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COMPARISON OF DIFFERENT APPROACHES IN FREE SURFACE MODELLING BY THE FINITE ELEMENT METHOD

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Abstract. *In the present work we approach the free surface formulation in three different ways and consider the unsteady laminar Newtonian isothermal flow of viscous incompressible liquid described by the Navier-Stokes equations. The governing equations are spatially discretised by the finite element method. The velocity correction method is used in the velocity-pressure segregation of the equations. The main advantages of this approach are the appearance of symmetric matrices of finite element coefficients and the equal order interpolation of velocity and pressure. In the Lagrangian approach a prediction-correction method updates the location of the free surface. The surface height method describes the independent mesh movement in the Arbitrary Lagrange-Eulerian formulation while the level set method traces the moving interface in the purely Eulerian description. The numerical experiments include the simulation of free oscillations in a tank filled by one or two liquids. The influence of initial wave amplitude and kinematic viscosity on the wave pattern is investigated as well as the influence of discretisation parameters on the accuracy of a numerical solution. A detailed quantitative comparison of the numerical results obtained by the different methods is presented. The efficiency, numerical accuracy and range of applicability of the investigated methods are also evaluated.*

1 INTRODUCTION

Many important industrial applications involve propagation of fluid-gas or fluid-fluid interfaces. Examples include diverse problems such as ship in wave analysis and sloshing in internal tank arrangements. The numerical simulation of free surfaces and moving interfaces presents great challenges to computational scientists since the underlying physical problem is singular and sensitive to small numerical perturbations. Numerical methods advocated for solving free boundary problems may be classified into three categories, depending on their description of motion: Lagrangian, Arbitrary Lagrange-Eulerian and Eulerian methods.

The Lagrangian approach renders the treatment of the free surface and interface conditions trivial. The free surface geometry at a given time step is advanced in terms of the instantaneous interface velocity field. The earliest fully Lagrangian treatments of viscous flow were advanced by Hirt *et al.*[1]. The Navier-Stokes equations are expressed in material co-ordinates. The material time derivative can be approximated by function values known at successive time instances. Ramasswamy *et al.*[2] subsequently updated mesh co-ordinates by iterative procedure. The main drawback of the Lagrangian approach is that the mesh deforms severely as the free surface moves, making remeshing and rezoning necessary. The last applications of the approach includes continues and adaptive remeshing at every time step[3].

The Arbitrary Lagrangian-Eulerian approach is a mixed variant of the Lagrangian and Eulerian approaches. The earliest applications of this approach are those of Hirt *et al.*[4], Hughes *et al.*[5]. The mesh deforms in accordance with an arbitrary velocity field that is independent of the flow velocities, except at the moving interface. The speed of the moving mesh is incorporated into the convective term reducing the mesh distortion. The latest works[6,7] show that the selection of the mesh velocity is not a trivial matter and requires complicated implementation. The position of the free surface could be obtained using the surface height method[8,9] and the free surface kinematic equation. However, the surface height method does not work for multi-valued surfaces and the steepness of the surface waves is limited.

The Eulerian approach is based on a so-called front capturing technique. A stationary mesh is generated to cover the spatial domain at all time instances. The flow velocities obtained from the Navier-Stokes equations propagate the location of the interface. The marker-and-cell (MAC) method[10] and the volume-of-fluid (VOF) method[11] lie in this category. In this paper we present an initial attempt in modelling moving interfaces by the level set method[12-16] and finite elements. The fluid interface is modelled as the zero set of a smooth function defined on the entire physical domain. The nodal values of a level set function are obtained by solving a convection dominated transport equation. This formulation has potential abilities to simulate moving interfaces, that develop corners, cusps and undergoes large topological changes. Most of the applications on level set methods are based on finite difference schemes. The spatial discretisation by the finite element method is believed to be more flexible and accurate. Therefore, problems with complicated geometry can be solved more efficiently. The first publications presenting attempts to combine the level set method and finite elements appeared recently[17-20].

2 EQUATIONS GOVERNING THE FLOW

Newtonian isothermal and laminar flow of viscous incompressible liquid is considered. The flow is described by the Navier-Stokes equations formulated in the Lagrangian, Eulerian and Arbitrary Lagrange-Eulerian reference frames respectively:

$$\rho \frac{Du_i}{Dt} = \rho F_i - \frac{\partial p}{\partial x_i} + \mu \Delta u_i \tag{1}$$

$$\rho \left(\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \right) = \rho F_i - \frac{\partial p}{\partial x_i} + \mu \Delta u_i \tag{2}$$

$$\rho \left(\frac{\partial u_i}{\partial t} + (u_j - w_j) \frac{\partial u_i}{\partial x_j} \right) = \rho F_i - \frac{\partial p}{\partial x_i} + \mu \Delta u_i \tag{3}$$

$$\frac{\partial u_i}{\partial x_i} = 0 \tag{4}$$

where u_i are components of the velocity, p is the pressure, w_i are components of the mesh velocity, ρ is the density, μ is the dynamic viscosity coefficient and F_i are components of the gravity force. The equations (1-3) represent conservation of momentum in different reference frames while the continuity equation (4) refers to conservation of mass.

In the present work we apply the velocity correction method[21-24], where velocity pressure segregation is performed at the differential equation stage. The resulting equation systems have diagonally dominant, positive defined and symmetric coefficient matrices. Conjugate gradient or conjugate residual iterative linear equation solvers are extremely efficient on this type of equation systems. Very important advantage of this method is equal order interpolation of the velocity and pressure. In the Eulerian formulation the solution u^{n+1} at a given time step $t+dt$ is obtained in four steps:

$$\hat{u}_i = u_i^n - \frac{dt}{2} u_j^n \frac{\partial u_i^n}{\partial x_j} \tag{5}$$

$$\boldsymbol{\rho u}_i^* = \boldsymbol{\rho u}_i^n - dt \rho u_j^n \frac{\partial \hat{u}_i}{\partial x_j} + dt\, \boldsymbol{\rho F}_i + dt \mu \Delta u_i \tag{6}$$

$$\Delta p^{n+1} = \frac{\rho}{dt} \frac{\partial u_j^*}{\partial x_j} \tag{7}$$

$$\rho u_i^{n+1} = \rho u_i^* - dt \frac{\partial p^{n+1}}{\partial x_i} \tag{8}$$

Essential boundary conditions for the velocity components are applied in the last step.

3 THREE APPROACHES FOR FREE SURFACE OR MOVING INTERFACE MODELLING

In this work free surface and moving interfaces are simulated by three highly different numerical methods based on three approaches. The prediction-correction method based on the Lagrangian approach, the surface height method based on the Arbitrary Lagrange-Eulerian approach and the level set method based on the Eulerian approach are investigated.

