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STUDY ON ACCURACY OF EIGENPAIR BY THE HIERARCHICAL COMPONENT MODE SYNTHESIS FOR LARGE SCALE EIGEN ANALYSIS OF STRUCTURES

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Abstract. *We have proposed a component mode synthesis (CMS) for structural eigen analysis using both eigenmodes of single-component structures and those of two-component structures in the synthesis. The algorithm is applied to the case where the model to be analyzed is divided into components in one direction. Using the basic algorithm hierarchically, the proposed CMS can be extended to the case where the model to be analyzed is divided into components in two directions. Likewise by applying the algorithm to three steps, it is also possible to expand it into three-direction division. We have shown that the proposed algorithm can be applied to the eigen analysis of structures with large scale such as ten million degrees of freedom (DOFs) and of general configuration.*

In this paper, we investigate the accuracy of the eigenpair obtained by the proposed CMS and hierarchical CMS (HCMS) in detail. By using appropriate number of eigenmodes in the synthesis, we could obtain eigenpairs with an extremely high accuracy. By increasing the size of the components, the eigenpairs were obtained in higher accuracy using less number of eigenmodes in the sysnthesis. There was little deterioration in accuracy of eigenpairs by multiplying the number of components. However, if the size of the components is decreased without decreasing the interface DOFs, the accuracy of eigenpairs deteriorated. The accuracy of eigenpairs obtained by the HCMS was also satisfactory. Thus, the proposed CMS and HCMS are proved to be appropriate for very large scale eigen analysis of general configuration.

1 INTRODUCTION

The computer performance improvement in the past decade has been dramatic. Besides, the solution technique of large linear systems has largely advanced and those with more than one million DOFs can be now solved in practical cost and computation time. As a result, large-scale analyses of structures or fluids which are concluded to the solution of linear systems are not uncommon nowadays [1][2][3][4].

The determination of eigenfrequencies and the associated eigenmodes of structures is acquiring importance in many of the industrial fields [5]. However, such analyses, which are concluded to the solution of eigenvalue problems, are not common if the analysis scale is extremely large even now. This is due to the difficulty of the solution of large eigenvalue problems. The well-known and effective solution algorithms based on the Krylov subspace such as Lanczos method require several solutions of linear systems of large scale.

Contrary to the above, eigen analysis by CMS [6][7][8][9][10][11] is believed to be quite effective in executing eigen analysis of a large scale owing to the fact that solutions of large eigenvalue problems or large linear systems are not required. However, most of the present algorithms depend on the reduction of the DOFs by the Guyan's static reduction [12]. As a result, applying those algorithms to extremely large-scale problems creates difficulties as follows: (1) As the scale of the analysis increases, the number of computations increases very rapidly. (2) Similarly, memory size increases very rapidly. (3) The accuracy of the eigenfrequencies to be obtained is, in general, not so good.

The authors have proposed a CMS using only eigenmodes of components in the synthesis [13][14][15][16][17]. By avoiding the Guyan's static reduction, the proposed algorithm turned out to be quite appropriate for extremely large-scale eigen analysis. The new improved method is excellent in comparison with the conventional method in the following points: (1) the increase in the number of computations relative to the increase in the scale of the analysis is small. (2) Similarly, increase in the required memory size is small. To the best advantage, accuracy of the solutions obtained is improved. In addition, program is made easier and it is appropriate for parallel computation. Furthermore, using the proposed CMS hierarchically, structures of general configuration can be analyzed.

In this paper, we investigate the accuracy of the eigenpair obtained by the proposed CMS and HCMS in several cases. The accuracy of the eigenpairs using different number of eigenmodes, different number of components, different size of components and different number of interface DOFs is investigated in detail.

Accordingly this paper is composed of the four chapters as follows: In chapter 2, the algorithms of the proposed CMS and HCMS are introduced. In chapter 3, the accuracy of the eigenpairs obtained by the proposed algorithm is investigated. In chapter 4, conclusions are shown.

2 ALGORITHM

2.1 Basic algorithm

At first, we start with the generalized eigenvalue problem

$$\mathbf{Kx} = \lambda\mathbf{Mx} \tag{1}$$

where **K** and **M** are the stiffness and mass matrices, respectively, **x** is the eigenvector or the displacement vector and λ is the eigenvalue or the frequency parameter. The basic approach of the CMS is to provide a subspace **S** and approximate the eigenvectors in the subspace. The subspace **S** consists of eigenmodes of components and/or some other vectors which approximate the eigenvector partially. Hence, the solution process is first to provide the subspace **S**, and then project the eigenvalue problem on the subspace as,

$$\mathbf{S}^{\mathrm{T}}\mathbf{KS}\boldsymbol{f} = \tilde{\lambda}\mathbf{S}^{\mathrm{T}}\mathbf{MS}\boldsymbol{f} \tag{2}$$

and at last to solve the projected eigenvalue problem. The eigenvalue $\tilde{\lambda}$ of the projected eigenvalue problem is the approximation of the eigenvalue λ of the original problem and the product $\boldsymbol{f}$**S** approximates the eigenvector **x** of the original problem.

Next, we introduce the proposed CMS using the model described in Fig. 1. The model is a bar divided into several number of finite elements, are divided into four components. The components are designated by A, B, C, and D. Furthermore, let the DOFs of the non-interface of A, B, C, and D be expressed by the suffix 1, 3, 5, and 7, respectively. In addition, let the degrees of freedom of the interface among A-B, B-C, and C-D be expressed by the suffixes 2, 4, and 6, respectively.

