NUMERICAL MODELLING OF SWIRLING TURBULENT WAKES

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Abstract. The longitudinal component of excess momentum $J$ and angular momentum $M$ are the most important integral characteristics of the hydrodynamic wake past a body moving in unbounded fluid. The laws of turbulent motion in the wake essentially depend on these parameters. The information on experimental and numerical modelling of swirling turbulent wake is scant [1]-[4]. The present study deals with the numerical simulation of the wake when both the momentum and the angular momentum take zero values. The flow pattern is calculated within the framework of the thin shear layer approximation for nonclosed system of the motion and continuity equations. The closed system of equations is written for two different formulations of the closure relations. The numerical solution of the problem is performed with the use of the finite-difference algorithm realised on moving grids. The algorithm is conservative with respect to the laws of conservation of the momentum and the angular momentum. The experimentally measured distributions are used as the initial conditions. Both the models described agree well with the experimental data [5]. However, the second model shows better agreement. It is demonstrated that at the large distances downstream from the body the solution of the problem approaches the self-similar one. Simplified models of far turbulent wake behind self-propelled body have been constructed.
1 INTRODUCTION

This paper deals with the numerical modelling of the development of turbulent wake with a zero values of total excess momentum and angular momentum. This wake is formed in a fluid for example, behind uniformly moving a body with a propeller. The momentumless wake behind a slender axisymmetric body with nonzero vorticity was previously analysed numerically in the framework of a simplified $\varepsilon$-model [1, 2]. A survey of the cited papers and some subsequent papers can be found in [3]. The self-similar solutions for the swirling wake behind a self-propelled body are obtained analytically and numerically by the classical $e - \varepsilon$ model [4]. Below we give the calculation results obtained by two mathematical models and compare them with the experimental data [5]. The case of the wake flow with a zero value of total excess momentum and a nonzero value of angular momentum was considered in [6].

2 BASIC EQUATIONS

To describe the flow the following system of averaged equations for the motion and continuity in the thin shear layer approach is used

$$ U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial r} = - \frac{1}{r} \frac{\partial}{\partial r} r \langle u'v' \rangle + \frac{\partial}{\partial x} \int_r^\infty \frac{[W^2 + (\langle w'^2 \rangle - \langle v'^2 \rangle)]}{r} dr - \frac{\partial (\langle u'^2 \rangle - \langle v'^2 \rangle)}{\partial x}, \quad (1) $$

$$ U \frac{\partial W}{\partial x} + V \frac{\partial W}{\partial r} + VW \frac{1}{r} = - \frac{1}{r} \frac{\partial}{\partial r} r \langle v'w' \rangle - \langle v'w' \rangle \frac{1}{r}, \quad (2) $$

$$ \frac{\partial U}{\partial x} + \frac{\partial V}{\partial r} + \frac{V}{r} = 0. \quad (3) $$

Here $x, r, \varphi$—is cylindrical coordinate system with the origin at the trailing edge of the body; $x$—axis is directed opposite to the direction of the body motion; $U, V, W, u', v', w'$—are relevant velocity components of averaged and fluctuating motion; $\langle u'^2 \rangle, \langle v'^2 \rangle, \langle w'^2 \rangle, \langle u'v' \rangle, \langle u'w' \rangle, \langle v'w' \rangle$—Reynolds stresses; the symbol $\langle \rangle$ denotes the averaging. In the right-hand sides of Eqs. (1), (2) the terms containing co-factor in the form of the coefficient of the laminar viscosity have been omitted under assumption of their smallness.

3 MODELS OF TURBULENT MOTION

The closed system of equations is written for two different formulations of the closure relations. Mathematical Model 1 includes the following equations [7] for determination of Reynolds stresses in addition to Eqs. (1)–(3)

