NUMERICAL SIMULATION OF GRANULAR MATERIAL SILOS UNDER EARTHQUAKE EXCITATION

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Abstract. The investigation of bulk material filled silos under seismic excitation is just at the beginning. In this paper a numerical model for a silo will be described. This model consists of three components: the granular material, an interface element between the granular material and the silo shell itself. Some first results dealing with the dynamic behaviour of the granular material are presented in this paper. The bulk material behaviour is described in four different ways: hypoplasticity theory, two special versions of it which use time history functions and finally the intergranular strain approach.
1 INTRODUCTION

Silos for granular materials subjected to strong earthquake motions should remain operative after earthquakes in order to avoid disruptions in the flow of materials needed for direct aid to the population and also to provide materials needed for the rebuilding process.

Recent earthquakes have shown that the number of silo failures is still quite high although the safety of structures against structural collapse is steadily increasing. While the stiffness and mass distributions are well defined for normal building structures, a number of problems arise with granular material. The contact problem between the granular material and the silo wall and the non-linear material behaviour of the granular material itself leads to a highly non-linear interaction problem. Linear models have shown to be inadequate for a reliable simulation of the overall problem. While the dynamic behaviour during the charging and discharging process is well-known, the dynamic behaviour during an earthquake still poses some problems. The current codes are not very helpful for determining the effective mass to be used for the earthquake design.

2 MODELING

2.1 Discretisation

The silo has been modelled as a three-dimensional structure with three components: the granular material, the contact area between the granular material and the silo shell, and the silo shell itself.

![Discretisation of the silo](image)

2.2 Used elements

The silo shell is modelled as a 9-node shell element based on the theory of Huang and Hinton. The interface element used to simulate the interaction between the silo shell and the granular material is an 18-node volume element that consists of two layers of nodes. The element links the adjacent nodes of the shell element and the granular material. This direct coupling of the two nodes permits a local decoupling of the granular material from the shell. The Mohr-Coulomb law has been used as a friction law.
To calculate the stiffness-matrix, a surface matrix has to be determined, which describes the corresponding area of each node from the overall area. The corresponding area of the node $k$ results from:

$$A_k = \int_A N_k(\xi, \eta) \, dA = \int_{\xi, \eta} N_k(\xi, \eta) |J_A| \, d\xi d\eta = \sum_{i=1}^{m} \sum_{j=1}^{n} N_k |J_A| \alpha_i \alpha_j$$

(1)

The stiffness matrix is assembled from the individual spring stiffnesses connecting the nodes $k$ and $k'$. The spring stiffness depends on the relative displacement and the used friction law. This spring stiffness is a product of an absolute term $c_d$ and the corresponding area $A_k$ and is given by the following formulas:

$$c_n^k = c_d A_k \quad \text{for} \quad \Delta u_n^k \leq 0$$

$$c_n^k = 0 \quad \text{for} \quad \Delta u_n^k > 0$$

(2)

(3)

The relative displacements in the normal direction ($\Delta u_n^k$) and in the two tangential directions ($\Delta u_{t1}^k$, $\Delta u_{t2}^k$) are given by

$$\Delta u_n^k = (\bar{u}^k - \bar{u}^k)^{'} \bar{v}_n^k$$

$$\Delta u_{t1}^k = (\bar{u}^k - \bar{u}^k)^{0} \bar{v}_1^k$$

$$\Delta u_{t2}^k = (\bar{u}^k - \bar{u}^k)^{0} \bar{v}_2^k.$$  

(4)

(5)

(6)

The stiffness in the tangential direction results in a manner analogous to the stiffness in normal direction:
The tangential stiffness is iteratively approximated by the Mohr-Coulomb friction law
\[
\tau \leq \sigma \tan \varphi = \sigma \mu_0
\]
\[\tau^k_n = c^k\Delta u^k_n \quad \text{for } i=1, 2 \]  
\[\sigma^k_n = c_n \Delta u^k_n \]  
leading to
\[
c^k_u = \max(c_u) \quad \text{for } \Delta u^k_u < \mu_0 \Delta u^k_u \frac{c^k_u}{\max(c_u)} \quad \text{for } i = 1, 2 \]  
\[
c^k_u = c^k_u \mu_0 \frac{\Delta u^k_u}{\Delta u^k_u} \quad \text{for } \Delta u^k_u \geq \mu_0 \Delta u^k_u \frac{c^k_u}{\max(c_u)} \quad \text{for } i = 1, 2 \]  

The 27-node volume element for the granular material describes the material behaviour by using a hypoplastic material law, which is modified according to the two history functions and the intergranular strain theory.