We compose the subspace by an appropriate number of lowest eigenmodes of each single-component structure and each two-component structure. Here, single-component structure means a structure of a component and two-component structure means a structure of which two adjacent components connected to each other. In concrete, we use the eigenmodes of 1) structure A with interface A-B fixed, 2) structure B with interface A-B and B-C fixed, 3) structure C with interface B-C and C-D fixed, 4) structure D with interface C-D fixed, 5) structure A+B with interface B-C fixed, 6) structure B+C with interface A-B and C+D fixed and 7) structure C+D with interface B-C fixed, in the synthesis. Suppose $\boldsymbol{\phi}_1$, $\boldsymbol{\phi}_3$, $\boldsymbol{\phi}_5$ and $\boldsymbol{\phi}_7$ presents the eigenmodes of structures A, B, C and D, respectively. Suppose the eigemodes of structure A+B be presented as,

$$\begin{bmatrix} \boldsymbol{\phi}_1^{(AB)} \\ \boldsymbol{\phi}_2^{(AB)} \\ \boldsymbol{\phi}_3^{(AB)} \end{bmatrix} \tag{3}$$

the eigemodes of structure B+C be presented as,

$$\begin{bmatrix} \phi_3^{(BC)} \\ \phi_4^{(BC)} \\ \phi_5^{(BC)} \end{bmatrix} \tag{4}$$

and the eigemodes of structure C+D be presented as,

$$\begin{bmatrix} \phi_5^{(CD)} \\ \phi_6^{(CD)} \\ \phi_7^{(CD)} \end{bmatrix} \tag{5}$$

Using these expressions the subspace $\mathbf{S}_1$ is given as,

$$\mathbf{S}_1 = \begin{bmatrix} \phi_1 & 0 & 0 & 0 & \phi_1^{(AB)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \phi_2^{(AB)} & 0 & 0 \\ 0 & \phi_3 & 0 & 0 & \phi_3^{(AB)} & \phi_3^{(BC)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \phi_4^{(BC)} & 0 \\ 0 & 0 & \phi_5 & 0 & 0 & \phi_5^{(BC)} & \phi_5^{(CD)} \\ 0 & 0 & 0 & 0 & 0 & 0 & \phi_6^{(CD)} \\ 0 & 0 & 0 & \phi_7 & 0 & 0 & \phi_7^{(CD)} \end{bmatrix} \tag{6}$$

Accordingly the eigenvalue problem is projected on the subspace $\mathbf{S}_1$ as,

$$\mathbf{S}_1^{\mathrm{T}}\mathbf{K}\mathbf{S}_1 f\rangle = \tilde{\lambda}\mathbf{S}_1^{\mathrm{T}}\mathbf{M}\mathbf{S}_1 f\rangle \tag{7}$$

The approximated eigenvector is given by,

$$\begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \\ \mathbf{x}_4 \\ \mathbf{x}_5 \\ \mathbf{x}_6 \\ \mathbf{x}_7 \end{bmatrix} = \mathbf{S}_1 \begin{bmatrix} f\rangle_1 \\ f\rangle_2 \\ f\rangle_3 \\ f\rangle_4 \\ f\rangle_5 \\ f\rangle_6 \\ f\rangle_7 \end{bmatrix} = \mathbf{S}_1\boldsymbol{\xi} \tag{8}$$

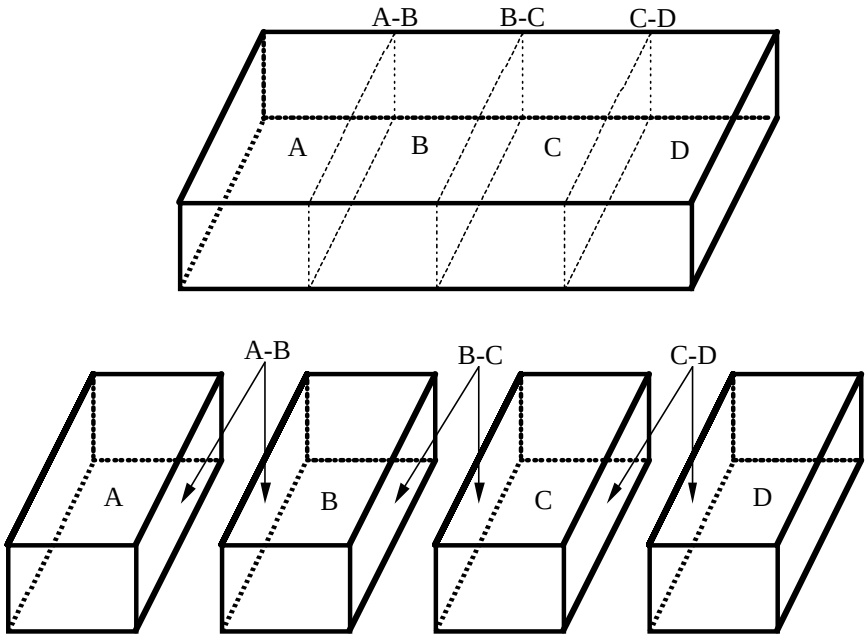


Fig. 1 Schematic view of the model and its division into components

2.2 The hierarchical component mode synthesis

The basic algorithm is applied to the case where the model is divided in a direction as illustrated in Fig. 1. However, it is not always appropriate to divide the model in such manner, if the DOFs of the model is extremely large or the model is complicated in configuration. Therefore, explanation is made with a method for expansion of the basic algorithm to be applied in case that the model described in Fig. 2(a) is divided into two directions as descibed in Fig. 2(b).

