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MAY TRANSIENT GROWTH THEORY EXPLAIN ISOLATED ROUGHNESS INDUCED TRANSITION ?

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Abstract. *In many practical situations, laminar-turbulent transition occurs at lower Reynolds numbers than those predicted by the classical linear stability theory. This process, called transient growth, is observed when the boundary layer is subjected to intense external disturbances such as high free-stream turbulence level or large three-dimensional surface imperfections. In such cases, perturbations in the laminar region take the form of streamwise elongated structures, commonly named streaks or Klebanoff modes, which may be strongly amplified resulting in an early "Bypass" transition. In this paper, a semi empirical model has been developed for the prediction of three-dimensional roughness induced transition. It is based on the transient growth theory and coupled with a "Bypass" transition criterion. This model has first been validated through a comparison with experimental data for a tripped transition on an ONERA airfoil. In a second step, the model has been confronted to experimental correlations : in particular, compressibility effects such as Mach number and wall temperature influences have been investigated.*

1 Introduction

Many experiments have demonstrated that roughness elements could have a deep impact on the laminar-turbulent transition process. Many surface imperfections (such as rivets, insects ...) are unavoidable. So it is necessary to estimate their effects on the stability of the boundary layer. Roughness elements have a dual-influence on the flow. First, they modify the mean flow and have an effect on its stability properties. Secondly, they can introduce small perturbations inside the boundary layer : they act on the receptivity process which is a key stage for stability studies. It has been shown that two-dimensional surface imperfections can be analysed with the linear stability theory : the transition location moves upstream towards the surface default for increasing height of steps or gaps. Physically, two-dimensional defaults strongly amplify the unstable TS waves [3]. Numerically, this problem is solved increasing the value of the N factor by a quantity ΔN depending on the height of the surface imperfection [14, 28].

Nonetheless for three-dimensional protuberances, the transition mechanism seems to be different. Reshotko highlights the fact that attempts to find a TS explanation for three-dimensional roughness, both discrete and distributed, have failed [15]. Indeed, contrary to two-dimensional steps for which an height increase leads to an earlier transition breakdown, for three-dimensional roughness elements, there is a critical height below which the transition location keeps unchanged. When this critical value is exceeded the transition is triggered at an abscissa close to the roughness element location. Van Driest and Blumer [22] studied the effect of three-dimensional roughness elements (spheres) on boundary layer transition on cones. They deduced a relationship for the effective height corresponding to the transition tripping depending on the displacement thickness and on the external Mach number. In the same way, von Doenhoff and Braslow [6, 8] have correlated experimental data and showed that transition occurred when the Reynolds number based on the roughness height R_{kk} reached a threshold : this value depending on the ratio between the diameter and the height of the roughness. Acarlar and Smith [1] have shown that an isolated roughness element induces an horseshoe vortex whose legs consist in two counter-rotating stationary vortices. Optimal perturbation theory [2, 12, 18] has pointed out that the most amplified perturbations inside both incompressible and compressible boundary layer were also created by stationary streamwise vortices. It is the "Lift-up effect" introduced by Landahl [11] : a vortex superimposed to the boundary layer shear pushes up low speed particles from the wall to the top of the boundary layer, and pulls down high speed particles towards the wall leading to a spanwise alternation of backward and streamwise jet streaky structures called the Klebanoff modes [10]. Klebanoff modes can undergo a 'transient growth' process meaning that the amplitude of the streaks can be heavily amplified. If the energy of the Klebanoff modes significantly raises, an early laminar turbulent transition can be triggered : this is the so-called Bypass, a term introduced by Morkovin [13], meaning that the natural transition process, driven by the TS waves, has been short-circuited. Transient growth is an attractive mechanism to consider

with respect to roughness induced transition [15].
As a matter of fact, Fransson et *al.* [9] have observed a transient amplification of the longitudinal velocity fluctuation behind an array of roughness elements. The streamwise velocity fluctuation clearly showed a transient amplification for the fundamental mode corresponding to the roughness array periodicity. Fransson also noted that the streaks induced by the roughness were suboptimal ones. This may be due to the fact that vortices don't match with the ones predicted by optimal perturbation theory and are closer to the wall. White et *al.* [25, 27] performed many experiments of roughness induced transient growth. They found out that the higher harmonics $\lambda_0/3$ and $\lambda_0/4$ have transient amplification just aft the element and that the amplification of the fundamental spanwise wavenumber perturbation started downstream. A striking characteristic is that Fransson obtained a positive u'-fluctuation in the centerline of the roughness whereas White observed a negative one. The discrepancy has been explained by Tumin and Reshotko [20] : using bi-orthogonal decomposition, they showed that behind a hump, there was a deficit velocity region, called 'wake region', surrounded by the legs of the horseshoe vortex : if the magnitude of the vortices is high enough the wake region may be filled in and cancelled by the wash motion induced by the vortices and the sign of the u'-fluctuation may be switched over. We can think that the larger the roughness is, the more extended the wake region would be. So, the topology of the flow behind a roughness default is not only linked to the height but also depends on the diameter (or in the same way on the shape) of the roughness element.
This paper aims at presenting a model, based on transient growth theory, to compute the amplification of Klebanoff modes behind a three-dimensional roughness element. This model is coupled with a criterion in order to predict the early roughness induced transition. In the first section the numerical model will be introduced. Then it will be calibrated and compared to experimental data. The influence of compressibility, wall temperature and Mach number, effects on the roughness induced transition will be then analysed comparing our numerical results to existing empirical correlations.

