NUMERICAL STUDY OF A PULSATING PLANE JET
IN QUIESCENT ENVIRONMENT

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Key words: Jet, Pulsed, Unsteady, Plane, Pulsation Amplitude, Pulsation Frequency, Laminar Regime.

Abstract. The pulsating free air jet issues from a plane nozzle in quiescent environment have been numerically investigated to determine the influence of several parameters on the flow. The studied parameters include Reynolds number, Re, Grashof number, Gr, Strouhal number, St, and the pulsation amplitude A. At the exit of the nozzle, the pulsating flow is imposed with a uniform temperature $T_0$ and a velocity $u=u_0(1+A \sin(\omega t))$. The unsteady Navier-Stokes equations are solved numerically with a finite difference schema. The results obtained show that the pulsation affects the flow in a near region of the nozzle to reach the same asymptotic regime than the steady jet established by Mhiri and al [10]. The results also show to deduct that the initial development of the jet is considerably faster and the entrainment in the first diameters is enhanced.
1 INTRODUCTION

Due to the fact of the diversity of their aspects and their applications, typical jet flows present a constant interest. Indeed, many applications are met in industry such as, the pulverization, the cooling by film, aeronautic propellers, the welding, etc...[1]

Among the numerous works on this vast subject, some have treated the influence of an initial perturbation on the development of the jet [2]. The motivations have been of two orders:

♦ On the practical plan, it consists on accelerating the jet expansion and increasing the exterior fluid entrainment.

♦ On the fundamental plan: the superposition of the periodic perturbation to the flow was proved to be a very fruitful method of investigation in order to understand the transition to the turbulence.

Most of works dealing with pulsating jet are originally experimental [3-9]. These studies present different approaches based either on a cyclic pulsation of the flow upstream the ejection, or on the utilization of an acoustic emission generating a field perturbation of the external static pressure.

Hussain and al [2,5], as well as Kelmanon [3] and Vulis [4], studied experimentally the plane jet submitted to a controlled sinusoidal perturbation in the initial region of the jet. They found that the entrainment of the jet in the first diameters increased at much faster rate in pulsating jets than in steady jets. Farrington and al [6] as well as Rockwells and al[7], treated pulsed plans jets experimentally, they found that the vortex formation occurred closer to the nozzle and the vortices were larger than in the steady-jet. Therefore, the entrainment increased, and the length of the potential core decreased.

A flow of type pulsed jet has been treated therefore experimentally; the complexity of the present phenomena gives back a purely difficult theoretical analysis of the problem otherwise it can be presented under a simplified form. The existence of such difficulty opens an interesting field although it is delicate to the utilization of numerical resolution methods.

In this present work, a numerical approach has been adopted to study the evolution during the time of a plane free pulsed jet in laminar regime.

2 MATHEMATICAL FORMULATION OF THE PROBLEM

2.1 Hypotheses

We consider vertical jet issues in quiescent ambient, from a rectangular plane nozzle with small dimensions in comparison to enclosure, or the ambient environment in which emerges the flow. The jet and the ambient fluid are constituted of the same fluid. The friction forces are of similar magnitude as inertia forces and thus we have boundary layer type of flow. The fluid is supposed incompressible, in this sense , the fluid density is supposed constant for the used range of the Grashof numbers, except in the buoyancy forces, where one considers a linear variation of this last with the temperature: it is the Boussinesq hypotheses. The width of the nozzle e is assumed to be very wide compared to its thickness in order to neglect edge
effects and to have a two-dimensional problem. The jet is submitted to a longitudinal and periodic perturbation to a unidirectional character of the ejection velocity. The flow is considered in a laminar unsteady regime.

2.2 Governing equations

The Navier-Stokes and the energy equations, in properly nondimensionalized form, can be written, using standard notation, as

Continuity:

\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0
\]  

Momentum equation:

\[
\frac{\partial U}{\partial \tau} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = \frac{1}{Re} \frac{\partial^2 U}{\partial Y^2} \pm \frac{Gr}{Re^2} \theta
\]  

Energy equation:

\[
\frac{\partial \theta}{\partial \tau} + U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{1}{Re Pr} \frac{\partial^2 \theta}{\partial Y^2}
\]  

\(\gamma\) takes values 0 and 1, for \(\gamma = 0\) the first two equations formulate the isothermal plane jet. When \(\gamma = 1\), one is in the buoyant jet case, the sign + represents the case of a ascending buoyant jet or cold descendant, whereas the sign - represents the case of a descendant buoyant jet or cold ascending.