First let it be considered with Fig. 2(c) what if the model illustrated in Fig. 2(a) is divided once in a single direction. In such a state, it is possible to apply the basic algorithm. To be concrete with this, the eigenmodes of sigle-component structures are obtained from the structures A, B, C, and D. In the meantime, the eigenmodes of two-component structures are obtained from the structures A+B, B+C, and C+D. By synthesizing these types of the eigenmode, it is possible to obtain the eigenmode of the whole model. Secondly let it be considered what if all of these structures are divided into the directions other than the ones referred to above. In this occasion, for example to obtain the vibration mode of the structure A, the basic algorithm is applied again. To be concrete with this, the eigenmodes of sigle-component structures are obtained from the structures A1, A2, A3, and A4. In the meantime, the eigenmodes of two-component structures are obtained from the structures A1+A2, A2+A3, and A3+A4. By synthesizing these types of the mode, it is possible to obtain the eigenmode of the structure A. The eigenmode of the structures B, C, and D can likewise be

obtained. Moreover to obtain the eigenmode of the structure A+B , the basic algorithm is applied again. The eigenmodes of single-component structures are obtained from the structures A1+B1, A2+B2, A3+B3 and A4+B4 and the eigenmodes of two-component structures are obtained from the structures A1+B1+A2+B2, A2+B2+A3+B3, and A3+B3+A4+B4. By synthesizing these types of the mode, it is possible to obtain the eigenmode of the structure A+B. The eigenmode of the structures B+C and C+D can likewise be obtained.

Here is the conclusion of the above statements. The eigenmode of the model in Fig. 2 can be obtained by synthesizing all the types of the eigenmode given below.

1) Eigenmode of the structures A1, A2, A3, A4, B1, B2, ... D3 and D4.

2) Eigenmode of the structures A1+A2, A2+A3, A3+A4, B1+B2, B2+B3, B3+B4, ..., D1+D2, D2+D3, and D3+D4.

3) Eigenmode of the structures A1+B1, A2+B2, A3+B3, A4+B4, B1+C1, ..., C1+D1, C2+D2, C3+D3 and C4+D4.

4) Eigenmode of the structures A1+B1+A2+B2, A2+B2+A3+B3, A3+B3+A4 +B4, B1+C1+B2+C2, B2+C2+B3+C3, B3+C3+B4+C4, C1+ D1+C2+D2, C2+D2+C3+D3, and C3+D3+C4+D4.

In this occasion, it is unnecessary to obtain once the eigenmode of the intermediate structures (the structures A, B, C, D, A+B, B+C, and C+D). It is possible to directly obtain the eigenmode of the whole model by means of the synthesis of the eigenmode referred to above. The subspace $\mathbf{S}_2$ is given as,

$$\mathbf{S}_2 = \begin{pmatrix} f\phi_{A1}^{A1} & 0 & 0 & \cdots & f\phi_{A1}^{A1+A2} & 0 & 0 & \cdots & f\phi_{A1}^{A1+B1} & 0 & \cdots & f\phi_{A1}^{A1+A2+B1+B2} & \cdots \\ 0 & f\phi_{A2}^{A2} & 0 & \cdots & f\phi_{A2}^{A1+A2} & f\phi_{A2}^{A2+A3} & 0 & \cdots & 0 & f\phi_{A2}^{A2+B2} & \cdots & f\phi_{A2}^{A1+A2+B1+B2} & \cdots \\ 0 & 0 & f\phi_{A3}^{A3} & \cdots & 0 & f\phi_{A3}^{A2+A3} & f\phi_{A3}^{A3+A4} & \cdots & 0 & 0 & \cdots & 0 & \cdots \\ 0 & 0 & 0 & \cdots & 0 & 0 & f\phi_{A4}^{A3+A4} & \cdots & 0 & 0 & \cdots & 0 & \cdots \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & f\phi_{B1}^{A1+B1} & 0 & \cdots & f\phi_{B1}^{A1+A2+B1+B2} & \cdots \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & f\phi_{B2}^{A2+B2} & \cdots & f\phi_{B2}^{A1+A2+B1+B2} & \cdots \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \cdots \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots \end{pmatrix} \tag{9}$$

where the superior character after ϕ indicates the structure and the inferior character indicates the degrees of freedom in each component.