3.1 Description of the prediction-correction method

The prediction-correction method is proposed by Ramaswamy *et al.*[2] and belongs to the category of the Lagrangian methods. In the Navier-Stokes equations the material time derivative is approximately defined by the functions at deformed and undeformed positions. The values of unknown functions at the deformed position are subsequently updated through iterative procedure in every time step. At the initial step of iterations and at the *m*-th step of iterations the location of a nodal point $x_i^{n+1(m)}$ at the time $t+dt$ is computed as

$$x_i^{n+1(0)} = x_i^n + dtu_i^n\ ,\ x_i^{n+1(m)} = x_i^n + (dt/2)(u_i^n + u_i^{n+1(m-1)}) \tag{9}$$

where dt is a time increment, x_i^n is the location of a nodal point at the time t, u_i^n is the velocity of a nodal point at the time t. The superscripts in brackets mean the number of iteration. The three steps of the velocity correction method are performed at each iteration and velocity $u_i^{n+1(m-1)}$ is updated in (9) formula. As usually convergence of unknowns is achieved after 3-4 iterations. The prediction-correction method is implemented on a mesh with fixed topology and is limited to cases of small geometrical distortion. Remeshing procedures are not applied in order to clarify the basic characteristics of the method.

3.2 Description of the surface height method

In finite element areas the surface height method was successively developed by several groups of researches[8-9]. A position of a free surface is defined by the surface height variables h, that are computed from the free surface kinematic equation:

$$\frac{\partial h}{\partial t} + u_1 \frac{\partial h}{\partial x_1} = u_2 \tag{10}$$

When surface height values are computed and free surface position is predicted, the solution domain could be updated. The surface height method combined with structured meshes allows a very simple algorithm for the mesh movement. A mesh does not move in x_1 direction. The locations of nodal points that are equally distributed in x_2 direction could be computed as

$$x_2^{n+1} = x_2^n + (h^{n+1} - h^n)\frac{x_2^n}{h^n} \tag{11}$$

The unstructured finite element meshes require more sophisticated strategy for mesh

movement[6,7]. The surface height method does not work for multi-valued surfaces and the steepness of surface waves is limited.

3.3 Description of the level set method

The level set method for modelling two-phase flow is introduced by Osher and Sethian[12]. The basic idea of the method is to embed the moving interface as the zero level set of a function ω defined on the whole solution domain. ω is initialized to be the signed normal distance from the interface. An convective transport equation

$$\frac{\partial \varphi}{\partial t} + u_j \frac{\partial \varphi}{\partial x_j} = 0 \tag{12}$$

moves the zero level set of ω exactly as the actual interface moves. Since ω is a smooth function, unlike ρ or μ, the equation (12) is more easily solved numerically than the continuity equations for ρ or μ.

The level set function ω is initially a distance function, but this property does not hold after several time steps. While equation (12) moves the zero level set at the correct velocity, the gradient of the function ω is not equal unity and ω may become irregular after some period of time. Conventional routines for re-initializing a distance function are based on explicitly finding the contour ω =0 and resetting the function ω near the moving front. This geometrical procedure can distort the front and might cause the developing of corners and loss of mass. A numerical re-initialization procedure[14] is implemented in the present work. The non-linear initial value problem

$$\frac{\partial \psi}{\partial t} = \operatorname{sign}(\varphi)\left(1 - \sqrt{\psi_{x_i}\psi_{x_i}}\right) \tag{13}$$

$$\psi(x_1, x_2, 0) = \varphi(x_1, x_2) \tag{14}$$

is solved until steady state. The zero level set of the function ψ is the same as ω. The function ψ remains unchanged at the interface and away from the interface it converges to $|\nabla \psi| \to 1$, when $t \to \infty$.

The solution of the Navier-Stokes equations yields unwanted instabilities at the ω =0, especially for large density ratios. This is due to the discontinuous density at the interface. A simple procedure for smoothing ρ is implemented:

$$\rho(\varphi) = \begin{cases} \rho_2 & \text{if } \varphi > \alpha \\ \rho_1 & \text{if } \varphi < -\alpha \\ 0.5(\rho_1 + \rho_2) + 0.5(\rho_1 - \rho_2)\sin\left(\dfrac{\pi\varphi}{2\alpha}\right) & \text{if } -\alpha < \varphi < \alpha \end{cases} \tag{15}$$

The above formula gives the interface of finite thickness 2α for all times. This procedure can

only be applied if ω is a distance function.

4 FINITE ELEMENT FORMULATION

The solution domain S is discretised by quadrilateral finite elements. Equal order bilinear shape functions $\boldsymbol{N}$ are used for both the pressure and velocity components as well as for the level set function

$$a_e = \sum_{i=1}^{4} N_i a_i \equiv \boldsymbol{N}^T \boldsymbol{A} \tag{16}$$

where a_e is any variable in the finite element e. The weak formulation of segregated Navier-Stokes equations (5-8) is derived by the standard Galiorkin finite element method and could be found in the references[17, 24]. The Green-Gauss theorem is applied to both sides of equation (7) of pressure calculation step[23]. In this particular case the pressure satisfies the consistent boundary conditions and solution should be more exact near the boundaries of a solution domain[22]. The weak point of the applied segregated approach is strict limitations to the time step. This is due to explicit formulation of the velocity prediction step in time.

4.1 Formulation of the prediction-correction method

The formulation of the prediction-correction method does not involve any additional equation for surface shape computations. Three equations of the velocity correction method are enclosed in iterative loop. The velocity prediction stage does not include convective terms and could be performed by a single step. The mesh co-ordinates are updated by the simple explicit procedure (9). This iterative method smoothes fields of unknowns and reduces deformations of a mesh.

4.2 Discretisation of the equation of the surface height method

The 1D linear shape functions are used for surface height approximation. The free surface kinematic equation (10) is discretised in time using the implicit Cranc-Nicolson scheme (θ=0.5):

$$(\tilde{\boldsymbol{M}} + \theta dt \tilde{\boldsymbol{C}}^{n+1})\boldsymbol{H}^{n+1} = (\tilde{\boldsymbol{M}} - dt(1-\theta)\tilde{\boldsymbol{C}}^{n})\boldsymbol{H}^{n} + dt\tilde{\boldsymbol{M}}(\theta \boldsymbol{U}_2^{n+1} + (1-\theta)\boldsymbol{U}_2^{n}) \tag{17}$$

where

$$\tilde{\boldsymbol{C}}^{n} = \int_{\Gamma_e} \tilde{\boldsymbol{W}}(\tilde{\boldsymbol{N}}^T \boldsymbol{U}_1^{n}) \frac{\partial \tilde{\boldsymbol{N}}^T}{\partial x_1} d\Gamma, \;\; \tilde{\boldsymbol{M}} = \int_{\Gamma_e} \tilde{\boldsymbol{W}} \tilde{\boldsymbol{N}}^T d\Gamma \tag{18}$$

Here $\tilde{\boldsymbol{M}}$ and $\tilde{\boldsymbol{C}}^{n}$ are non-symmetric mass and convection matrices, respectively. $\tilde{\boldsymbol{W}}$ are weighting functions of the standard streamline upwind Petrov-Galiorkin method[25]. Sophisticated stabilization schemes proposed by Dawson[26] or O⟩ate[27-28] are not applied in order to check how accurate could be standard simply implementable schemes.

