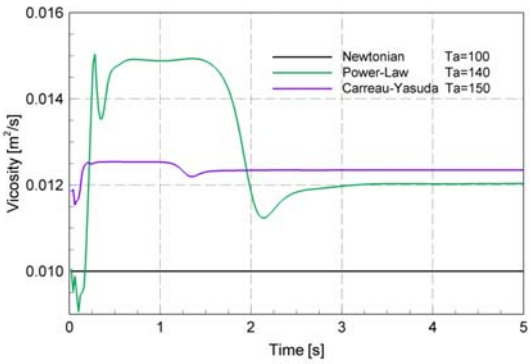
The viscosity function, which depends of the strain rate, should be modeled to close the equations system that represents the problem. Therefore, the power-law[7] and Carreau-Yasuda[8] models are used. The first model evaluates the viscosity with the expression $\eta = k\, S^{n-1}$, where k is consistency coefficient and n is the flow behavior index or power-law exponent. The expression of Carreau-Yasuda model, $\eta - \eta_\infty / (\eta_o - \eta_\infty) = [1 + (\lambda S)^a]^{(n-1)/a}$, depends of more predefined parameters, where η_o is the zero shear rate viscosity, η_∞ is the infinite shear rate viscosity, λ is a parameter with units of time and a is a dimensionless parameter.

The governing equations were discretised following the finite volume method[9] with staggered grid on Cartesian coordinate system and temporal and spatial second order of approximation. The pressure-velocity coupling was realized using the fractional steps method[10], where the fluctuating pressure field was solved iteratively with the Strongly Implicit Procedure[11].

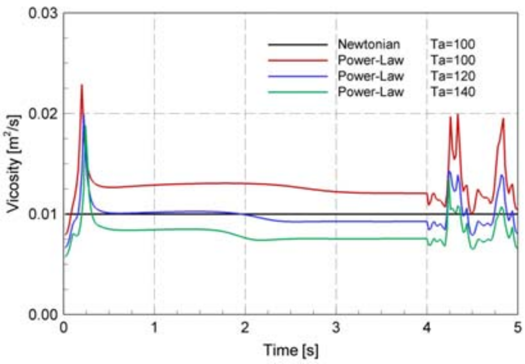
3 RESULTDS

Simulations were performed with the purpose of confirm the correct implementation of the models and for validate the numerical code. Therefore, the Taylor-Couette flows with several configurations have been carried out.

The geometric configuration associated with the problem is defined by the gap, distance between the surfaces of the inner and outer cylindrical channels of radius R_i and R_o and axial length L_a. The inner surface has anticlockwise rotational velocity ω. In eccentric configuration appear the eccentric radius R_{ex} and the eccentric rotational velocity ω_{ex}. The dimensionless parameter these problem, Taylor number, is defined as[12] $Ta = \omega R_i (R_e - R_i)/\eta_c$, where η_c is the characterist viscosity. For simulations, the following definitions were realized: R_i, R_o, L_a, R_{ex}= 0.125, 0.4, 0.4, 0.05 m, ω_{ex} = $4\pi\ s^{-1}$; parameters for the power-law model[13], k=0,221 $Pa.s^n$, n=0,704; parameters for the Carreau-Yasuda model[13,14], $\eta_{o,\infty}$=0,004, 0,0001 $Pa.s$, λ=0,1 s, a=2 e n=0,704; Eulerian grid of 42x42x24 (x, y, z).



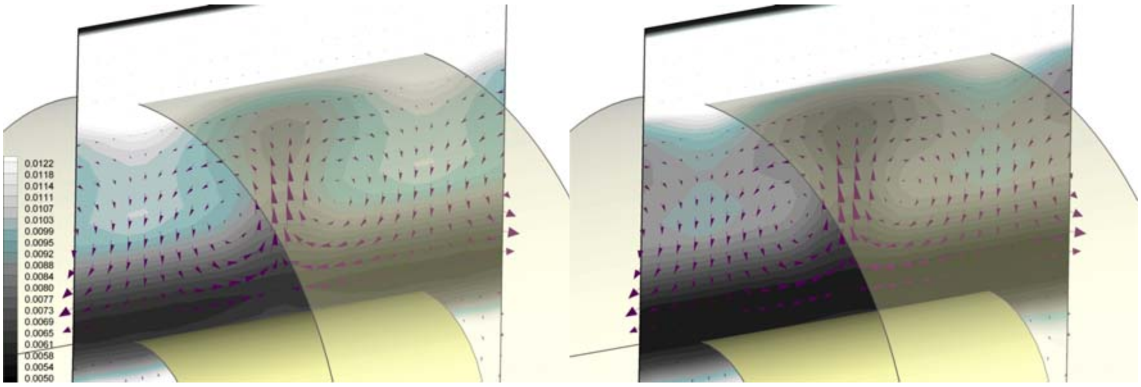
(a)



(b)

Figure 1. Temporal distribution of kinematic viscosity for Newtonian and non-Newtonian flows; (a) concentric configuration, (b) eccentric configuration.