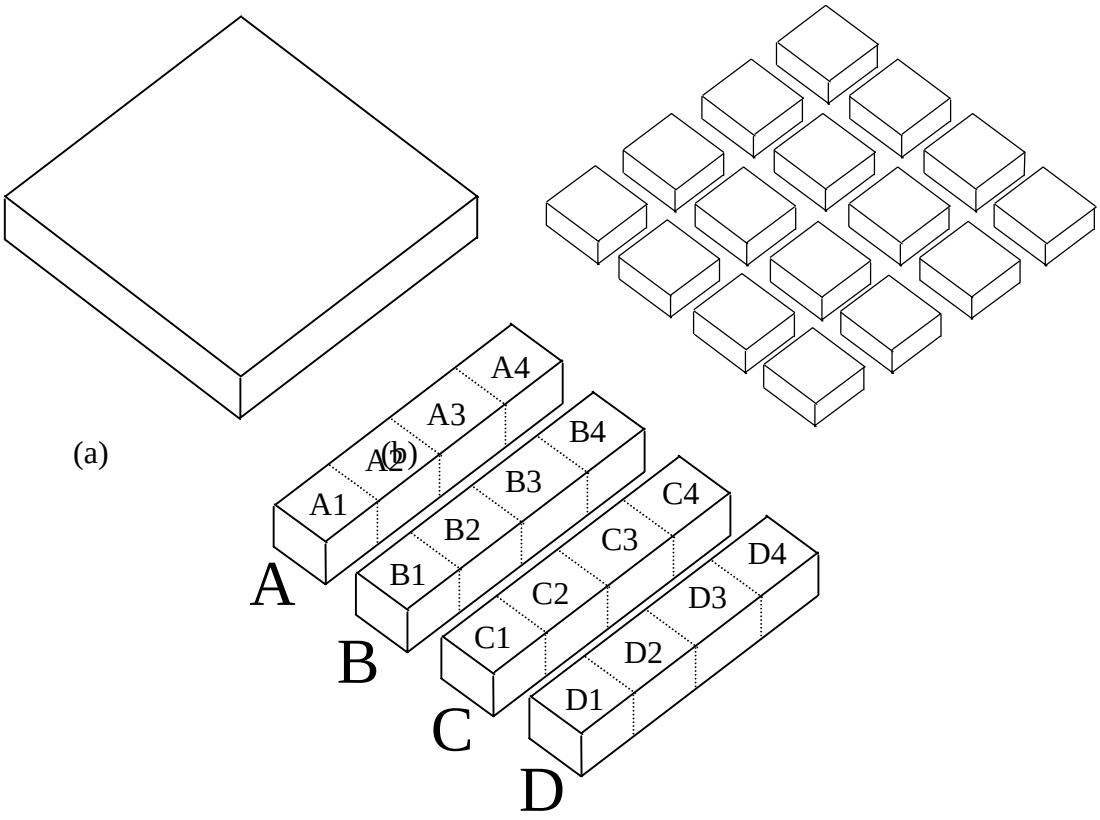
Accordingly the eigenvalue problem is projected on the subspace as,

$$\mathbf{S}_2^{\mathrm{T}}\mathbf{K}\mathbf{S}_2 f\tilde{\mathbf{u}} = \tilde{\lambda}\mathbf{S}_2^{\mathrm{T}}\mathbf{M}\mathbf{S}_2 f\tilde{\mathbf{u}} \tag{10}$$

The approximated eigenvector is given by,

$$\begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \\ \mathbf{x}_4 \\ \mathbf{x}_5 \\ \vdots \\ \mathbf{x}_{49} \end{bmatrix} = \mathbf{S}_2 \begin{bmatrix} f\grave{\mathrm{I}}_1 \\ f\grave{\mathrm{I}}_2 \\ f\grave{\mathrm{I}}_3 \\ f\grave{\mathrm{I}}_4 \\ f\grave{\mathrm{I}}_5 \\ \vdots \\ f\grave{\mathrm{I}}_{49} \end{bmatrix} = \mathbf{S}_2 \xi \tag{11}$$

As hereby described, it is possible, it is shown, to expand the basic algorithm into two-direction division by applying it to two steps. Likewise by applying the basic algorithm to three steps, it is also possible to expand it into three-direction division. Thanks to the expansion of the algorithm made in this manner, it is possible to apply the proposed CMS to eigen analysis of the structures with large scale and general configuration.



(c)

Fig. 2 Schematic view of (a) analysis model, (b) divided into components in two directions and
(c) model is divided once in a single direction

3 NUMERICAL EXAMPLE

3.1 Estimation of accuracy of the eigenpair

In this chapter, we investigate the accuracy of the eigenfrequency and the eigenvector obtained by the proposed CMS in several cases. For this purpose, we introduce the following two error measures. The first one is the normalized error of the eigenfrequency defines as,

$$\text{normalized error} = \frac{|f_C - f_L|}{f_L} \tag{12}$$

where f_C is the eigenfrequency obtained by the CMS and the f_L is the eigenfrequency obtained by solving the whole eigenvalue problem by the Lanczos method. For f_L, values with which full conversion is made are used. The second one is the error bound [18] defined as,

$$\text{error bound} = \left\{1 - \frac{(\rho(\bar{\mathbf{x}}))^2}{\hat{\mathbf{x}}^T\mathbf{M}\hat{\mathbf{x}} / \bar{\mathbf{x}}^T\mathbf{M}\bar{\mathbf{x}}}\right\}^{\frac{1}{2}} \tag{13}$$

where $\rho(\bar{\mathbf{x}})$ is the Rayleigh quotient defined as,

$$\rho(\bar{\mathbf{x}}) = \frac{\bar{\mathbf{x}}^T\mathbf{K}\bar{\mathbf{x}}}{\bar{\mathbf{x}}^T\mathbf{M}\bar{\mathbf{x}}} \tag{14}$$

where $\bar{\mathbf{x}}$ is the eigenvectors obtained from the CMS. Vector $\hat{\mathbf{x}}$ is obtained by,

$$\mathbf{K}\bar{\mathbf{x}} = \mathbf{M}\hat{\mathbf{x}} \tag{15}$$

The models used in this paper are all rectangular quartz plates with different size and number of elements. The parameters of the models are shown in Table 1. The elemnts are all cube shaped 8-node hexahedral elements. The models are divided into identical components.

