GENERIC AND EFFICIENT IMPLEMENTATION TECHNIQUES FOR HIGH ORDER FINITE ELEMENTS

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ABSTRACT

The author is concerned with the development of the finite element software Netgen/NGSolve. It is open source under the LGPL license, and available from http://sourceforge.net/projects/ngsolve.

Netgen is an automatic mesh generator. It can import geometric models in various formats including IGES and Step. NGSolve is a finite element library providing elements, integrators for bilinear-forms and linear-forms, and linear algebra. Key features of NGSolve are elements of arbitrary and variable order for continuous, hybrid-discontinuous, and vector-valued finite elements. NGSolve is a general purpose library, and specific application classes such as flow solvers or electromagnetic field solvers are added by means of dynamic libraries.

The focus of this talk is on efficient techniques for high order elements. In the standard procedure for element matrix integration in 3D, the computational costs are $O(p^9)$. By sum-factorization techniques, i.e. utilizing tensor product structure of the integration points as well as of the basis functions, one can reduce the costs to $O(p^7)$. Similarly, one can reduce a matrix-free operator application from $O(p^6)$ to $O(p^4)$ operations. These techniques are quite straight forward for hexahedral elements, but more advanced for simplicial finite elements using the Dubiner basis.

In this talk we give a representation of tensor product basis functions for all common shapes of elements, for scalar and vectorial elements which allow a generic implementation of matrix assembling and operator application. Experiments show a speed-up of more than 10 for moderate polynomial orders of 5 to 10.