$$ U \frac{\partial \langle u'^2 \rangle}{\partial x} + V \frac{\partial \langle u'^2 \rangle}{\partial r} = -2(1 - \alpha) \langle u'v' \rangle \frac{\partial U}{\partial r} - \frac{2}{3} \varepsilon - C_1 e \left( \frac{\langle u'^2 \rangle}{2 e} - \frac{2}{3} \varepsilon \right) + \frac{2}{3} \alpha P + $$
\[ U \frac{\partial (v'^2)}{\partial x} + V \frac{\partial (v'^2)}{\partial r} - 2 \frac{W}{r} \langle v'w' \rangle = 2(1 - \alpha) \langle v'w' \rangle \frac{W}{r} - \frac{2}{3} \varepsilon - C_1 \frac{\varepsilon}{e} \left( \langle v'^2 \rangle - \frac{2}{3} e \right) + \frac{2}{3} \alpha P + \]
\[ + \frac{C_s}{r} \frac{\partial}{\partial r} \left[ \frac{r e}{\varepsilon} \left( \langle v'^2 \rangle \frac{\partial (v'^2)}{\partial r} - \frac{2}{r} \langle v'w' \rangle \langle v'w' \rangle \right) \right] \]
\[ - \frac{2 C_s e}{r \varepsilon} \left[ \langle v'w' \rangle \frac{\partial (v'w')}{\partial r} + \langle w'^2 \rangle \frac{(\langle v'^2 \rangle - \langle w'^2 \rangle)}{r} \right]. \]
\[ U \frac{\partial \langle w'^2 \rangle}{\partial x} + V \frac{\partial \langle w'^2 \rangle}{\partial r} + 2 \frac{W}{r} \langle v'w' \rangle = -2(1 - \alpha) \langle v'w' \rangle \frac{W}{r} - \frac{2}{3} \varepsilon - C_1 \frac{\varepsilon}{e} \left( \langle w'^2 \rangle - \frac{2}{3} e \right) + \frac{2}{3} \alpha P + \]
\[ + \frac{C_s}{r} \frac{\partial}{\partial r} \left[ \frac{r e}{\varepsilon} \left( \langle v'^2 \rangle \frac{\partial \langle w'^2 \rangle}{\partial r} + \frac{2}{r} \langle v'w' \rangle \langle v'w' \rangle \right) \right] \]
\[ - \frac{2 C_s e}{r \varepsilon} \left[ \langle v'w' \rangle \frac{\partial \langle v'w' \rangle}{\partial r} + \langle w'^2 \rangle \frac{(\langle v'^2 \rangle - \langle w'^2 \rangle)}{r} \right]. \]

The turbulent shear stress \( \langle u'w' \rangle \) is derived from the nonequilibrium algebraic approximations introduced by Rodi [8]:
\[ \langle u'w' \rangle = \alpha_1 \left( \langle u'w' \rangle \frac{\partial W}{\partial r} + \langle v'w' \rangle \frac{\partial V}{\partial r} \right). \]

Here \( \alpha_1 = -\lambda_1 \frac{e}{\varepsilon}, \lambda_1 = \frac{1 - C_2}{C_1 + P/\varepsilon - 1}. \)

To determine the values of the rate of dissipation \( \varepsilon \) we make use of the relevant differential equation.
\[ U \frac{\partial \varepsilon}{\partial x} + V \frac{\partial \varepsilon}{\partial r} = C_\varepsilon \frac{\partial}{\partial r} \left( \frac{r \langle v'^2 \rangle \partial \varepsilon}{\varepsilon} \right) + \frac{\varepsilon}{e} (C_{\varepsilon 1} P - C_{\varepsilon 2} \varepsilon). \]  

(10)

The value \( P \) is the energy production caused by averaged motion

\[ P = - \left( \langle u'v' \rangle \frac{\partial U}{\partial r} + \langle v'w' \rangle r \frac{\partial (W/r)}{\partial r} \right). \]

In Model 2 we determine the Reynolds shear stresses by the Rodi nonequilibrium relations:

\[ \langle u'v' \rangle = \alpha_1 \langle v'^2 \rangle \frac{\partial U}{\partial r}, \]

(11)

\[ \langle v'w' \rangle = \alpha_1 \left( \langle v'^2 \rangle r \frac{\partial}{\partial r} \left( \frac{W}{r} \right) + \frac{W}{r} \langle v'^2 \rangle - \langle w'^2 \rangle \right). \]

(12)

The normal Reynolds stresses are obtained from the transport differential equations (4)—(6) and \( \langle u'w' \rangle \) from relationship (9).

The quantities \( C_s, C_\varepsilon, \alpha, C_1, C_2, C_{\varepsilon 1}, C_{\varepsilon 2} \) are empirical constants. Their values are taken to be equal 0.22, 0.17, 0.93, 0.6, 2.2, 1.45, 1.92.