	Size (mm)	Number of elements	Degrees of freedom
Model 1	1,000 x 100 x 300	100 x 10 x 30	103,323
Model 2	2,000 x 100 x 300	200 x 10 x 30	205,623
Model 3	1,000,000 x 100 x 300	10,000 x 10 x 30	10,231,023
Model 4	1,000 x 100 x 200	100 x 10 x 20	69,993
Model 5	1,000 x 100 x 1,000	100 x 10 x 100	336,633

Table 1 Parameter of the models

3.2 Number of eigenmodes

The normalize error of eigenfrequency using different number of eigenmodes in the synthesis was investigated. For this purpose, Model 1 (103,323 DOFs) was divided into four components and the eigenpairs were computed by the CMS using 1) 5 eigenmodes of each single-component structure and 25 eigenmodes of each two-component structure, totally 95 eigenmodes, 2) 15 eigenmodes of each single-component structure and 35 eigenmodes of each two-component structure, totally 165 eigenmodes and 3) 25 eigenmodes of each single-component structure and 45 eigenmodes of each two-component structure, totally 235 eigenmodes. Figure 3 shows the normalized error of eigenfrequency in the three cases. In case 1) from second to 39th eigenfrequency (39th eigenfrequency = 10.760051 MHz), in case 2) 71 lowest eigenfrequencies (71st eigenfrequency = 16.010654 MHz), and in case 3) 92 lowest eigenfrequency (92nd eigenfrequency = 18.221408 MHz), were obtained with normalized error better than about 1.0×10^{-2}. By increasing the number of eigenmodes in the synthesis, the normalized error of the eigenfrequency was improved and the number of eigenfrequencies to be obtained was also increased.

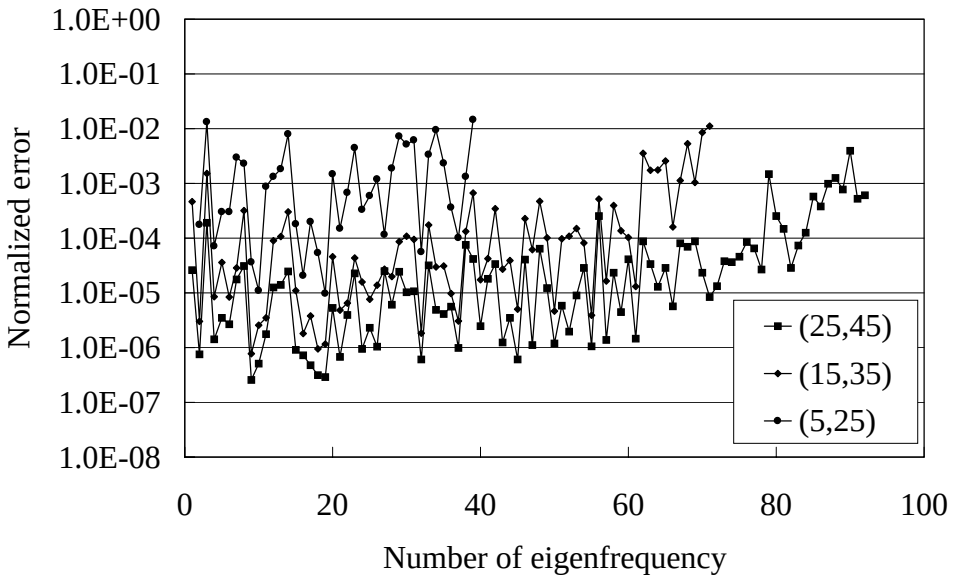


Fig. 3 Normalized error of eigenfrequency of Model 1 using different number of eigenmodes in the synthesis

3.3 Number of components

The change of the normalized error and the error bound of the eigenpair by increasing the number of components without changing the size of each component was investigated. For this purpose, at first, Model 1 (103,323 DOFs) was divided into four components and the eigenpairs were computed by the CMS. Here, 25 lowest eigenmodes of each single-

component structure and 45 lowest eigenmodes of each two-component structure, totally 235 eigenmodes were used in the synthesis. The normalized error and the error bound are shown in Fig. 4. We obtained 92 lowest eigenmodes with normalized error better than 1.0×10^{-2}. Note here that all 92 lowest eigenpair was obtained one-to-one correspondence with that by the Lanczos solution.

Next, Model 2 (205,623 DOFs) was divided into eight components and the eigenpairs were computed by the CMS. Here, 25 lowest eigenmodes of each single-component structure and 45 lowest eigenmodes of each two-component structure, totally 515 eigenmodes were used in the synthesis. The normalized error and the error bound are shown in Fig. 5. We obtained 178 lowest eigenfrequency (178th eigenfrequency = 18.040134 MHz) with normalized error better than 1.0×10^{-2}. Here also all 178 lowest eigenmodes were obtained with one-to-one correspondence with that by the Lanczos solution.