3 MATERIAL LAW FOR THE GRANULAR MATERIAL

3.1 Hypoplasticity

The hypoplasticity theory describes the material behaviour without additional terms like a yield surface or a plastic potential. It does not distinguish between elastic and plastic deformation. The hypoplasticity theory describes the stress rate \( \dot{T} \) subjected to the strain-rate \( D \), the effective stress \( T \) and a linear and non-linear stiffness-matrix \( L \) and \( N \).

\[
\dot{T} = L(\dot{T}) : D + N(\dot{T}) \| D \|
\]

A comprehensive introduction is given by Kolymbas. The modified hypoplasticity by von Wolffersdorff is the base of the following investigation.

3.2 Bauer’s History function

The hypoplasticity theory is able to describe the energy dissipation during cyclic excitation only very roughly. The material law cannot describe the stiffness increase at the beginning of the reloading process. The stiffness does not depend only on the effective stress but also on the stress time history. Bauer added to the hypoplasticity a function describing the time dependent behaviour of granular material:

\[
\dot{T} = L(\dot{T}) : D + N(\dot{T}) \| D \| [I + f] [I + f] \]

The function \( f_t \) limits the compressibility by way of:

\[
 f_t = r_0 \exp \left[ -r_1 l_{cr} \right] = r_0 \exp \left[ -r_1 \frac{e - e_{\text{min}}}{e_{cr} - e_{\text{min}}} \right].
\]  

(15)

Here, \( r_0 \) and \( r_1 \) are material parameters and \( e_{cr}, e_{\text{min}} \) and \( e \) are the critical, the minimal and the effective void ratio.

Bauer formulates the time history function \( f_m \) as a function of the displacement capacity:

\[
 f_m = \mu_1 \frac{A}{A_{\text{max}}} \frac{\text{tr}(\mathbf{T}D) + \text{tr}(\mathbf{T}D)}{\|\mathbf{T}\|\|\mathbf{D}\|}.
\]

(16)

The history parameter \( A \) is dependent on the load path.

\[
 \dot{A} = \mu_2 \left[ \|\text{tr}(\mathbf{T}D)\| - \text{tr}(\mathbf{T}D) \right] - \mu_3 \frac{A}{A_{\text{max}}} \left[ \|\text{tr}(\mathbf{T}D)\| + \text{tr}(\mathbf{T}D) \right] [1 + f_t]^{-1}
\]

(17)

\[
 A \geq 0
\]

(18)

\[
 A_{\text{max}} = 1 \quad \text{for} \quad t = 0
\]

(19)

\[
 A_{\text{max}} = A \quad \text{for} \quad \text{tr}(\mathbf{T}D) < 0
\]

(20)

\[
 \dot{A}_{\text{max}} = 0 \quad \text{for} \quad \text{tr}(\mathbf{T}D) \geq 0
\]

(21)

For \( \text{tr}(\mathbf{T}D) < 0 \) the parameter \( A \) will increase, otherwise \( A \) will decrease.

### 3.3 Modified history function

Braun\(^2\) modified the time history by using the energy criteria of Chen and Mizuno\(^3\) instead of the displacement capacity. The energy concentration depends on

\[
 d\Omega = \varepsilon_{ij} d\sigma_{ij}
\]

(22)

and the modified history function \( f_m \) is given by:

\[
 f_m = \mu_1 \frac{A}{A_{\text{max}}} \frac{d\Omega}{\|d\mathbf{T}\|\|\mathbf{D}\|}
\]

(23)

and

\[
 \|d\mathbf{T}\| = \sqrt{d\sigma_{11}^2 + d\sigma_{22}^2 + d\sigma_{33}^2 + 2(d\sigma_{12}^2 + d\sigma_{13}^2 + d\sigma_{23}^2)}
\]