2 Description of the Model

2.1 Equations

If the amplitude of the streaks is too high, their influence on the mean flow cannot be neglected. A natural approach consists in applying the boundary layer turbulent equations to describe the laminar zone, even though the fully turbulent state has not been reached yet. Total quantities (velocities, pressure and temperature) are split into a mean part and a fluctuating one :

$$Q = \tilde{Q} + q' \tag{1}$$

In order to make the equation dimensionless, typical length and velocity scales are introduced. We know that subcritical roughness induce streaky streamwise elongated structures : therefore, a typical scale of the geometry L (for instance the length of the flat plate or the chord of the wing) is used to normalize the streamwise coordinate. In the wall normal direction, a typical boundary layer thickness $\delta = L/\sqrt{Re_L}$ is applied to characterize the diffusion process. The continuity equation, satisfied by the Klebanoff modes implies that if u' is in order of U_∞, $v' = O(U_\infty/\sqrt{Re_L})$. These scales were introduced by Prandtl to study laminar boundary layers. These assumptions for two-dimensional incompressible boundary layers lead to the following system of equations :

$$\tilde{U}_x + \tilde{V}_y = 0 \tag{2a}$$

$$\tilde{U}\tilde{U}_x + \tilde{V}\tilde{V}_y = U_e U_{e_x} + \nu \tilde{U}_{yy} + \overline{u'u'}_x + \overline{u'v'}_y \tag{2b}$$

$$u'_t + \left(\tilde{U}u'\right)_x + \tilde{V}u'_y + v'\tilde{U}_y = \nu\left(u'_{yy} + u'_{zz}\right) \tag{2c}$$

The terms $\overline{u'u'}_x$ and $\overline{u'v'}_y$ describe the influence of the streaks on the mean flow. The Klebanoff modes consist in u'-fluctuations and verify the equation (2c). For compressible flows, two similar equations, one for the mean temperature and one for its fluctuation, can be developed. In order to close the system (2) in the laminar region, the wall normal velocity fluctuation v' has to be modelled. This aims at expressing the "Lift-up" effect in accordance with the fact that a wall normal velocity perturbation in a shear flow brings about the emergence and the amplification of streamwise velocity fluctuations. Owing to the Prandtl scales, all dimensionless equations are parabolic in nature. So if the wall normal velocity is modelled, u' can be computed by a marching numerical procedure using the values of the mean flow at the previous upstream station. So the averaged equation (2b) can be solved taking into account the terms which describe the influence of the streaks on the mean flow. The key stage now, is to model the wall normal velocity disturbance induced by the roughness element.

2.2 Wall normal velocity fluctuation modelling

2.2.1 Fransson experiment

Fransson et *al.* [9] performed streaks hot wire measurements behind an array of cylinders. Experiments were performed at four different free stream velocities $U_\infty = 5, 6, 7$ and 8 [m/s] in order to control the amplitude of the streaks changing the ratio k/δ which varied between 2.24 and 2.84. Hot wire measurements have clearly shown that the streamwise velocity fluctuation inside the boundary layer underwent a transient amplification as illustrated by the symbols of the figure 1(a). In this figure, Fransson et *al.* have used a non dimensional X coordinate, given by relation (3), and found out an universal streaks behaviour.

$$X = \left(\frac{\beta*}{\beta_{opt}}\right)^2 \cdot \nu \cdot \frac{1}{U_{\inf}} \cdot x \tag{3}$$

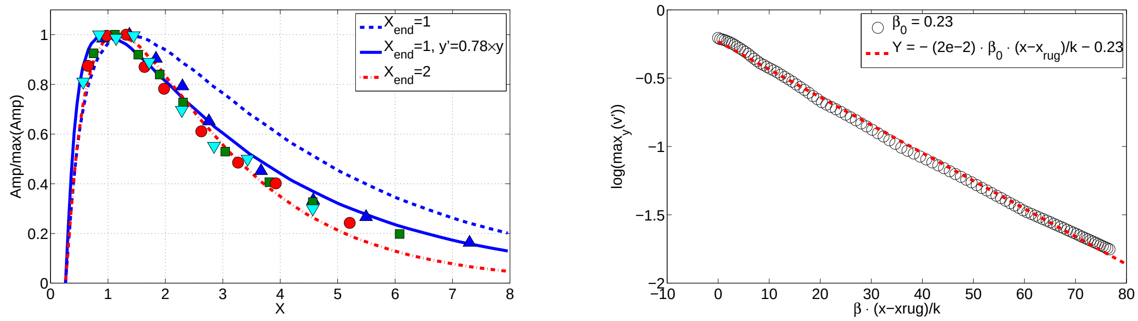
The relation implies that at $X = 1$, the spanwise wavenumber matches with the optimal one $\beta = \beta_{opt} = 0.45$ [2, 12]. They computed the optimal disturbance which corresponded with the most amplified streak at $X_f = 1$; the corresponding curve is the dashed line on figure 1(a) : discrepancies between experiment and optimal perturbation theory (OPT) have been justified by the fact that the cylinders should induce sub-optimal initial disturbances closer to the wall. Therefore, Fransson et *al.* computed an artificial initial perturbation by compressing the wall normal coordinate y by a factor c : for $c = 0.78$, *ie.* an initial disturbance closer to the wall, the numerical result is in good agreement with the measurements (solid line on figure 1(a)).