The problem variables can be made dimensionless by using the characteristic length \( D \) (the body diameter) and the velocity scale \( U_0 \). At a distance \( x = x_0 \) from the body the initial conditions for \( U, W, \langle u'u' \rangle, \varepsilon \) are specified as functions consistent with the experimental data. At \( r \to \infty \) the free stream conditions are specified, the boundary values at \( r = 0 \) are determined from conditions of symmetry for functions \( U, \langle u'^2 \rangle, \langle v'^2 \rangle, \langle w'^2 \rangle, \langle u'v' \rangle, \varepsilon \) and antisymmetry for \( W, V, \langle u'u' \rangle, \langle u'w' \rangle \).

4 NUMERICAL ALGORITHM

The problem of the solution algorithm is similar to the one presented in [9]. Beforehand the equations of models are reduced to conservative form in which the laws of momentum and angular momentum are a consequence of the integration of equations (1), (2) over the total cross section of the wake. First order finite difference approximations of equations and boundary conditions conservative with respect to these laws have been constructed. Numerical algorithm testing was performed using Loitsiansky asymptotic solution of the problem of laminar submerged swirling jet degeneration.

5 MAIN COMPUTATIONAL RESULTS

The results of calculations are presented below. At a distance \( x = 10D \) the initial values were specified using experimental data [5]. For the sake of simplicity the computations were performed on uniform stationary grids in variable \( r \). A step value \( h_x^{n+1} \) is determined
from the relation $h_{x}^{n+1} = h_{x}^{n} \times 1.006$ (here $n$ is a layer number with respect to $x$). The grid parameters were chosen in the course of the numerical experiments so that if the grid steps for variables $r$ and $x$ are halved, then the deviations in the grid solutions are no larger than 1 % in the uniform norm.

Figure 1: Comparison of the calculated dimensionless profiles of the defect of the velocity $U_{1}$ (a) and the tangential velocity $W$ (b) with experimental data [5].

Figure 2: Comparison of the calculated dimensionless profiles of fluctuation intensities of the velocity components with experimental data [5].

Dimensionless mean velocity defect profiles $U_{1} = U - U_{0}$ and tangential velocity component $W$ are presented in Fig. 1 a,b. Figure 2 shows the turbulent fluctuation intensities of velocity components $\sigma_{u} = \sqrt{\langle u'^{2} \rangle}$, $\sigma_{v} = \sqrt{\langle v'^{2} \rangle}$, $\sigma_{w} = \sqrt{\langle w'^{2} \rangle}$. In Figs. 1-2 the calculation results are presented by the lines; the experimental data by dots. As is seen in Figs. 1-2, Models 1,2 adequately describe the experimental data. Analyzing the results
of the comparison between calculated and experimental data in Fig. 1, one can see considerably more disagreement when the results of calculation are based on Model 1. Figure 3 demonstrates the decay of centerline values of velocity defect $U_{10}/U_0$, maximum values of tangential velocity component $|W|_{\text{max}}/U_0$, axial values of the turbulence energy $e_0/U_0^2$ and characteristic dimension of the wake $r_{1/2}/D$. The value $r_{1/2}$ is determined by the relation $\sigma_u(x, r_{1/2}) = \sigma_u(x, 0)/2$. Calculations are performed on the base of Model 2. As shown in this Fig., the results of numerical modelling (curves) are in a good agreement with experimental data (markers). Figure 3 demonstrates also the laws of decay of far wake $r_{1/2} \sim x^{0.2}$; $U_{10} \sim x^{-1.9}$; $|W|_{\text{max}} \sim x^{-2.5}$; $e_0 \sim x^{-1.45}$. Figure 4 shows the behavior of normalized functions $U_1/U_{10}$, $e/e_0$, $W/W_{\text{max}}$ versus the distance from the body. The results of Figs. 3, 4 give the evidence of the self-similarity of far swirling turbulent wake behind self-propelled body.

Based on results of numerical experiments simplified models of far turbulent wake have been constructed. The main simplified model is far wake model [10]. Some computational results demonstrates that this model is applicable for $x/D > 100$. 

![Figure 3: Centerline values of velocity defect $U_{10}/U_0$, maximum values of tangential velocity component $|W|_{\text{max}}/U_0$, axial values of the turbulence energy $e_0/U_0^2$ and characteristic dimension of the wake $r_{1/2}/D$ depending on the distance $x/D$.](image)
Figure 4: Normalized functions $U_1/U_{10}$, $e/e_0$, $W/W_{max}$ versus the distance from the body.

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