The nondimensionalized equation (1) to (3) are as follows, using the nondimensionalized quantities:

\[
X = \frac{x}{e}, \quad Y = \frac{y}{e}, \quad U = \frac{u}{u_0}, \quad V = \frac{v}{u_0}, \quad \theta = \frac{T - T_m}{T_0 - T_m}, \quad \tau = \frac{tu_0}{e}
\]

\[
Re = \frac{u_0 e}{v}, \quad Gr = \frac{g \beta (T_0 - T_m)e^3}{v^2}, \quad St = \frac{f e}{u_0}, \quad Pr = \frac{v}{\alpha}, \quad \omega t = 2\pi St\tau
\]

The associated boundary condition to the equations (1) to (3) are described as

At \(X > 0\):

\[
\left\{ \begin{array}{ll}
\frac{\partial U(X,Y,\tau)}{\partial Y} = V(X,Y,\tau) = \frac{\partial \theta(X,Y,\tau)}{\partial Y} = 0 & \text{at } Y = 0 \\
U(X,Y,\tau), \theta(X,Y,\tau) \to 0 & \text{at } Y \to \infty
\end{array} \right.
\]  

The nondimensionalized conditions at the ejection of the nozzle is adapted in this study as:
\[ X = 0 \rightleftharpoons V(X,Y,\tau) = 0 \text{ and } \begin{cases} * \text{Si} & 0 \leq Y < 0.5 \\ U(X,Y,\tau) = 1 + A \sin(2\pi St\tau) \\ \theta(X,Y,\tau) = 1 \\ * \text{Si} & Y \geq 0.5 \\ U(X,Y,\tau) = 0, \theta(X,Y,\tau) = 0 \end{cases} \] (6)

2.3 Numerical method of resolution

In this work, equations (1) to (3) associated to their boundary conditions are solved numerically by a finite difference schema. The used mesh is staggered: the equation of continuity is discretized at node \((i+1/2,j+1/2)\) whereas momentum and energy equations are discretized at node \((i, j+1/2)\). This method used in a previous work \([10]\), has been adopted for stability numerical reasons in relation to the nonstaggered mesh. The used mesh is non-uniform in following \(X\). One indeed, the step is taken very thin near the nozzle \((\Delta X_1=10^{-4}\) for \(0 < X < 2)\); a few farther, the calculation step increases \((\Delta X_2=10^{-3}\) for \(2 < X < 10)\); to be able to go farther in the jet, bigger step is adapted \((\Delta X_3=10^{-2}\) for \(X > 10)\). In the transversal direction, the used mesh is uniform, the calculation step is constant \((\Delta Y=0.01)\), and his value imposes a number of \(N\) points in this direction so that the jet is not cut. The distance \(Y=(n-1)*\Delta Y\), for Reynolds and Grashof numbers used, is 24. The resolution time was such that one pulsating period was divided by 120 time steps. In most cases, periodically temporal solution was obtained after 6-12 cycles of pulsation.

3 RESULTS AND DISCUSSION

The main objective of this study is to examine the influence of pulsation on the behavior of a plane jet in a laminar unsteady regime.

3.1 Isothermal jet

The numerical calculation code elaborated permits us to determine the dynamic characteristics of a pulsed plan isothermal air jet \((Pr=0.71)\) in a laminar regime.

On the figure 1, one represents the profiles of centerline velocity of the jet \((Y=0)\) for different Reynolds numbers and the other parameters are constantly maintained. On this same figure, one remarks in a given instant \((\omega t=9\pi/2)\) and for fixed pulsation amplitude and pulsation frequency \((Strouhal number)\), the flow presents oscillations that disappear at the same distance for different Reynolds number. One can also remark that the centerline velocity of pulsed jet presents fluctuations in the zone of the potential core, the amplitude of these last is raised more with the low Reynolds numbers. That permits us to deduce that the impact of the pulsation is more important when the flow is in a low movement.

