SCALABLE, NONLINEAR, IMPLICIT ALGORITHMS FOR EXTENDED MAGNETOHYDRODYNAMICS

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ABSTRACT

The extended magnetohydrodynamics (XMHD) model is a two-fluid description of quasineutral, fully ionized gases (plasmas) in the presence of electromagnetic fields. Physically, XMHD is well-known to support very disparate time and length scales. When discretized, XMHD results in extremely ill-conditioned algebraic systems. As a result, and due to their inherent numerical stability (CFL) constraints, explicit methods are unsuitable to model long-frequency XMHD phenomena.

It follows that temporal implicitness is needed to improve efficiency. Partially implicit approaches (in which only some of the terms in XMHD are made implicit) have been effective efficiency-wise, but they suffer from accuracy degradation over long time spans due to the numerical approximations involved [1]. Fully implicit methods, on the other hand, hold the promise of both efficiency and accuracy, as they ensure numerical consistency at all times while being free of numerical instabilities [1].

A fully implicit implementation of XMHD is non-trivial, however, as a very large, sparse, and stiff nonlinear algebraic system needs to be inverted. Furthermore, it is desirable to do this scalably in parallel and optimally algorithmically (i.e, with work increasing linearly with the number of degrees of freedom). With the advent of modern nonlinear solvers, this goal is now within reach. The focus in this presentation is on Newton-Krylov methods (NK), which can be implemented Jacobian-free (i.e., without forming or storing the Jacobian to proceed), and they can be preconditioned [2].

Proper preconditioning of NK is key for efficiency. Preconditioning allows one to exploit simpler solver strategies to accelerate the convergence of the overall nonlinear iteration. Furthermore, approximations in the preconditioner do not affect the quality of the converged solution. A particularly promising approach is the use of multigrid methods (MG) in the preconditioner, as they can deliver algorithm optimality [2]. However, XMHD is strongly hyperbolic, and thus unsuitable for a naive MG treatment.

In this talk, we describe our progress over the last few years towards a fully implicit, scalable solver for XMHD based on classical multigrid strategies. At the core of our approach is the reformulation of the original XMHD model in a way that yields several smaller block-diagonally dominant systems, which are amenable to a multigrid treatment. Numerical results will demonstrate the excellent behavior (both algorithmic and in parallel) of our approach in 2D [3,4] and 3D XMHD [5,6] examples.

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