4.3 Discretisation of equations of the level set method

In this paper modelling of moving interfaces by the level set method is combined with finite elements. Equation (12) governing the evolution of the front is stabilized using a one step Taylor-Galerkin procedure[29]

$$\boldsymbol{M}\boldsymbol{\Phi}^{n+1} = \boldsymbol{M}\boldsymbol{\Phi}^{n} - dt\boldsymbol{C}^{n+1}\boldsymbol{\Phi}^{n} + \frac{dt^2}{2}\boldsymbol{T}\boldsymbol{\Phi}^{n} \tag{19}$$

which is second order accurate in time. $\boldsymbol{M}$ is the mass matrix, $\boldsymbol{C}^n$ is the matrix of convective terms and $\boldsymbol{T}$ is the stabilization matrix.

The equation of re-initialization procedure (13) is stabilized using the SUPG methods[30]. Experiments performed in this work showed that various Taylor-Galiorkin procedures did not give good results for the investigated problem. The discretisation in time is performed using the Crank-Nicolson scheme ($\theta = 0.5$). The system of unsteady, non-linear equations may be written as

$$(\hat{\boldsymbol{M}} + \theta dt\hat{\boldsymbol{C}}^{n+1})\boldsymbol{\Psi}^{n+1} = (\hat{\boldsymbol{M}} - dt(1-\theta)\hat{\boldsymbol{C}}^{n})\boldsymbol{\Psi}^{n} + dt\hat{\boldsymbol{M}}\mathrm{S}(\varphi) \tag{20}$$

here $\hat{\boldsymbol{M}}$ an $\hat{\boldsymbol{C}}^n$ are mass and convection matrices:

$$\hat{\boldsymbol{M}} = \int_{S_e} \hat{\boldsymbol{N}}\boldsymbol{N}^T dS \quad , \hat{\boldsymbol{C}}^n = \int_{S_e} \hat{\boldsymbol{N}}(\boldsymbol{N}^T\boldsymbol{W}_j^n)\frac{\partial \boldsymbol{N}^T}{\partial x_j} dS \tag{21}$$

S is a smoothed sign function; $\boldsymbol{W}$ is a unit normal pointing outward from the zero level set.

The formulation of the level set method is capable of simulating moving interfaces, that develop corners, cusps and undergoes large topological changes. The level set function automatically takes care of merging and breaking of interfaces. The level set formulation joined with the finite element method could be very universal and promising. However, some procedures of the method are not sufficiently developed in the finite element areas. Actually the re-initialisation procedure is the bottleneck of the investigated method. Therefore, the best performance of the level set method is not achieved in this work.

5 THE RESULTS AND DISCUSSIONS

Free oscillations of the liquid surface in a tank are modelled in this work. All investigated methods are implemented and all numerical tests are performed using C++ library DIFFPACK[31]. The prediction-correction and surface height methods are applied for one phase flow in a rectangular tank (fig. 1). The density of a liquid ρ=1000 kg/m^3. The initial free surface is described by the sinusoidal curve $h(x_1, t=0) = 0.5 + A\cdot\sin(\pi x_1)$. The influence of the different initial amplitudes (A=0.01, 0.05, 0.1 m) to the flow pattern is investigated as well as the influence of different dynamic viscosity (μ=100, 10, 1, 0.1 kg/(m$\cdot$s)). Fig. 2 shows vertical displacements of three nodal points allocated on the free surface. The wave pattern (fig. 2a, 2b) is regular for small initial amplitudes. In this case the prediction-correction and

the surface height methods produce results that are in good qualitative agreement with the results obtained by Ramaswamy *et al.*[2]. The solution becomes highly non-linear and non-regular for waves with large amplitudes (fig. 2c-2f, 3). The mass loss is observed in these problems simulated by the surface height method.

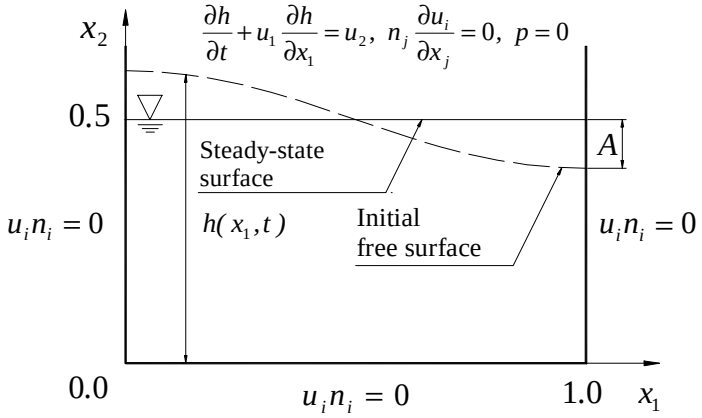

Figure 1. Free oscillations in a tank. Geometry and boundary conditions

The quantitative comparison of the results obtained by the investigated methods is performed for small and medium initial amplitudes. Fig. 4 shows the maximal vertical displacements of the free surface nodes. The results obtained by the prediction-correction method are more accurate. This method needs 2-2.5 times more computations in one time step, but it allows using coarser finite element meshes and larger time steps. Denser finite element meshes reduce the numerical error of the free surface kinematic equation and results of the surface height method become more accurate. In case of initial amplitude A=0.01 m the mass loss is decreased from 0.24 to 0.06% of the total liquid mass in a tank. In case of A=0.05 m the mass loss is reduced from 1.3 to 0.42% of the total mass. The prediction-correction method based on the Lagrangian approach is not applicable to flows with large displacements. Therefore, free oscillations with A=0.1 m are simulated using only the surface height method based on the Arbitrary Lagrange-Eulerian approach. The influence of the numerical parameters to the accuracy of the numerical solution and the mass loss is investigated. Decreasing the size of time step the numerical error of the amplitude is reduced by 10% of the initial amplitude height for μ=1 kg/(m·s). Reducing the mesh size the mass loss is decreased from 1.2 to 0.5% of the total mass for μ=10 kg/(m·s).