One notices likewise that for the different Reynolds numbers considered, fluctuations created by pulsation disappear to the same distance \(X=10\), the variation of the Reynolds number doesn't influence therefore the extended of the region where there are creation and dissipation of these fluctuations.
The evolutions of the longitudinal centerline velocity of the jet for different instants are presented in figure 2. One notes that the sustained pulsation of the velocity generates the progressive waves that propagate, in the jet, to distances more downstream the nozzle when the time increases. For the elevated distances these waves will be amortized completely.

The profiles of the vertical centerline velocity of the jet (Y=0) are shown in figure 3 according to the longitudinal distance X, for different pulsation amplitudes (figure 3a) and for different Strouhal numbers (figure 3b).

One notes that, when the pulsation amplitude is low (of the order of 5%), the profile of the centerline velocity of the pulsed jet tends down towards the one of the permanent jet established by Mhiri and al [10]. When the amplitude increases, one distinguishes three regions: the first near the nozzle, in which the centerline velocity profiles are almost parallel to the permanent jet one (region I), more downstream, in the third region (region III), profiles of the pulsed jet coincident with those of the no pulsed jet. A zone of transition (region II) joins these two regions in which appear the oscillations of the velocity on the jet axis that accompanied by a disappearance of the potential core. One also notices that, these oscillations amplify themselves where the pulsation amplitude increases. For a fixed time, these oscillations propagate in the axial region over a vicinity distance of 8 to 9 width of the nozzle and attenuate to disappear completely to 10 width of the nozzle.

One records on the figure 3b, that when the number Strouhal increases, the creation of oscillations takes place more and more near the nozzle and therefore, the length of the potential core decreases and in this case the region III begins in vicinity of the nozzle.

On the figure 4, one represents the evolution of the dynamic half-thickness of the jet in a certain given instant for Re=100, A=30% and for different Strouhal numbers. One notice that the expansion of the jet takes place more rapidly when the Strouhal number increases.

3.2 Buoyant jet

In this section one presents, the results obtained for a heated plane pulsed jet in a laminar regime. Our results are presented for the case of an ascending hot jet or descendant cold jet (γ= +1).

On the figure 5, one distinguishes a variable extent transition zone that is the place of movement pulsed dissipation, this region decreases in thickness when the inertia effects become dominating (increasing Re). Indeed, at great Reynolds number, fluctuations created by the pulsation disappear quickly, one has interest thus to work with a low Reynolds number to look better at the pulsation effect on the flow in a larger region.

On the figure 6, one note that the increase of the Grashof number drags the widening of the region of fluctuation dissipation, the heating is therefore also an important factor for the study of the behavior of the pulsed flow.

In following, one fixed the Reynolds number at 100 and the Grashof number equal to 500 to work in a mixed convection case.

One starts with representing on the figure 7, the evolution of the centerline temperature of the jet (at Y=0). The figure 7a shows that in the near region of the nozzle (region I), the temperature of the pulsed jet profiles coincident with his non-pulsed homologues, even for the
high pulsation amplitudes of the jet. On the other hand in the transition region (region II), the centerline temperature profiles present fluctuations. These fluctuations are at first amplified when the pulsation amplitude of jet increases, on a distance of 10 to 12 times of the nozzle width, than decay and vanish completely to 12 times of the nozzle width (region III).

According to the figure 7b, one notes that the transition zone (region II) is larger when the Strouhal number decreases. Therefore the increases of the Strouhal number drags the oscillations creation closer to the nozzle. One can also mention that when the Strouhal number is big enough (superior to 1.6), profiles of centerline temperature coincide with those of the non-pulsed jet dice the nozzle exit. It permits us to conclude that beyond St=1.6, pulsation doesn't influence anymore the characteristics of the pulsed heated jet.

The evolutions of the longitudinal centerline temperature of the jet for different instants are presented in figure 8. Similar variation can be observed that figure 2.

The figure 9a represents the transverse limit velocity of the jet. One remarks that, the transverse limit velocity increases with the pulsation amplitude in the core potential zone. Therefore the entrainment of the exterior fluid in relation to the non-pulsed jet increases. On the other hand while moving away of the nozzle (plume region) the transverse limit velocity profiles of the pulsed jet coincide with the non-pulsed jet. What confirms the fact that pulsation doesn't influence the flow only for the distances closer to the nozzle.

The same observations can be done in the case where the Strouhal number varies (figure 9b).

