AN EFFICIENT HIGH-ORDER IMPLICIT ALGORITHM FOR 3D MAGNETOHYDRODYNAMIC STUDIES OF STRONGLY MAGNETIZED PLASMAS USING C^1 FINITE ELEMENTS

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ABSTRACT

We describe the problem formulation and some initial results from the M3D-C^1 code [1,2] applied to magnetic fusion devices in toroidal geometry. The set of scalar variables and projection operators employed for the vector momentum and magnetic field evolution equations has several unique and desirable properties, making it a preferred system for solving the magneto-hydrodynamics equations in a torus with a strong toroidal magnetic field. The “weak form” of these equations explicitly conserves energy, making it suitable for a Galerkin finite element formulation provided the basis elements have C^1 continuity. The full system admits two energy conserving reduced systems of equations that substantially reduce computational requirement. An implicit (split) time advance is presented that implements the method of differential approximation [3] by adding diagonally dominant self-adjoint energy terms to the mass matrix to obtain numerical stability and provide rapid convergence in the iterative solution. We illustrate the algorithm by calculating two-dimensional steady-states of a resistive two-fluid plasma, self-consistently including flows and anisotropic viscosity (including gyroviscosity) for diverted plasmas in geometries typical of the National Spherical Torus Experiment (NSTX) [4]. A linear three-dimensional capability is described that calculates unstable linear eigenmodes of an arbitrary two-dimensional equilibrium, including flow. Extensive benchmarking results are presented showing that the growth rates calculated by M3D-C^1 in the ideal limit agree with those calculated by ideal MHD codes. Extension of this code to fully nonlinear three-dimensional problems uses Hermite cubic finite elements in the toroidal direction. The sparse matrix equation to be solved each time-step assumes a block-tridiagonal structure that is exploited in the pre-conditioner.

References

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