The level set method based on the Eulerian approach is used for modelling two-phase flow in a square tank filled by two liquids. Both liquids occupy 50% of the tank. In presented examples $\rho_1 = 1000\,\mathrm{kg/m^3}$, $\rho_2 = 100\,\mathrm{kg/m^3}$, dynamic viscosity coefficient μ=100, 10, 1 kg/(m·s) and initial amplitude A=0.05, 0.1 m. The numerical results of free oscillations in the tank are presented in fig. 5, 6. The accuracy of the interface capturing procedures is limited by the mesh size. After the long time interval the front deviates from the middle of the tank and is inside the smallest finite element (fig. 5d). It means that our numerical results are in

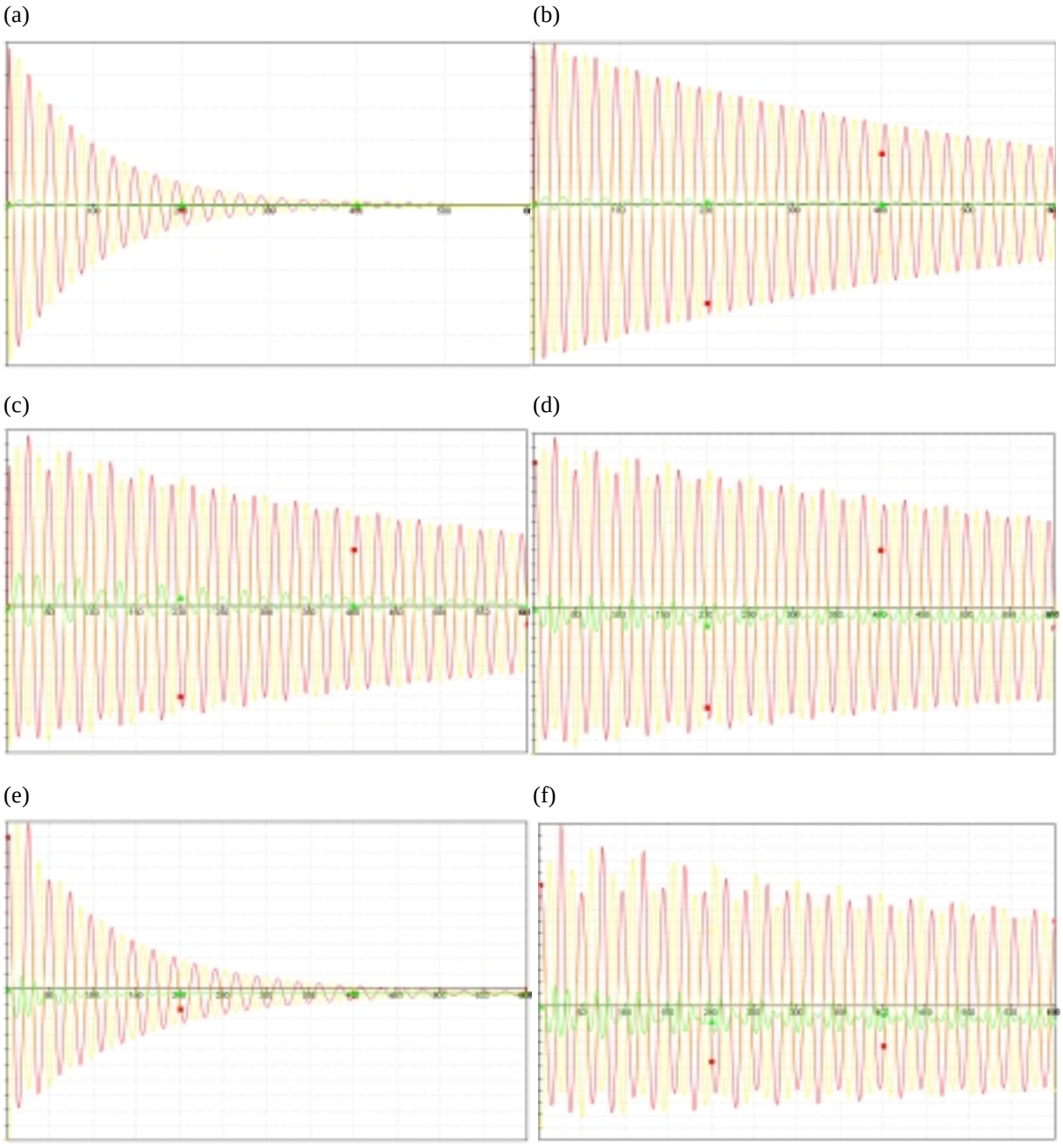

Figure 2. The time histories of the amplitude of free oscillations in the tank T=[0, 30] s.
The Lagrangian approach: (a) A=0.01 m, $\propto$=10 kg/(m $\cdot$ s), finite element mesh 20x10, time step dt=0.005 s,
(b) A=0.01 m, μ=1 kg/(m $\cdot$ s), finite element mesh 60x30, time step dt=0.0025 s,
(c) A=0.05 m, μ=1 kg/(m $\cdot$ s), finite element mesh 60x30, time step dt=0.001 s.
The ALE approach:(d) A=0.05 m, μ=1 kg/(m $\cdot$ s), finite element mesh 64x32, time step dt=0.0005 s,
(e) A=0.1 m, μ=10 kg/(m $\cdot$ s), finite element mesh 40x20, time step dt=0.0025 s,
(f) A=0.1 m, μ=1 kg/(m $\cdot$ s), finite element mesh 60x30, time step dt=0.001 s