Here we propose another approach : we have calculated the initial disturbance which maximises the amplitude of induced streaks at the station $X_f = 2$. This also provides a good agreement with experimental data as illustrated by the dash-dotted line on figure 1(a). The evolution of the corresponding wall normal velocity fluctuation is presented on figure 1(b) : v'-fluctuation amplitude is decaying exponentially in the streamwise direction.

2.2.2 v' profile

To model the wall normal fluctuating velocity profile in the boundary layer thickness we will use the same function as the one proposed by Biau [5] for the transient growth induced by free-stream turbulence. This function is continuously increasing from the wall to the boundary layer edge, and may not match with the wall normal velocity fluctuation profile really induced by roughness elements. Nonetheless, the goal is to impose a small disturbance in the wall normal direction able to create a lift-up effect and to lead to the formation of Klebanoff modes.

$$g(y) = \begin{cases} \dfrac{y^2 e^{-(2\pi/\delta_{99})\cdot y}}{\max_y \left|y^2 e^{-(2\pi/\delta_{99})\cdot y}\right|} & \text{if} \quad y \leq \delta_{99} \\ \text{cst} & \text{if} \quad y > \delta_{99} \end{cases}$$



(a) Evolution of the streaks amplitude. Comparison between experimental data [9](symbols) and Optimal Perturbation Theory (curves).

(b) Evolution of v' amplitude

Figure 1: Calibration on the Fransson experiment [9]

Previous works concerning the modelling of transient growth induced by free-stream turbulence have been performed [5, 24] : in this context, the amplitude of v' had been naturally chosen proportional to the free-stream turbulence. For roughness induced transient growth, we suppose that the amplitude of the streaks is linked to the roughness height. The question is to know if this dependence is linear or not. Reshotko and Tumin [16] have recovered the evolution of the transition Reynolds number induced by surface defaults in ballistic range experiment, assuming that the fluctuations were directly proportional to the height of the roughness. The present authors have in a first attempt use this assumption to model the transition amplification of streaks induced by a protuberance [23].

On the contrary, White and Ergin [26] have shown that the amplitude of the modes $\lambda_0/3$ and $\lambda_0/4$ was proportional to the Reynolds number based on the height of the roughness : $v' \propto R_{kk} \propto k^2$. Direct numerical simulations, performed by Choudhari [7], have confirmed these results for a Reynolds number range between $75 < R_{kk} < 250$. In the present paper we assume that the amplitude depends on the Reynolds number R_{kk}. Besides, optimal perturbation theory has revealed that v' was decaying exponentially (see figure 1(b)). Therefore, we propose the following relationship for v' :

$$\frac{v'}{U_e} = A_1 \cdot R_{kk} \cdot e^{\left(-A_2 \cdot \beta_0 \cdot \frac{x - x_0}{k}\right)} \cdot g(y) \tag{4}$$

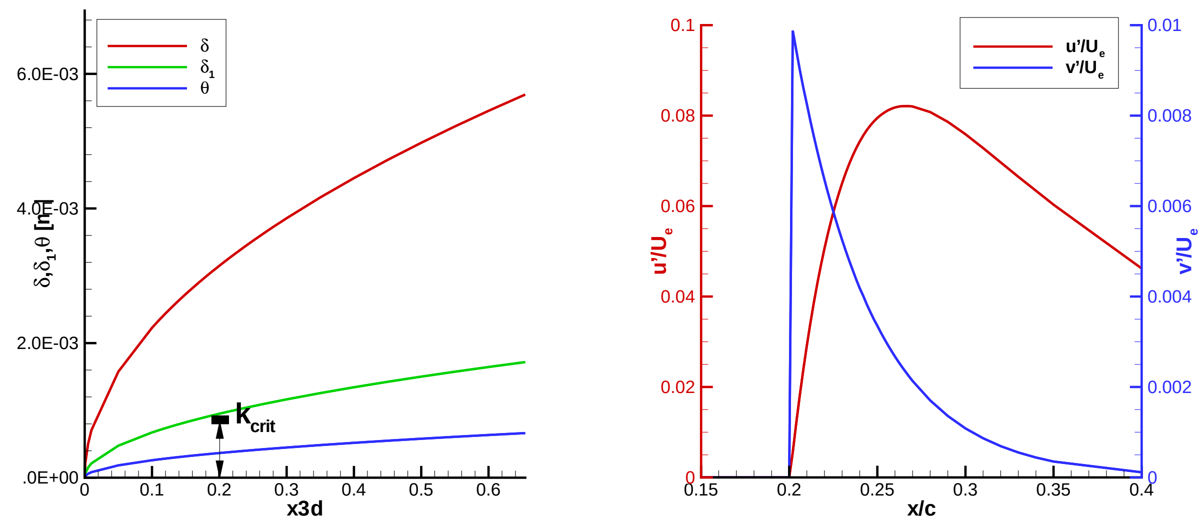
The constant A_2 is given by the optimal theory and corresponds to the slope of the v' evolution on figure 1(b) : $A_2 = 2 \cdot 10^{-3}$. The exponential formulation is a function of $\beta_0 = 2\pi/\lambda_0$: physically, this means that for high spanwise wavenumber, *ie.* $\lambda_0 \ll 1$, which corresponds to tightened vortices, the longitudinal dissipation will be higher. On the contrary, extended vortices will tend to propagate further downstream inside the boundary layer. The value of A_1 is fixed by calibration on a reference experimental case, which will be described in the following section 3.2.