Numerical probes located inside of the domain, as can be seen from Fig. 1, enable to monitor the temporal develop of the flow, for instance through of the kinematic viscosity. In the temporal distribution for concentric configuration (Fig. 1a), there are some important features of the non-Newtonian flows, such as, the Taylor-Couette flow with both models (Power-law and Carreau-Yasuda) reach stable steady just before the 4 s, as well as, that the local variations of viscosity are more intense with power-law model. On the other hand, the temporal distribution for the eccentric configuration (Fig. 1b, with power-law model) presents variations of the viscosity to reach regime with fixed eccentricity (in the position of Fig. 4a), for three values of Ta, from 4 s the rapid changes correspond to the two complete cycles of circular motion with variable eccentricity around the axis of the outer channel.



(a) (b)

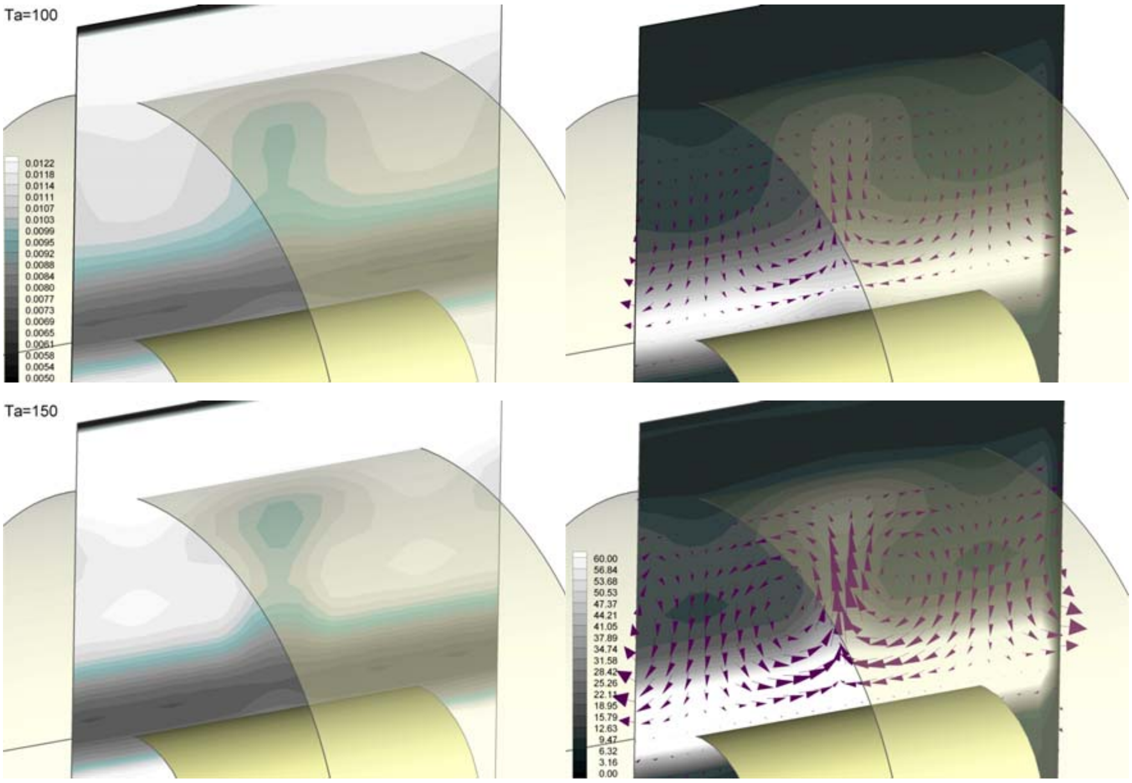
Figure 2. Vector velocities field and kinematic viscosity, power-law model and concentric configuration; (a) Ta =120, (b) Ta =140.

All simulated cases, using concentric configuration, revealed the presence of Taylor vortices, showing fields of velocity components similar to the Newtonian flows[5,15]. The Taylor vortices appear in stable regime, which as the Ta increases become more intense (higher velocity vectors, as observed in Fig. 2) and the vortex centers are displaced to approach more two adjacent vortex pairs, that are counter- rotating, but also to distance from near the inner surface is shorter. When compared with results from Newtonian flow, Table 1, the Taylor vortices show always shorter distances between their centers, as well as, between the vortex centers and the surface of the inner channel. The localization of the vortex centers as function of Ta, for Newtonian flow, is influenced basically by inertia effects, already in non-Newtonian flow is influenced by inertia effects and effects arising from the variable viscosity, as shown in Figs. 3 and 4.

Ta =100	Left vortice		Right vortice	
	$r\,[m]$	$z\,[m]$	$r\,[m]$	$z\,[m]$
Newtonian	0,259	0,220	0,259	0,386
Carreau-Yasuda	0,253	0,225	0,253	0,379
Power-law	0,247	0,229	0,247	0,376

Table 1. Vortex centers localization for Newtonian and non-Newtonian flows.