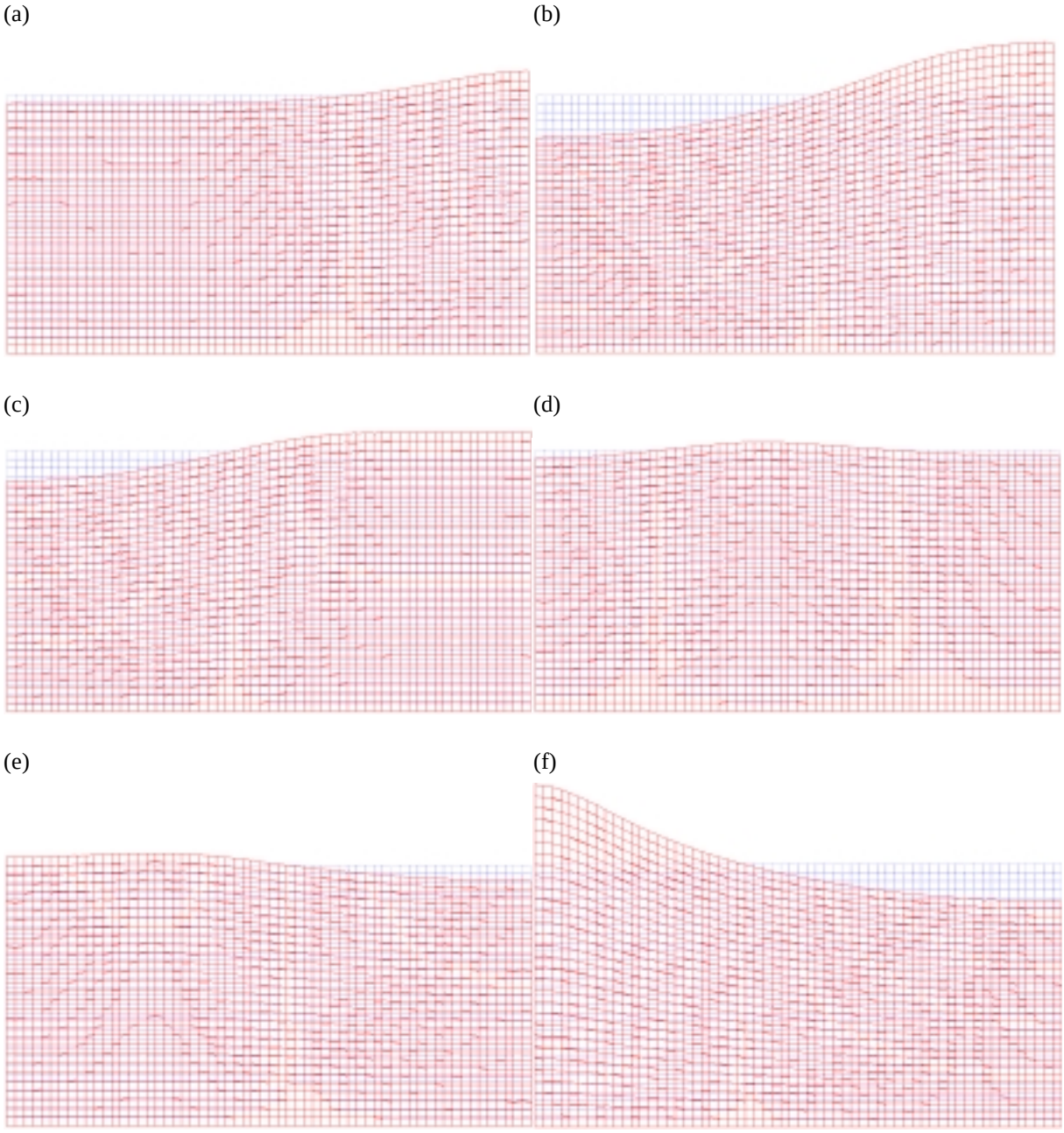

Figure 3. Moving finite element meshes. The Arbitrary Lagrange-Eulerian approach, A=0.1 m, μ=1 kg/(m $\cdot$ s), finite element mesh 60x30, time step dt=0.001s: (a) t=0.35 s, (b) t=0.65 s, (c) t=0.8 s, (d) t=0.9 s, (e) t=0.95 s, (f) t=1.2 s

agreement with the accuracy prescribed by Eulerian methods. Fig. 7 illustrates the evolution of the moving interface in time. It shows the values of the level set function at the points allocated on the centre line of the tank. It is not the exact position of the interface, but its numerical approximation, which is less accurate for large values and more accurate for values approaching the zero level set. Rising curves plotted in fig. 7 show that the total mass is not conserved in time. The most of numerical discretisations of the level set formulations do not preserve this property in general. It was observed that numerical diffusion introduces a normal motion proportional to the local curvature of the interface. In publications[15-16] new re-initialisation procedures are introduced in order to remedy this effect, but they are very complicated and hardly implementable in the first order finite element routines.

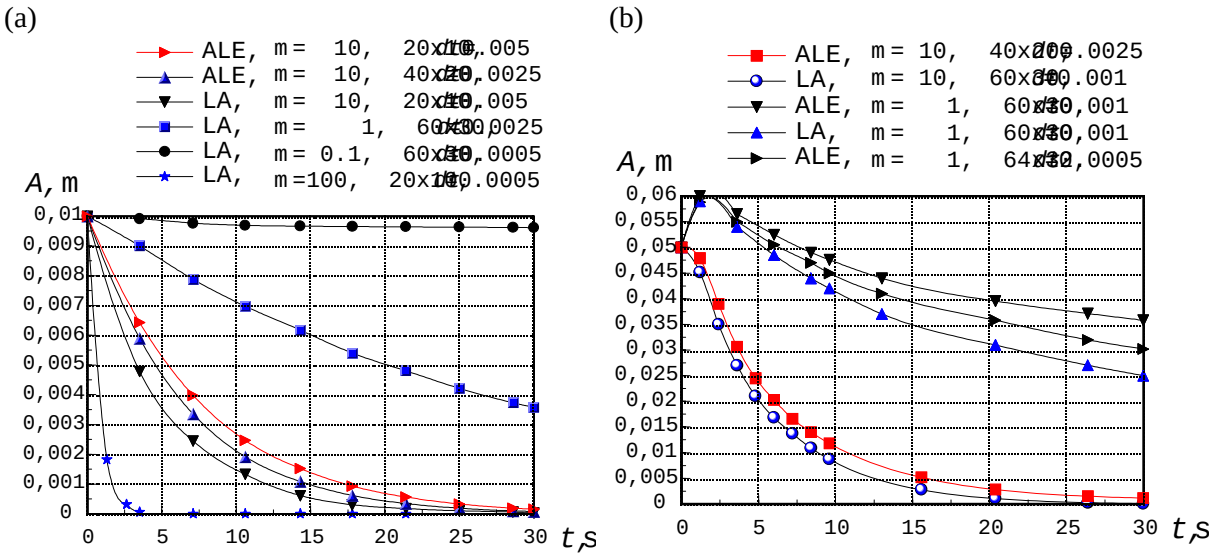

Figure 4. The quantitative comparison of the results. The curves of maximal amplitude for different methods, viscosity coefficients, finite element meshes and time steps (LA – Lagrangian approach, ALE – Arbitrary Lagrange-Eulerian approach): (a) A=0.01 m, (b) A=0.05 m

The material properties are discontinuous at the zero level set. Unwanted instabilities can be caused by large density ratios. The smoothing procedure described by formula (15) helps only for small density ratios of order 10. Instabilities occur after several thousands of time steps for $\rho_1 / \rho_2 = 100$ or 1000. We have tried to overcome this difficulty by introducing higher order interpolation functions or treating finite elements as occupied by only one liquid, but these attempts have been unsuccessful. It may be related with the particular implementation of the velocity correction method. The source term of the Poisson's equation in the pressure calculation step includes the discontinuous density. This discontinuity might cause oscillations in the solution of Poisson's equation.