2.3 Bypass transition Criterion

Determination of the transition location is important to determine the boundary layer properties. Indeed, it determines the streamwise length of the region where the boundary layer remains laminar and fixes the starting point of the turbulent area which develops downstream. The computation of the transition location is usually based on a criterion *ie.* a quantity resulting from the laminar boundary layer computation is compared to a threshold value. Van Driest and Blumer [21] proposed a criterion based on the local vorticity Reynolds number considering that the transition was triggered when the ratio of the local inertial stress to local viscous shear reached a limiting value. Adapting their approach to the transient growth phenomenon leads to the following relationship :

$$\max_{\forall y} \left| \frac{-\rho \cdot \overline{u'v'}}{\mu \frac{\partial U}{\partial y}} \right| = C \tag{5}$$

From a physical point of view, this relationship expresses the fact that transition occurs only when the ratio between the driving term of streak formation $\overline{u'v'}$ and the dissipative one $(\nu \times \partial U / \partial y)$ reaches a certain value. This criterion has been calibrated and successfully applied to predict Bypass transition for boundary layers subjected to significant free stream turbulence (FST) level [5, 24]. Even though the receptivity process induced by surface defaults is different from the FST one, we keep the same transition threshold $C = 0.65$.



(a) Boundary layer thicknesses. The critical roughness height is represented by the arrow.

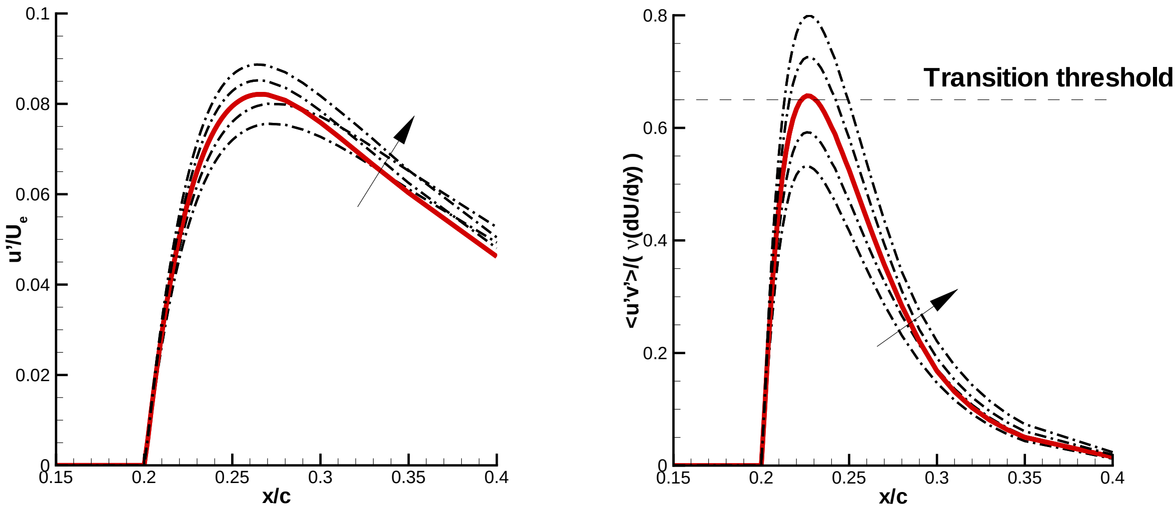
(b) Evolution of the amplitude of the streamwise u' and normal v' velocity fluctuations.

Figure 2: Boundary layer thicknesses and spatial evolution of the amplitude of the fluctuations. $U_\infty = 10$ [m/s]. The roughness element is located at $x_0 = 0.2$ [m] and the diameter is $d = 5$ [mm].

3 Numerical results

3.1 Incompressible flow on a flat plate

The first computation has been carried out for a transition induced by a roughness element located on a flat plate submitted to an incompressible flow. The protuberance is located at $x_0 = 0.2$ [m], has a diameter of $d = 5$ [mm] and the free-stream velocity is fixed to $U_\infty = 10$ [m/s]. The height of the roughness element is progressively increased up to the critical value for which the transition criterion (5) is verified. For this configuration the critical height is $k_{crit} = 860$ [μm] *ie.* is the order of magnitude of the displacement thickness as represented on figure 2(a). The fluctuations of the normal and streamwise velocities are represented on figure 2(b). We can see that our modelling induces a maximum for the v' fluctuation (given by the relationship (4)) at the roughness location. Then v' is exponentially decreasing but forces by "Lift-up" effect, numerically represented by equation (2c), the emergence and the transient amplification of u' (red line on figure 2(b)). The longitudinal velocity fluctuation reaches a maximum around 8% of the external velocity at $x \approx 0.28$ [m]. Besides, the maximum value of u' is by a factor 10 higher than the amplitude of v'. On the figures 3, the amplitude of the Klebanoff modes and the corresponding value of the criterion for various roughness heights are represented. The criterion $\overline{u'v'}/(\nu dU/dy) = 0.65$ is verified for $k_{crit} = 860$ [m] as illustrated on figure 3(b). The figure 3(a) also demonstrates that the u' amplification is restricted to a limited re-