The temporal behavior of centerline velocity $U$ and of centerline temperature $\theta$ for different axial distances is plotted in figure 10. One finds that these two sizes present a sinusoidal variation propagating downstream the nozzle. One also observes a tightening of the wave form towards 3.09 times of the nozzle width (figure 10a). More downstream any oscillation is detectable far from the nozzle. Closer to the nozzle, no period is detectable in temperature profiles (figure 10b). Beyond of an equal distance to 0.6 times of the nozzle width these profiles present a periodical part with a weak amplitude will be thereafter amortized.

4 CONCLUSION

The effects of the important governing parameters, such as Reynolds numbers, Grashof number, Strouhal number and the pulsation amplitude on the pulsed plane jet in unsteady regime have been studied numerically.

The obtained results show that the influence of pulsation is especially observed in the jet region (in the vicinity of the nozzle) and the introduction of a perturbation that drags the creation of fluctuations closer to the nozzle. It also found that the disappearance of these last occurs at the same distance (at $X=10$) when the pulsation amplitude changes and on the different distances when the Strouhal number varies. It is shown that when the pulsation amplitude is low, the velocity and the temperature profiles resemble much to the quasi-steady solutions established by Mhiri and al.[10].

At the end, we retain on one hand that pulsation doesn't therefore modify the parameters of the flow in the plume region (far from the nozzle), on the other hand it accelerates the initial
development of the jet and ameliorate the diffusion and the entrainment of the ambient air in the first diameters.

Figure 1: Centerline velocity profiles

Figure 2: Centerline velocity profiles

Figure 3: Centerline velocity of the jet
Figure 4: dynamic half-thickness of the jet

Figure 5: Centerline velocity profiles

Figure 6: Centerline velocity of the jet

Figure 7: Centerline temperature of the jet
Figure 7: Centerline temperature of the jet

Figure 8: Centerline temperature of the jet

Figure 9: Transverse limit velocity of the jet
REFERENCES


NOMENCLATURE

\( A \) \hspace{1cm} \text{dimensionless pulsation amplitude of the axial velocity.}
\( e \) \hspace{1cm} \text{width of the nozzle, m}
\( f \) \hspace{1cm} \text{dimensional frequency, s}^{-1}
\( \text{Gr} \) \hspace{1cm} \text{Grashof number, } \frac{g \beta (T_0-T_{\infty}) e^3}{\nu^2}
\( \text{Pr} \) \hspace{1cm} \text{Prandtl number, } \frac{\nu}{\alpha}
\( \text{Re} \) \hspace{1cm} \text{Reynolds number, } \frac{u_0 e}{\nu}
\( \text{St} \) \hspace{1cm} \text{Strouhal number, } \frac{f e}{u_0}
\( t \) \hspace{1cm} \text{dimensional time, s}
\( T \) \hspace{1cm} \text{dimensional temperature, K}
\( u, v \) \hspace{1cm} \text{dimensional axial and transverse components of the velocity, m.s}^{-1}
\( U, V \) \hspace{1cm} \text{dimensionless axial and transverse components of the velocity.}
\( x, y \) \hspace{1cm} \text{dimensional axial and transverse coordinates, m}
\( X, Y \) \hspace{1cm} \text{dimensionless axial and transverse coordinates}
\( Y_{0.5U} \) \hspace{1cm} \text{dimensionalless dynamic half-thickness of the jet}

Symboles grecs

\( \alpha \) \hspace{1cm} \text{Thermal diffusivity of fluid, m}^2\text{s}^{-1}
\( \beta \) \hspace{1cm} \text{coefficient of thermal expansion of the fluid}
\( \nu \) \hspace{1cm} \text{Kinematic viscosity of fluid, m}^2\text{s}^{-1}
\( \tau \) \hspace{1cm} \text{dimensionless time, } \tau/(e/u_0)
\( \theta \) \hspace{1cm} \text{dimensionless temperature}
\( \omega \) \hspace{1cm} \text{angular velocity, } \omega=2\pi f$

Indices

\( c \) \hspace{1cm} \text{jet centerline condition}
\( \ell \) \hspace{1cm} \text{limit}
\( 0 \) \hspace{1cm} \text{jet exit condition}
\( \infty \) \hspace{1cm} \text{ambient environment}