On the stability of numerical schemes modeling non-Newtonian fluids

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ABSTRACT

We study various numerical schemes for differential models of non-newtonian fluids, like the viscoelastic polymer fluids. Our motivation is to give some tracks to understand at least some of the numerical instabilities that have been observed for decades by rheologists in simulations, which are generically known as High-Weissenberg Number Problems. Our starting point among many differential viscoelastic models is the prototypical Oldroyd-B model, for which a *free energy estimation* holds [3]. The free energy is indeed the right positive quantity that allows to control the long-time behaviour of solutions to a Cauchy problem for the Oldroyd-B model [4]. We will show under which assumptions a free energy estimate is also satisfied by discretizations of the Oldroyd-B model [1,2]. Some numerical observations obtained with different schemes will support our viewpoint that it may be important for the stability of numerical schemes to satisfy a discrete free energy estimation.

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