(a) Evolution of the amplitude of Klebanoff modes u' as a function of the roughness height

(b) Evolution of the criteria

Figure 3: Spatial evolution of the streamwise velocity and the criteria as a function of the roughness height. The red line corresponds to the critical height : $k_{crit} = 860$ [μm]. The arrow indicates increasing k from 820 [μm] to 900 [μm] by 20 [μm] step.

gion just downstream of the protuberance before the viscous decay occurs. The distance where the criterion is increasing is even more restricted (figure 3(b)). Thus if the Bypass transition has to happen, it will be triggered close to the roughness location.

3.2 Comparison with experimental data

A set of experiments on both natural and tripped transition has been performed by Seraudie et *al.* [17] on the upper side of an Onera-D airfoil. The geometry of the profile is plotted on figure 4 with the corresponding velocity distribution for a zero angle of attack and a free-stream velocity $U_\infty = 35$ [m/s]. The upper side of the profile is covered by an heating skin to modify the wall temperature from the adiabatic temperature $T_{ad} = 300$ [K] to $T_w = 340$ [K]. The transition position is determined using infra red thermography technique. Measurements demonstrate that in the case of natural transition (smooth surface) a wall heating has a strong destabilizing effect moving up the transition location towards the leading edge as illustrated by the two pictures of figure 5(a). The tripping of the transition is made using glass bead with diameter of $d = 0.2$ or 0.3 [mm] placed at 10% or 20% of chord from the leading edge : the corresponding locations are specified by the vertical arrows of the figure 4. For each configuration, two measurements have been performed corresponding to the two distinct wall temperatures $T_w = T_{ad} = 300$ [K] and $T_w = 340$ [K]. The test section velocity is increased up to determine the critical velocity

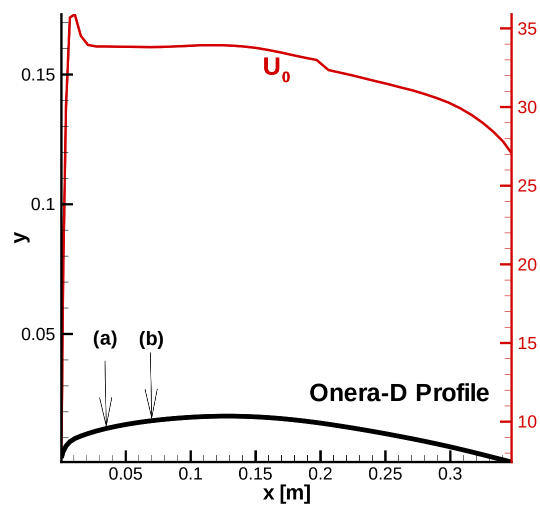


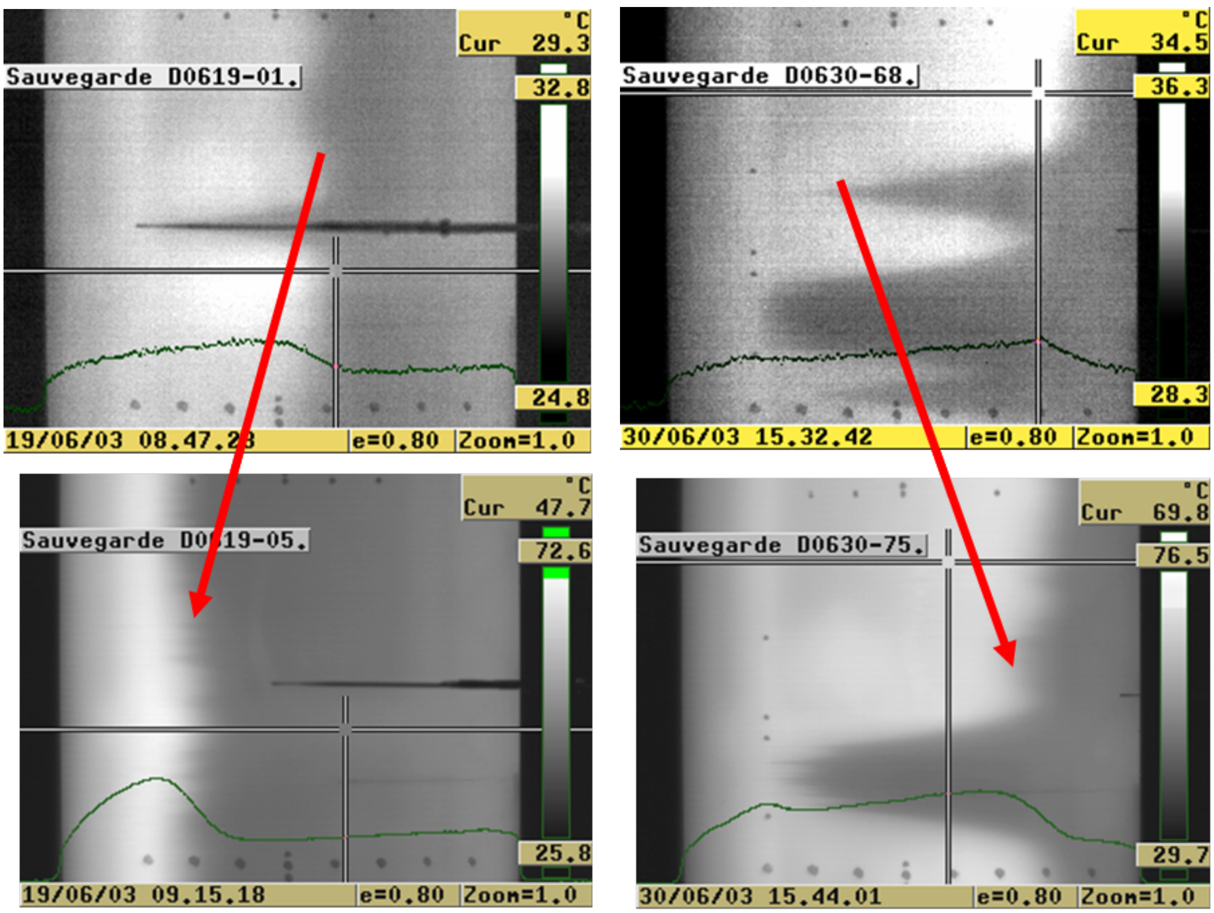
Figure 4: Upper side of the Onera-D profile and velocity distribution for zero angle of attack