The bottleneck of the level set method is a numerical implementation of re-initialisation procedure. Solution of non-steady and non-linear problem in every time step takes a lot of computational resources. In this work it was observed that convergence of re- initialisation

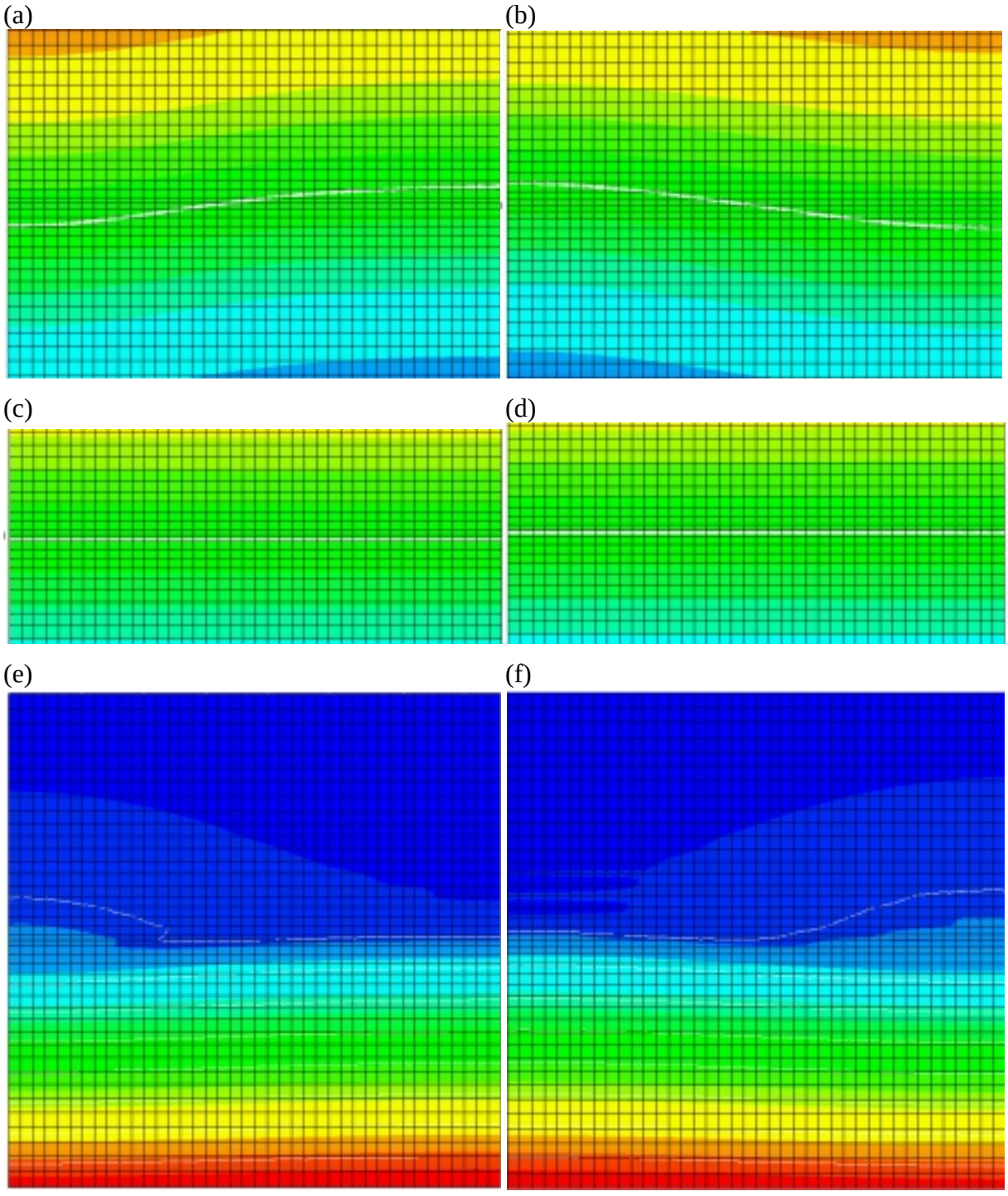

Figure 5. Free oscillations in the tank simulated by the level set method (A=0.05 m, Re=100):
(a) the level set function at t=0.1, (b) the level set function at t=0.7,
(c) the level set function at t=4.2, (d) the level set function at t=8.3,
(e) pressure at t=0.1, (f) pressure at t=0.7

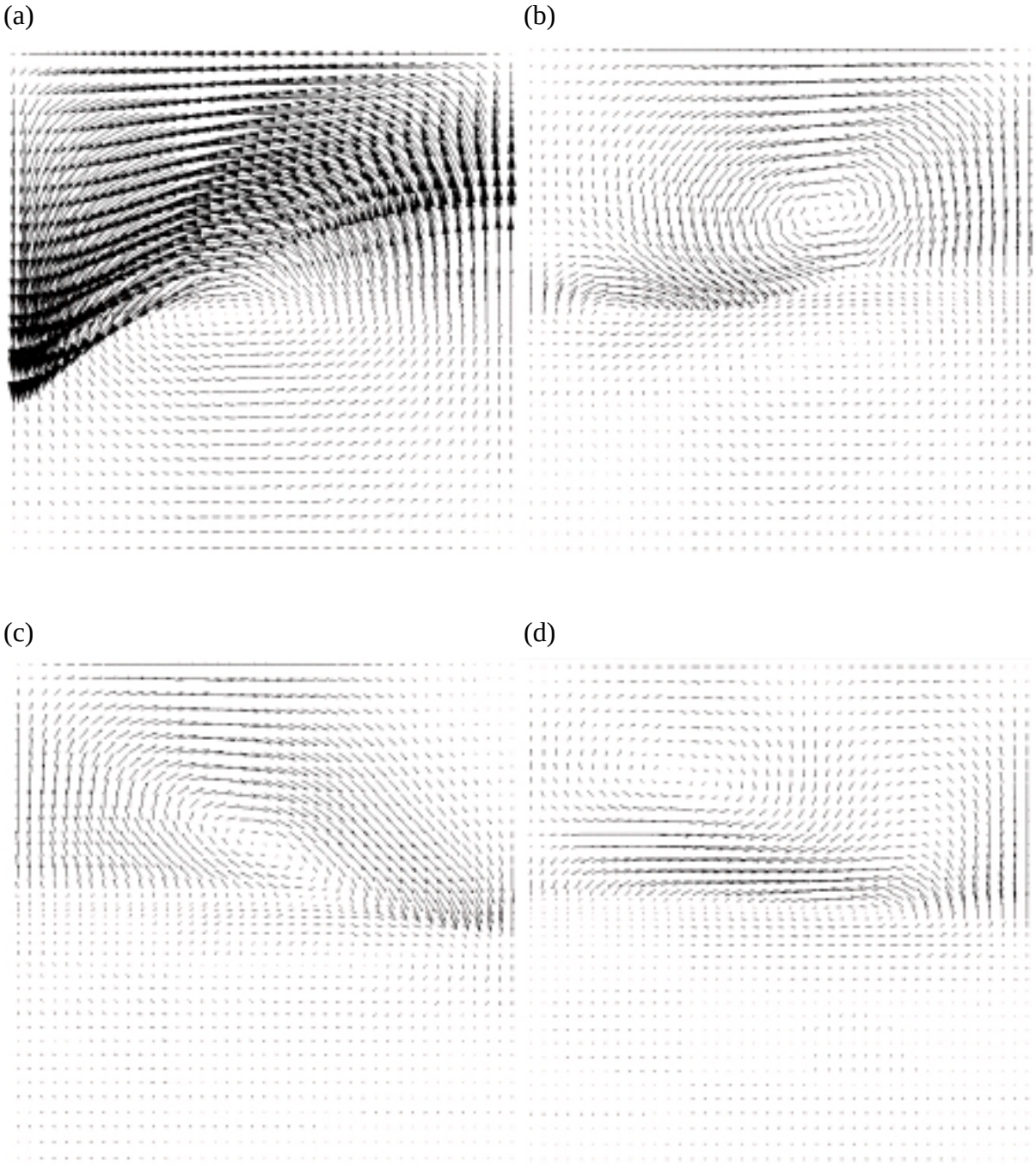

Figure 6. Velocity vectors of the two-phase flow: (a) t=0.4 s, (b) t=0.7 s, (c) t=1.4 s, (d) t=1.5 s