At last, Model 3 (10,321,023 DOFs) was divided into 400 components and the eigenpairs were computed by the CMS. Here, 25 lowest eigenmodes of each single-component structure and 45 lowest eigenmodes of each two-component structure, totally 27,955 eigenmodes were used in the synthesis. The eigenvalue problem of the whole model is extremely great in scale, which prevented the computation by the Lanczos method from being made. As results, normalized error of the solutions could not be obtained. As the number of eigenmodes was too large, the error bound was examined in respect of the closest eigenmode in the range 1.0MHz to 17.0MHz in 1.0MHz steps. The error bound for these eigenmodes is shown in Fig. 6. In the figure, the error bound in the case of Model 1 and 2 are also described for comparison. The error bound is believed to be as high as the ones in case of Models 1 and 2. Accordingly the accuracy of the eigenfrequency is expected to be as high as the ones in case of Models 1 and 2.

The normalized error and the error bound did not deteriorate by increasing the number of components (without changing the size of each component).

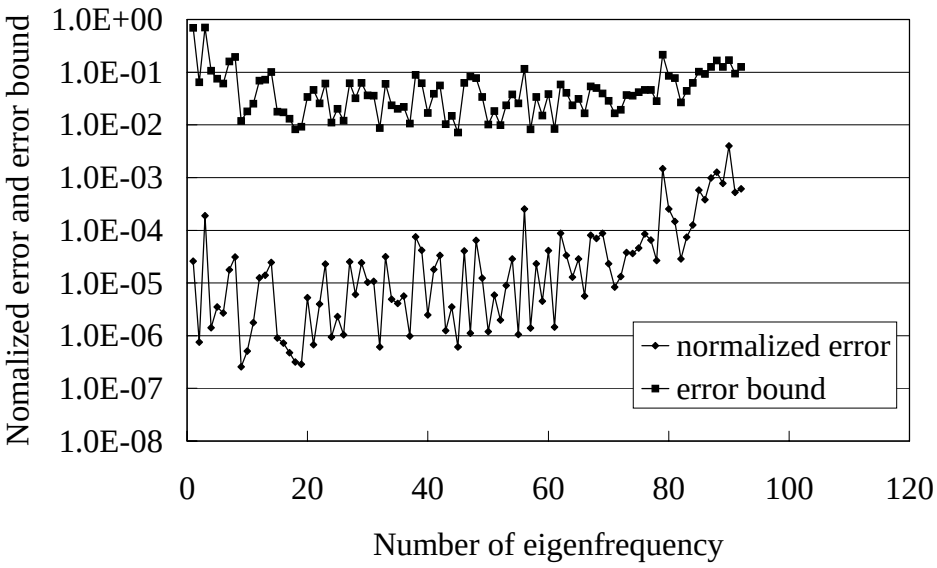


Fig. 4 Normalized error and error bound of eigenpairs of Model 1divided into four components

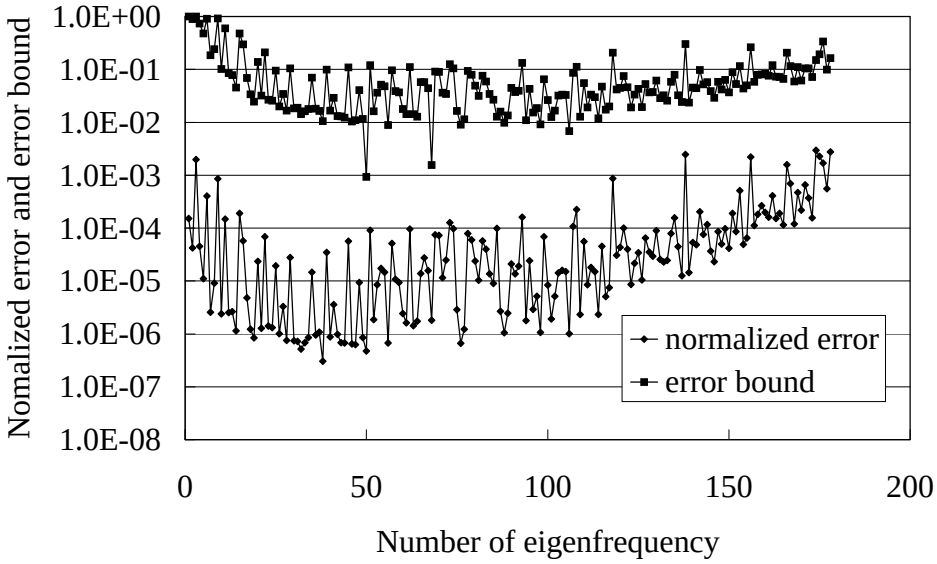


Fig. 5 Normalized error and error bound of eigenpairs of Model 2 divided into 8 components