corresponding to the transition onset : the experimental accuracy concerning the critical velocity is around $\Delta U_0 \approx 2.5$ [m/s].

The v' modelling, *ie.* the constant $A_1 = 3.6 \times 10^{-5}$, has been calibrated taking as reference the tripping velocity for a 0.2 [mm] bead placed at 10% chord for an adiabatic wall. This reference configuration is pointed out by the arrow on figure 6(a). For the highest height $k = 0.3$ [mm], the numerical critical velocity is over evaluated compared to the experiment. The simulation recovers the moderate stabilizing effect of a wall heating : for $T_w = 340$ [K], numerical results correspond to the green line and are consistent with the measurements. Since heating destabilizes TS instabilities, this stabilizing effect, illustrated on figure 5(b), is a characteristic feature of Bypass transition triggered by Klebanoff modes [4, 19] and provides some evidence for a possible transient growth explanation for roughness induced transition. When the glass spheres are moved away from the leading edge, up to $x_0 = 0.07$ [m], the tripping velocities are higher than for the case $x_0 = 0.035$ [m]. This is due to the fact that when the bead moves downstream, the ratio k/δ becomes weaker and the boundary layer *"sees"* a smaller roughness element : the external velocity has to be increased to trigger transition. For the tallest bead, $k = 0.3$[mm], the numerical modelling still over estimates the critical velocity. Nonetheless, the order of magnitude of U_0, in the light of the measurement accuracy, keeps correct.

3.3 Comparison with experimental correlation

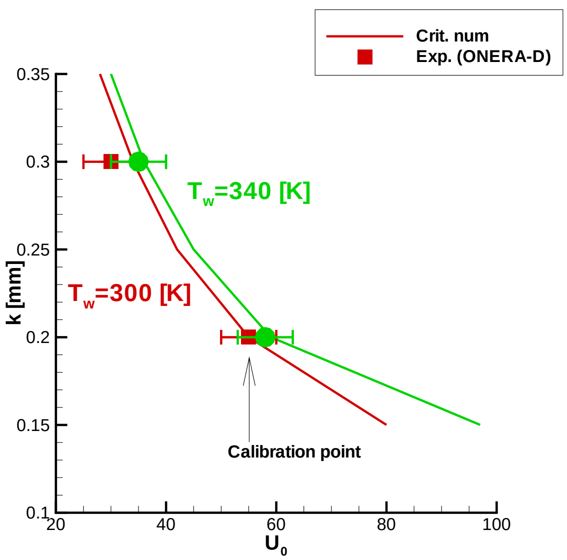
In the past many experiments have been performed dealing with transition induced by three-dimensional isolated surface imperfections. Van Driest and Blumer [22] studied the effect of spheres placed on a 10^o angle cone at a supersonic velocity $M = 2.71$. Using



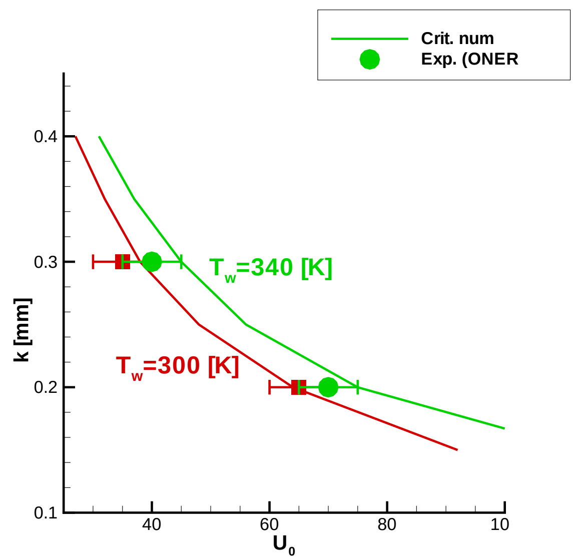
(a) **Natural** transition position.