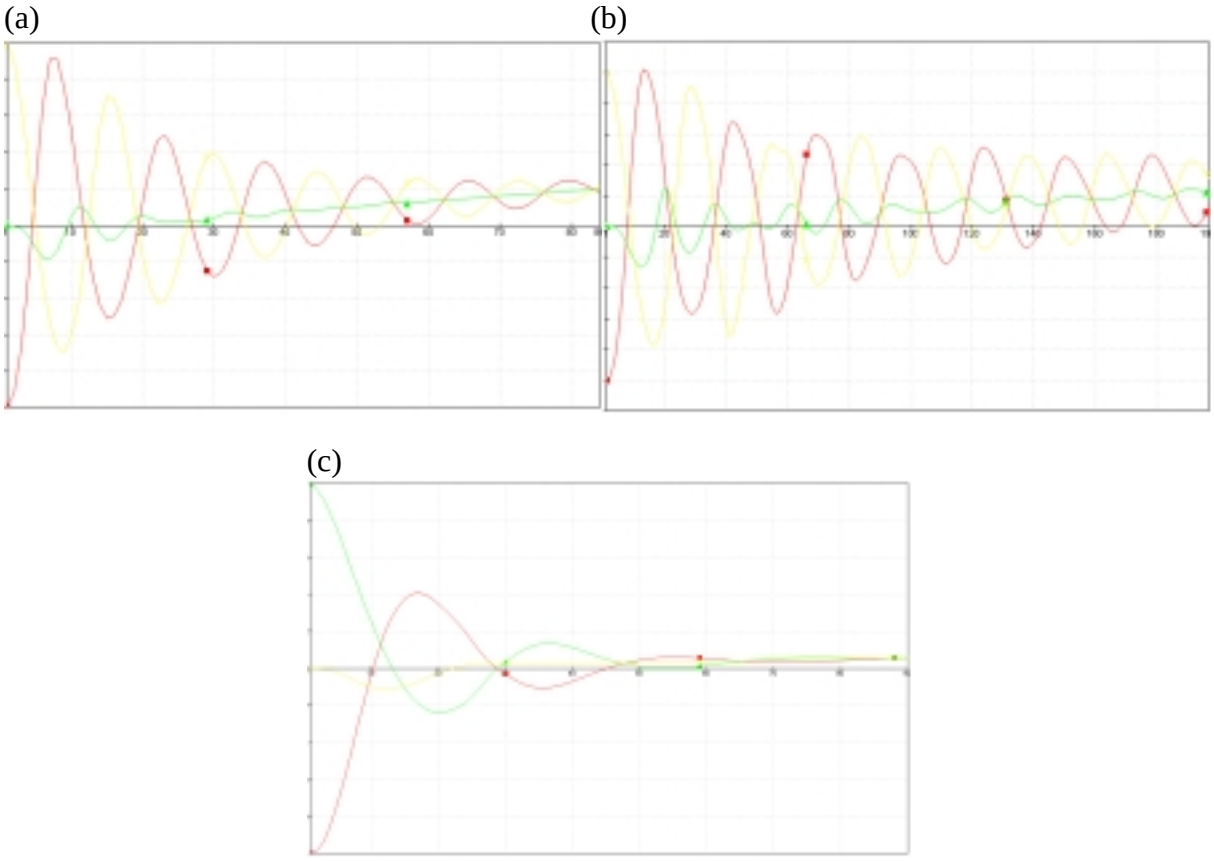

Figure 7. The computed time histories of the level set function at the middle of the tank: (a) A=0.05 m, μ=10 kg/(m·s), T=[0, 8.4]s, (b) A=0.05 m, μ=1 kg/(m·s), T=[0, 9.6]s, (c) A=0.1 m, μ=100 kg/(m·s), T=[0, 4.5]s

procedure implemented using standard finite element schemes is worse than convergence of procedures proposed in finite difference areas[15]. It might be related with non-monotonic or monotonic behaviour of various schemes. Therefore, implicit scheme is applied and its convergence is only checked in a band including the interface. It is quite difficult to find suitable values for such a numerical parameters as artificial time step, a bandwidth and a tolerance of the convergence criterion. The implementation of an efficient re-initialisation procedure presents a great challenge.

The Lagrangian approach to free surface modelling is the most natural and simplest. The results obtained by the prediction-correction method are very accurate. This method allows using coarser finite element meshes and larger time steps. The main drawback of the Lagrangian approach is that the mesh deforms severely as the free surface moves, making remeshing and rezoning necessary. The prediction-correction method was not able to simulate free oscillations with the initial amplitude equals 20% of the liquid depth in the tank. The Arbitrary Lagrange-Eulerian approach includes some advantages of both approaches and avoids some their disadvantages. The arbitrary velocity of the mesh is incorporated into the

convective term, making it more stable and reducing mesh distortion. The numerical results obtained by the surface height method are less accurate, but this method does not need a lot of computational resources in case of free oscillations with small amplitudes. The main problem of the method is to solve free surface kinematics equation accurately and to avoid numerical oscillations. The standard stabilization methods give large numerical error in case of large initial amplitudes. This error could be reduced using denser finite element meshes and smaller time steps. The formulation of the level set method is very flexible and universal, but its numerical implementation in finite element codes is not trivial. It seems that re-initialisation procedure needs advanced discretisation and stabilization methods in case of sophisticated flows. The numerical experiments show that this particular implementation is capable of simulating free oscillations with large amplitudes. The accuracy of the interface capturing method is still dependent on the mesh size at the moving interface.

6 CONCLUSIONS

Three highly different numerical methods for modelling free surface and moving interface flows were investigated and compared in this work. Free oscillations in a tank were simulated by the prediction-correction method based on the Lagrangian approach, the surface height method based on the Arbitrary Lagrange-Eulerian approach and the level set method based on the Eulerian approach. The quantitative comparison of the numerical results obtained by the prediction-correction and surface height methods was performed. In case of small initial amplitudes of oscillations the results obtained by the prediction-correction method were more accurate. Denser finite element meshes allowed reducing the numerical error of the surface height method. In case of large initial amplitudes the prediction-correction method was not able to solve problems that were solved by the surface height method based on the Arbitrary Lagrange-Eulerian approach. In this paper one of the first implementation of the level set method by finite element procedures was investigated. The mathematical model of the level set method is very universal and has potential abilities for describing wide range of moving interface problems. Computational resources and several procedures that are not sufficiently developed limit the actual performance of the method.