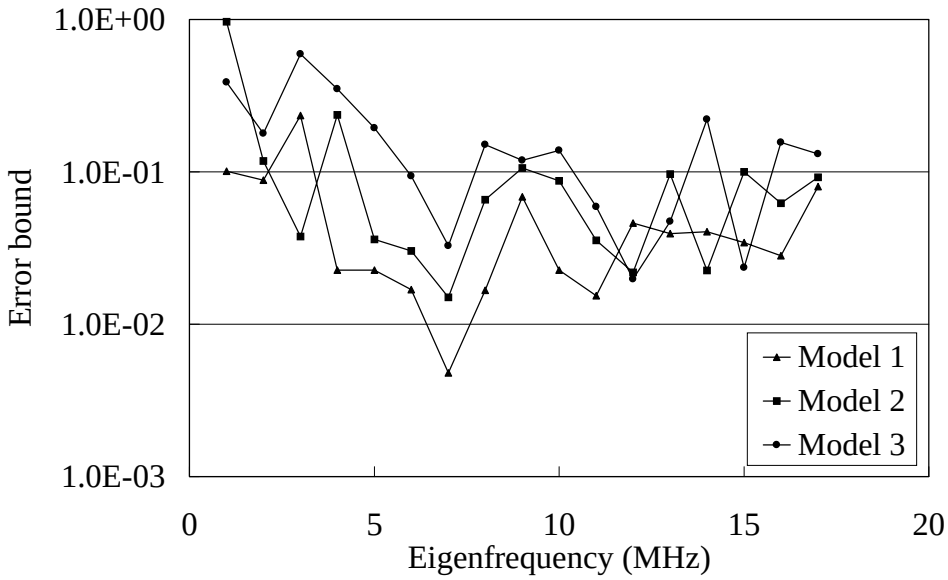


Fig. 6 Error bound of Model 1, 2 and 3 using components of same size

3.4 Number of division into components

The normalized error of the eigenfrequency dividing the model into different number of components was investigated. For this purpose, Model 1 (103,323 DOFs) was divided into four, five and ten components. In all cases, 25 lowest eigenmodes of each single-component structure and 45 lowest eigenmodes of each two-component structure were used in the synthesis. The total number of eigenmodes used in the synthesis was 235, 305 and 655, respectively. Figure 7 shows the normalized error of eigenfrequency in the three cases. In the case of dividing the model into four components, the average of the normalized error of 50 lowest eigenfrequency was 1.6×10^{-5}. In the case of dividing the model into five components, the average of the normalized error was deteriorated to 7.0×10^{-5}. In the case of dividing the model into ten components, the normalized error was seriously deteriorated.

The normalized error deteriorates by decreasing the size of components.

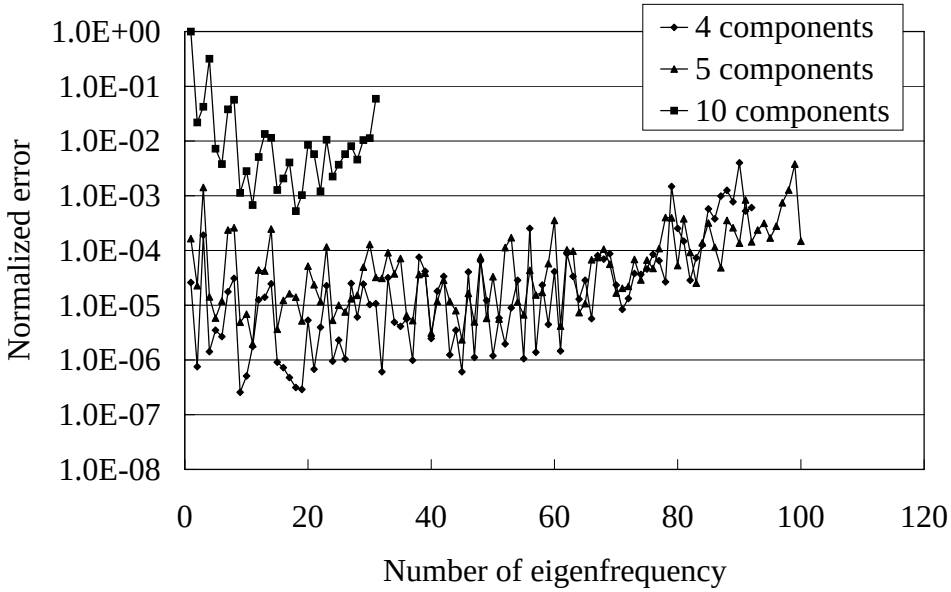


Fig. 7 Normalized error of eigenfrequency of Model 1
dividing into different number of components

3.5 Size of components

The normalized error of the eigenfrequency by changing the interface DOFs of the components was investigated. For this purpose, Model 1 (103,323 DOFs), Model 4 (69,993 DOFs) and Model 5 (336,633 DOFs) were divided into ten components. In all cases, 25 lowest eigenmodes of each single-component structure and 45 lowest eigenmodes of each two-component structure, totally 655 eigenmodes were used in the synthesis. Figure 8 shows the normalized error of eigenfrequency in the three cases. In the case of Model 1, 31 lowest eigenmodes were obtained one-to-on correspondence with the Lanczos solution. In case of Model 4, where the interface DOFs was reduced from 1,023 (Model 1) to 693, more than 100 lowest eigenfrequencies were obtained one-to-on correspondence with the Lanczos solution. In the case of Model 5, where the interface DOFs was increased to 3,333, we could not find good correspondence with the Lanczos solution.

The normalized error deteriorates by increasing the number of interface DOFs.