(b) **Roughness tripped** transition position.

Figure 5: Thermography visualisations of the transition position. The top pictures correspond to a wall temperature $T_w = 300$ [K] whereas the bottom ones to $T_w = 340$ [K]. The red arrows represent the transition position evolution due to a wall heating.



(a) Critical velocity as a function of the roughness height. $x_0 = 0.035$ [m].



(b) Critical velocity as a function of the roughness height. $x_0 = 0.07$ [m].

Figure 6: Triggering of the transition by roughness elements on the Onera-D airfoil. Roughness elements are located at $x_0 = 0.035$ [m] and 0.07 [m]. Measurements have been performed for two distinct wall temperatures $T_w = 300$ [K] (red) and $T_w = 340$ [K] (green). The lines correspond to the numerical results and the symbols to the measurements [17].

data obtained for other Mach numbers, they established a relationship for the critical roughness height given by :

$$\frac{k}{\delta_1} = 32.4 \times \sqrt{1 + 0.2 \times M_e^2} \times \left(\frac{U_e \cdot \delta_1}{\nu_e}\right)^{-1/2} \tag{6}$$

where δ_1 and M_e are the displacement thickness and the external Mach number at the roughness location. Van Driest and Blumer's correlation has been deduced for spherical surface defaults *ie.* roughness elements characterized by a specific ratio between the height k and the diameter d around 1.

Von Doenhoff and Braslow [6, 8] have gathered flight experimental data and have shown that the Reynolds number R_{kk}, given by the relation (7), was a relevant parameter for the roughness induced transition.

$$R_{kk} = \frac{\rho_k \cdot U_k \cdot k}{\mu_k} \tag{7}$$

U_k, ρ_k and μ_k respectively represent the velocity, the density and the viscosity for the undisturbed boundary layer at $y = k$. Experiments have shown that the critical value of the Reynolds number corresponding to the transition onset depended on the ratio d/k. Data gathered and used by von Doenhoff and Braslow are plotted on figure 9(a) and defined the hatched area of figure 9(b).

First of all, we have studied the influence of the roughness location on the critical height. This computation has been carried out for a flat plate with $U_e = 10$ [m/s] for three wall temperatures : $T_w = 193$, 293 and 393 [K]. Our numerical results are represented by the symbols on the figure 7 and the solid lines correspond to the van Driest and Blumer criteria (6). As expected, for a fixed wall temperature, the critical height increases when the sphere is moved away from the leading edge. Moreover, we recover the fact that a wall cooling ($T_w = 193$ [K]) has a destabilizing effect : the transition is triggered for smaller roughness elements. When the temperature is raised compared to the adiabatic temperature, the height of the surface default has to be increased to trigger transition. Whatever the wall temperature, the present model gives coherent results compared to the van Driest and Blumer correlation.

The effect of Mach number has then been analysed. The results are plotted on figure 8(a) where the red line represents the van Driest and Blumer criteria and the symbols the numerical results. All the results are obtained for fixed total pressure and temperature : $P_i = 10^5$ [Pa] and $T_i = 333$ [K]. The roughness element is located at $x_0 = 0.2$ [m]. First of all, in the subsonic range, when the velocity is increased from $M = 0$ to $M = 0.5$, the critical height rapidly decreases. From $M = 0.5$ to $M = 1$, the critical height is still decreasing but the slope is weak ; past $M = 1$, the critical height starts increasing.

On figure 8(b), the critical height of the roughness elements has been plotted as a function of the wall temperature for two subsonic Mach numbers $M = 0.1$ and $M = 0.5$.

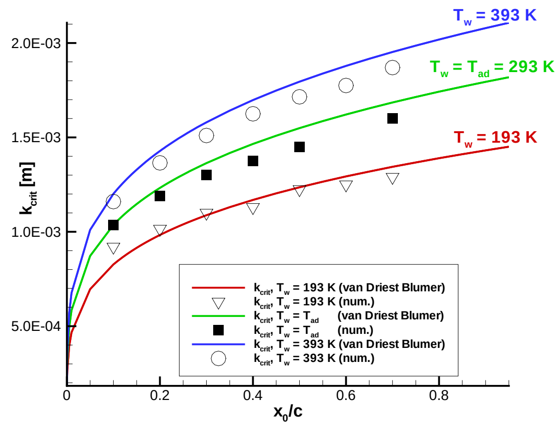
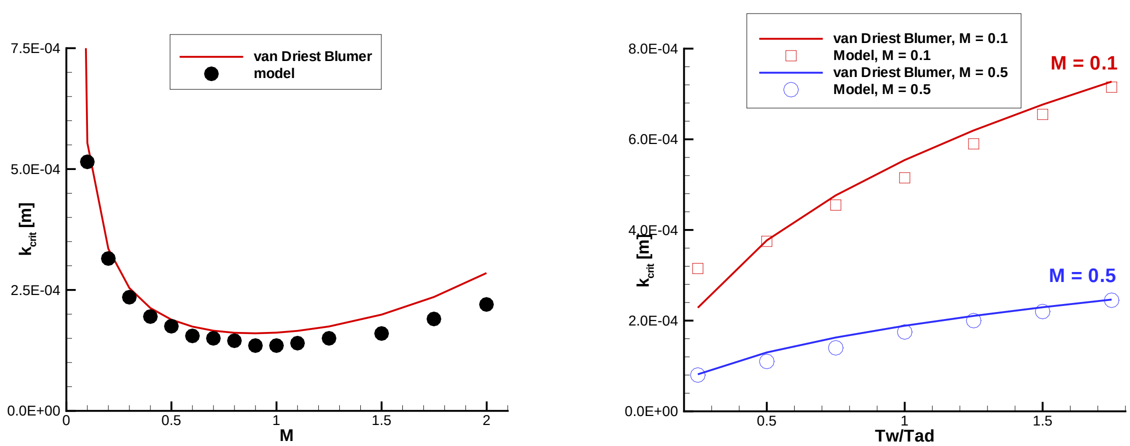