Figure 2 shows the pseudoplastic flow for Ta = 120 and 140, where the kinematic viscosity decreases with increasing strain rate, characteristic behavior when n is less than unity (one can notice same behavior for Carreau-Yasuda model, see Fig. 3). Distribution of viscosity has smaller values from near the surface of inner channel and higher near the outer surface, in intermediate region there is the influence of radial flow between the vortices. It is this distribution that explains the displacement of the vortex centers in relation to the vortices in Newtonian flow.



(a) (b)

Figure 3. Flows in concentric configurations, Carreau-Yasuda model; (a) kinematic viscosity, (b) velocity vectors and strain rate.

Ta	Left vortice		Right vórtice	
	$r\,[\,m\,]$	$z\,[\,m\,]$	$r\,[\,m\,]$	$z\,[\,m\,]$
100	0,253	0,225	0,253	0,379
120	0,255	0,229	0,255	0,375
150	0,259	0,231	0,259	0,373

Table 2. Vortex centers localization for non-Newtonian flows, using Carreau-Yasuda model.

The Taylor vortices predict with Carreau-Yasuda model, as Ta increases, maintains the same performance observed as when used for power-law model. The vortex centers ex flow of Carreau-Yasuda models (Table 2) have an intermediate position between the localization of the vortex centers for Newtonian flows and non-Newtonian flows with power-law model (Table 1), because to differences in magnitude of the kinematic viscosity and shear rate fields, as observed in Fig. 3.

An illustrative temporal sequence of the results with Carreau-Yasuda model is shown in Fig. 4, for eccentric configuration, where the instantaneous velocity iso-surfaces, that represent the counter-rotating structures, deform along a tangential direction as function of the space annular reduction, effects of inertia and the influence of non-Newtonian propierties.

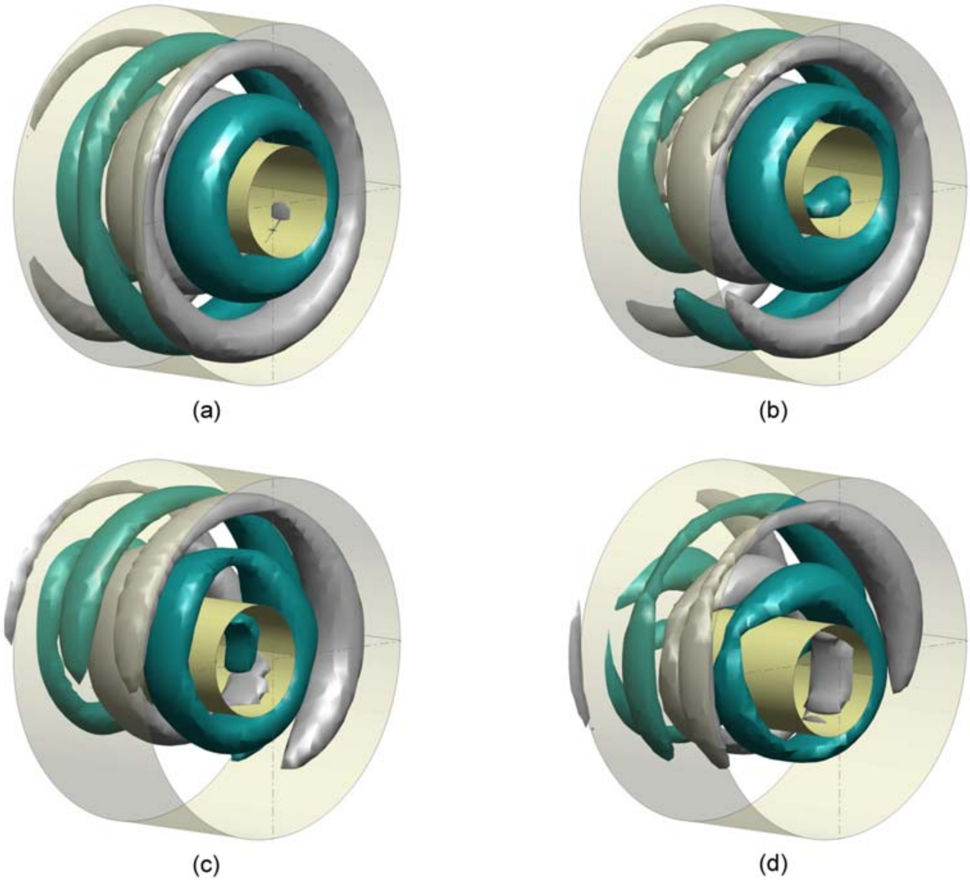


Figure 4. Instantaneous axial velocity iso-surfaces for Ta =100, power-law model; (a) 4 s, (b) 4.1 s, (c) 4.25 s, (d) 4.35 s.

4 CONCLUSIONS

Were present results of non-Newtonian flows simulations inside concentric and eccentric annular channels using power-law and Carreau-Yasuda models. The results of all the considered flows showed the presence of Taylor vortices, which characterizes such as Taylor-Couette flows. Thus, the results are physically consistent. The viscosity and shear rate field, among other features of the flows, agree with references. Therefore, the numerical code can be considered qualitatively validated. On the other hand, confirms the ability of IBM for represent stationary and moving interfaces.

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