CODE AND SOLUTION VERIFICATION IN CFD: EXAMPLES FOR RANS SOLVERS

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ABSTRACT

Computational Fluid Dynamics (CFD) has established itself as a valuable tool to analyze flow problems of various kind encountered in practical engineering. In the early years of CFD, the ability to obtain a numerical solution for a complex problem was already an achievement. However, with the growing responsibility of numerical solutions in engineering decisions, the credibility of the simulations must be established with Verification and Validation [1].

Verification and Validation have different goals [1]:

- Verification is a purely mathematical exercise focused on numerical and coding errors. As simply defined by Roache [1], it guarantees that we are *solving the equations right*.
- Validation is a science/engineering activity meant to show that the selected mathematical model is a good representation of the “reality”. In Roache’s words [1]: it checks if we are *solving the right equations*.

In this presentation we will focus only on the mathematical side of the problem, i.e. Verification. In fact, Verification is composed of two different activities:

1. Code Verification, intending to demonstrate the correctness of the code that contains the algorithm to solve a given mathematical model. It requires error evaluation. Therefore, an exact solution must be available.
2. Solution Verification, aiming at estimating the error/uncertainty of a given numerical solution, for which, in general, the exact solution is unknown.

Verification is basically concerned with numerical errors. Therefore, it is fundamental to know what are the different contributions to the numerical error and how can they be evaluated/estimated. This is exactly the focus of the first part of the presentation: round-off, iterative and discretization errors.

In many practical flow problems at high Reynolds numbers, the Reynolds Averaged Navier-Stokes (RANS) equations are still the only viable choice. Therefore, we will discuss Code and Solution Verification in RANS solvers. However, it is obvious that most of the problems discussed in this presentation are also relevant for other areas of Computational Mechanics.

The evaluation of errors required by Code Verification implies that the exact solution must be available. There are no analytical solutions for the RANS equations. However, this does not mean that Code Verification is an impossible task! The Method of the Manufactured Solutions (MMS) [2, 3] provides a framework to perform Code Verification of any numerical solver of systems of partial differential equations. The idea is very simple: construct a solution, i.e. specify all unknowns (including turbulence models) by selected mathematical functions; pass these functions through your equation system and find analytically for each equation the source term required to remove any imbalance; run your code with these source terms activated with suitable boundary conditions extracted from the constructed solution. The better your grid resolution, the closer you should then be able to reproduce the manufactured solution.

In principle, there are no restrictions for the construction of the exact solutions. However, as we discuss in the second part of the presentation, it is convenient to follow some basic rules in creating Manufactured Solutions. In particular, for eddy-viscosity turbulence models [4, 5] it is best to keep the proposed solutions close to physical realism to avoid unnecessary numerical problems.

Solution Verification intends to estimate the numerical error/uncertainty of a numerical solution, for which the exact solution is usually unknown. Unfortunately, a reliable error estimator for any level of grid refinement and complexity of the governing equations is not available in the open literature. Most of the existing methods [6, 7, 8] require data in the so-called "asymptotic range", i.e. a single dominant term in a power series expansion of the error. This means levels of grid refinement that are not normally used in practical applications [9]. In the last part of the presentation, we present a procedure to estimate discretization uncertainties based on grid refinement studies[10, 11] and several examples of its application. We also demonstrate that, when judging numerical solutions, misleading conclusions may easily be drawn if nothing is known about the numerical uncertainty.

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