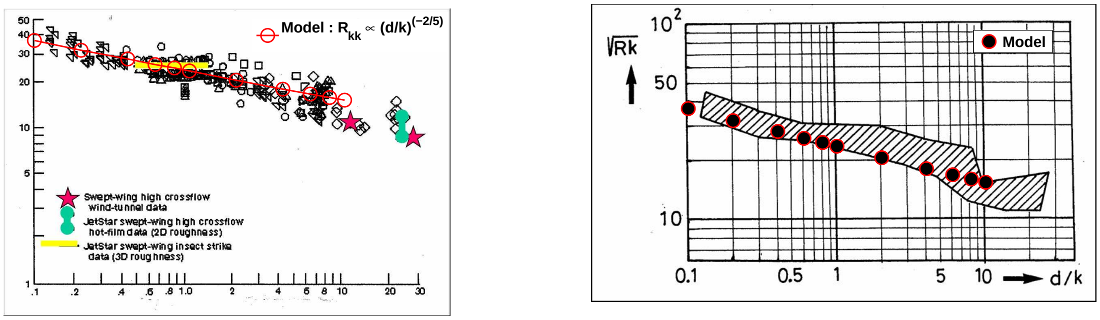
Figure 7: Critical height of three-dimensional roughness element as a function of the roughness location for three wall temperatures : $T_w = 193$, 293 and 393 [K].



(a) k_{crit} as a function of the Mach number

(b) k_{crit} as a function of the wall temperature

Figure 8: Compressibility effects (Mach number and wall temperature) on the critical height of three-dimensional roughness element



(a) experimental data of three-dimensional roughness induced transition

(b) Von Doenhoff and Braslow correlation

Figure 9: Critical value of R_{kk} corresponding to the transition onset as a function of the ratio d/k

For the two Mach numbers, the stabilizing (respectively destabilizing) effect of a wall heating (cooling) is recovered. We can note a small discrepancy between the numerical result and the van Driest and Blumer correlation for $M = 0.1$ and $T_w/T_{ad} = 0.25$. In the same way as the figure 8(a) in the subsonic region, we can see that increasing the Mach number from 0.1 to 0.5 has a destabilizing effect. Considering transition induced by spherical (*ie.* $d/k \approx 1$), the present model provides coherent results in close agreement with the van Driest and Blumer correlation.

The influence of the specific aspect d/k has then been analysed and compared to the von Doenhoff and Braslow criteria. The academic case of a flat plate is still considered. The diameter of the roughness element is fixed to $d = 1$ [mm] and the critical velocity is searched as a function of the ratio d/k. The corresponding Reynolds numbers R_{kk} provided by the model have been plotted on figure 9. These Reynolds numbers were obtained for $x_0/c = 0.1$. One more time, the numerical values of the Reynolds numbers corresponding to the transition onset match with von Doenhoff and Braslow correlation : in particular, the evolution of the critical Reynolds number $R_{kk} \propto (d/k)^{(-2/5)}$ is recovered as illustrated on figure 9(a).

4 Conclusion

In this paper, the early transition induced by isolated three-dimensional roughness elements has been investigated. A model, dedicated to boundary layers developing on walls characterized by surface imperfections has been introduced. This model is based on transient growth theory *ie.* on the production of streaks (u' fluctuations) induced by a wall normal velocity fluctuation (v') according to the "Lift-up" effect. The wall normal velocity fluctuation has been modelled assuming that the receptivity process was non-linear. A Bypass transition criterion has then been introduced in order to predict the boundary layer breakdown induced by three-dimensional roughness elements. This modelling provides results in close agreement with the van Driest and Blumer correlation and demonstrates a coherent sensitivity to free-stream Mach number. Moreover, the stabilizing effect of wall heating is well recovered compared both to van Driest and Blumer's correlation and measurements. Studying the influence of the specific ratio d/k, the numerical critical Reynolds numbers based on the height of the roughness R_{kk} are in the range provided by the von Doenhoff and Braslow criteria. Therefore, transient amplification of Klebanoff modes appears, at least partly, as a convenient explanation for the early transition due to three-dimensional surface imperfections.

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