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REFERENCES

[1] C.W. Hirt, J.L.Cook and T.D. Butler, "A Lagrangian method for calculating the dynamics of an incompressible fluid with free surface", *J. Comput. Phys.*, 5, 103-124 (1970).

[2] B. Ramaswamy, M. Kawahara and T. Nakayama, "Lagrangian finite element method for the analysis of two-dimensional sloshing problems", *Int. J. Numer. Meth. Fluids*, 6, 659-

670 (1986).
[3] R. Radovitzky and M. Ortiz, "Lagrangian finite element analysis of Newtonian fluid flows", *Int. J. Numer. Meth. Engrg.*, 43, 607-619 (1998).
[4] C.W. Hirt, A.A. Amsden and J.L. Cook, "An arbitrary Lagrangian-Eulerian computing method for all flow speeds", *J. Comput. Phys.*, 14, 227-253 (1974).
[5] T.J.R. Hughes, W.K. Liu and T.K. Zimmermann, "Lagrangian-Eulerian finite element formulations for incompressible flows", *Comput. Methods Appl. Mech. Engrg.*, 29, 329-349 (1981).
[6] C. Farhat, C. Degand, B. Koobus and M. Lesoinne, "Torsional springs for two-dimensional dynamic unstructured meshes", *Comput. Methods Appl. Mech. Engrg.*, 163, 231-245 (1998).
[7] T. Kvamsdal, K.M. Okstad, K. S∫rli and P. Pegon, "Two-level adaptive mesh movement algorithms for FSI computations", *In Computational Methods For Fluid-Structure Interaction* (eds. T. Kvamsdal *et al.*), Tapir Publishers, Trondheim, 121-131 (1999).
[8] S. Chippada, T.C. Jue, S.W. Joo, M.F. Wheeler and B. Ramaswamy, "Numerical simulation of free-boundary problems", *Int. J. Comput. Fluid Dyn.*, 7, 91-118 (1996).
[9] S.R. Idelsohn, E. O⟩ate and C. Sacco, *Finite Element Solution of Free Surface Ship-Wave Problems*, Report No. 119, CIMNE, Barcelona, (1997).
[10] F.H. Harlow and J.E. Welch, "Numerical calculation of time-dependent viscous incompressible flow", *J. Phys. Fluids*, 8, 2182-2189 (1965).
[11] C.W. Hirt and B.D. Nichols, "Volume of Fluid (VOF) method for the dynamics of free boundaries", *J. Comput. Phys.*, 39, 201-225 (1981).
[12] S. Osher and J.A. Sethian, "Fronts propagating with curvature-dependent speed: algorithms based on Hamilton-Jacobi formulation", *J. Comput. Phys.*, 79, 12-49 (1988).
[13] J.A. Sethian, *Level Set Methods*, Cambridge University Press, Cambridge, (1996).
[14] M. Sussman, P. Smereka and S.J. Osher, "A level set method for computing solutions to incompressible two-phase flow", *J. Comput. Phys.*, 114, 146-159 (1994).
[15] Y.C. Chang, T.Y. Hou, B. Merrimann and S.J. Osher, "A level set formulation of Eulerian interface capturing methods for incompressible fluid flows", *J. Comput. Phys.*, 124, 449-464 (1996).
[16] M. Sussman and E. Fatemi, "An efficient, interface-preserving Level Set redistancing algorithm and fluid flow", *SIAM J. Sci. Comput.*, 20, 1165-1191 (1999).
[17] **A. Kačeniauskas and G. Skeie, "Modelling of viscous incompressible flow with free** surface by level set method and finite elements", *In Computational Methods For Fluid-Structure Interaction* (eds. T. Kvamsdal *et al.*), Tapir Publishers, Trondheim, 305-314 (1999).
[18] O. Munthe and H.P. Langtangen, "Finite elements and object-oriented implementation techniques in computational fluid dynamics", accepted for publication in *Comput. Methods Appl. Mech. Engrg.*, (1998).

[19] N.H. Sharif, N.-E. Wiberg and M. Levenstam, "Free surface flow in dams with level set approach and adaptive FEM", *In CD-ROM proc. of I European Conference on Computational Mechanics (Solids, Structures and Coupled Problems in Engineering)*, Munich, Germany, (1999).

[20] P. R∑back, "Level-set approach to the finite element method". *In proc. of XII Nordic Seminar on Computational Mechanics* (eds. R. Kouhia and M. Mikkola), Helsinki, 12-15 (1999).

[21] M. Kawahara and K. Ohmiya, "Finite element analysis of density flow using the velocity correction method", *Int. J. Numer. Methods Engrg.*, 6, 659-672 (1986).

[22] R.W. Lewis, K. Ravindran and A.S. Usmani, "Finite element solution of incompressible flows using an explicit segregated approach", *Arch. Comp. Methods Engrg.*, 2, 69-93 (1995).

[23] S. Hassanzadeh, V. Sonnad and S. Foresti, "Finite element implementation of boundary conditions for the pressure Poisson equation of incompressible flow", *Int. J. Numer. Methods Engrg.*, 18, 1009-1019 (1994).

[24] **A. Kačeniauskas and R. Kačianauskas, "**Modelling of free surfaces using moving finite element meshes", *J. Mechanika*, 4, 29-37 (1999).

[25] A.N. Brooks and T.J.R. Hughes, "Streamline upwind/Petrov-Galerkin formulation for convection dominated flows with particular emphasis on the incompressible Navier-Stokes equation", *Comput. Methods Appl. Mech. Engrg.*, 32, 199-259 (1982).

[26] C.W. Dawson, "A practical computer method for solving ship-wave problems", *Proc. II Int. Conf. Num. Ship Hydrodynamics*, USA, (1977).

[27] E. O⟩ate, "Derivation of stabilized equations for advective-diffusive transport and fluid flow problems", *Comput. Methods Appl. Mech. Engrg.*, 58, 233-267 (1998).

[28] E. O⟩ate and S.R. Idelsohn, *Stabilization of the numerical solutions for the free surface wave equation in fluid dynamics*, Research Report No. 110, CIMNE, Barcelona, (1997).

[29] J. Donea, "A Taylor-Galerkin method for convective transport problems", *Int. J. Numer. Methods Engrg.*, 20, 101-119 (1984).

[30] T.J.R. Hughes, M. Mallet and A. Mizukami, "A new finite element formulation for computational fluid dynamics: II. Beyond SUPG", *Comput. Methods Appl. Mech. Engrg.*, 58, 341-355 (1986).

[31] H.P. Langtangen, "Computational Partial Differrential Equations Numerical Methods and Diffpack Programming", *Lecture Notes in Computational Science and Engineering*, 2, Springer-Verlag, (1999).