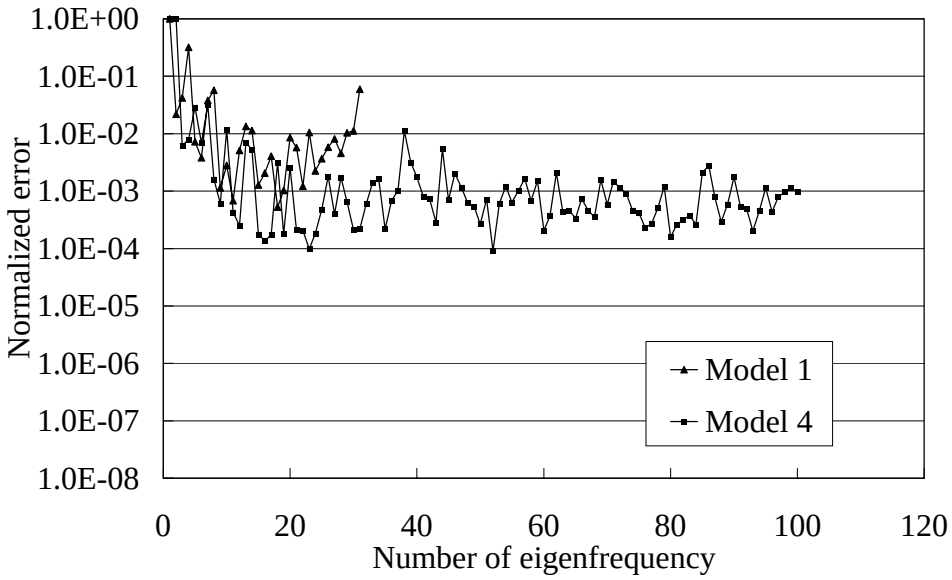


Fig. 8 Normalized error of eigenfrequency of Model 1 and Model 4 divided into ten components

3.6 Two-directional division into components

As shown in the previous section, it is difficult to obtain the eigenpairs in high accuracy by the CMS if the model is long and also wide. In such case the HCMS are to be applied. For this purpose, Model 5 was divided into four by four and ten by ten components and the eigenpairs were computed by the HCMS. In both cases, 15 lowest eigenmodes of each single-component structure, 30 lowest eigenmode of each two-component structure and 55 lowest eigenmode of each four-component structure were used in the synthesis. The total number of eigenmodes was 1,455 and 11,355, respectively. Figure 9 shows the normalized error of eigenfrequency in both cases. In the case of dividing the model into four by four components, the average of the normalized error was 7.1×10^{-6}. In the case of dividing the model into ten by ten components, the average of the normalized error was deteriorated to 2.5×10^{-4}.

The normalized error by the HCMS is satisfactory. Here also, by reducing the size of the components results in the deterioration of the normalized error.

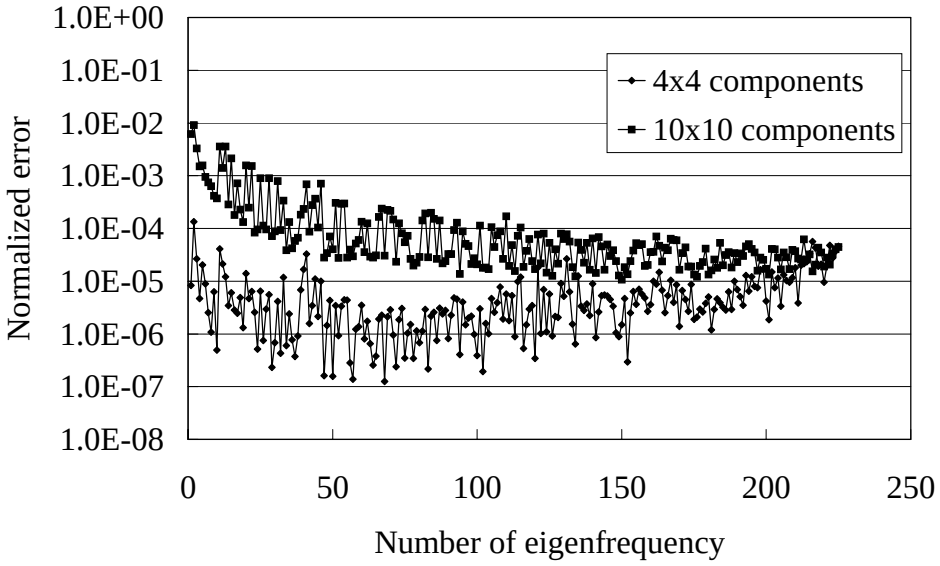


Fig. 9 Normalized error of eigenfrequency of Model 5
dividing into different number of components

4 CONCLUSION

We have investigated the accuracy of the eigenpair obtained by the proposed CMS and HCMS). The conclusions are:

1) Using appropriate number of eigenmodes in the synthesis, the eigenpairs was obtained in extremely high accuracy. We could obtain 92 lowest eigenfrequency of 100 thousand DOFs model with normalized error was better than 1.0×10^{-2} using only 235 components eigenmodes.
2) Using components of appropriate size, the eigenpairs was obtained in extremely high accuracy. 178 lowest eigenfrequencies of 200 thousand DOFs model was obtained with normalized error was better than 1.0×10^{-2}, using eigenmodes of components with 25,000 to 50,000DOFs.
3) There was little deterioration in accuracy of eigenpairs by multiplying the number of components. The error bound of 10 million DOFs model was as good as that of the 100 thousand DOFs model.
4) If the DOFs of the interface was increased, the number of eigenpairs to be obtained was decreased. More than 100 lowest eigenmodes of 70 thousand DOFs model were obtained using 235 eigenmodes of components with 693 interface DOFs. However, only 31 lowest eigenmodes of 100 thousand DOFs model were obtained the same number of eigenmodes of components with 1,023 interface DOFs.

5) The accuracy of eigenpairs obtained by the HCMS was also satisfactory. The normalized error of more than 200 lowest eigenmodes of 300 thousand DOFs model divided into four by four components was better than 1.0×10^{-4}.

As a result, we conclude that the proposed CMS and HCMS are quite appropriate for eigen analysis of structures with extremely large scale and of general configuration.

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