(24)

\[
 \|\mathbf{D}\| = \sqrt{\varepsilon_{11}^2 + \varepsilon_{22}^2 + \varepsilon_{33}^2 + 2(\varepsilon_{12}^2 + \varepsilon_{13}^2 + \varepsilon_{23}^2)}
\]

(25)

The history parameter \( A \) is, analogously to Eq. (17)

\[
 \dot{A} = \mu_2 \left[ \|d\Omega\| - d\Omega \right] - \mu_3 \frac{A}{A_{\text{max}}} \left[ \|d\Omega\| + d\Omega \right]
\]

(26)
\[ A \geq 0 \quad (27) \]
\[ A_{\text{max}} = 1 \quad \text{for } t = 0 \quad (28) \]
\[ A_{\text{max}} = A \quad \text{for } d\Omega < 0 \quad (29) \]
\[ \dot{A}_{\text{max}} = 0 \quad \text{for } d\Omega \geq 0 \quad (30) \]

### 3.4 Intergranular strain

By taking the relative displacements into account, the hypoplastic material laws can properly simulate the material behaviour but they are not able to simulate small load amplitudes in the case of cycling loading. In this material law, the deformations are accumulated unrealistically, causing the so-called ratcheting effect.

The intergranular strain considers not only the deformation of the granular skeleton due to grain rearrangement but also the deformation of the contact area of the bulk material. In the case of small cyclic loading before the granular skeleton is rearranged, the loading path has to cross the area of deformation of the contact area between the grains. This reproduces the material memory of the cyclic loaded grain. In this case the material will not show any ratcheting effect.

The material memory of the granular material is limited to the size of the strain path which is necessary to reach the stiffness of a monotonic loading. If this material stiffness is reached, the material memory of the granular material is „deleted“.

Niemunis and Herle explained the intergranular strain by means of the “brick analogy” of Simpson. A man is attached to a single brick by a string of the length R. The movement of the man is a symbol for the deformation of the granular material. The behaviour of the brick describes the material behaviour of the granular material. If the man moves while the brick rests, the material behaviour can be described with the intergranular strain approach. But if the brick moves the material behaviour can be described with the hypoplastic model.

Niemunis and Herle suggested the following expression for the calculation of the stiffness \( M \)

\[
\mathbf{M}_i = \left[ \rho^2 m_T + (1-\rho^2) m_R \right] \mathbf{L} + \begin{cases} 
\rho^2 (m_T - m_R) \mathbf{L} : \mathbf{\hat{\delta}} + \rho^2 \mathbf{N} \mathbf{\hat{\delta}} & \text{für } \mathbf{\hat{\delta}} : \mathbf{D} > 0 \\
\rho^2 (m_R - m_T) \mathbf{L} : \mathbf{\hat{\delta}} & \text{für } \mathbf{\hat{\delta}} : \mathbf{D} \leq 0
\end{cases}
\] 

\[ (31) \]

where \( m_T, m_R, r \) and \( \chi \) are material parameters.
The direction of the intergranular strain $\hat{\delta}$ and the factor $\rho$ are given in the following manner

$$
\hat{\delta} = \begin{cases} 
\delta / \|\delta\| & \text{for } \delta \neq 0 \\
0 & \text{for } \delta = 0
\end{cases}
$$

(32)

$$
\rho = \frac{\|\delta\|}{R}
$$

(33)

The development of the intergranular strain can be calculated with the following equation

$$
\delta = \begin{cases} 
(1 - \hat{\delta\delta\delta}\rho^\beta) \cdot \hat{D} & \text{für } \hat{\delta} : \hat{D} > 0 \\
\hat{D} & \text{für } \hat{\delta} : \hat{D} \leq 0
\end{cases}
$$

(34)

4 CYCLIC BEHAVIOUR

To estimate the cyclic behaviour of the investigated materials it will be helpful to simulate soil mechanic experiments. Triaxial and oedometric tests under cyclic loading have been singled out for this purpose.

The analysis of the results of the oedometric test shows that the hypoplastic model is not in a position to simulate the dynamic behaviour of the granular material. This version of hypoplasticity theory accumulates more strains than the modified versions and the intergranular strain approach. The material behaviour of the reloading is identical to the first loading behaviour (Fig. 3), a hardening of the material did not take place.

![Figure 3: Hypoplasticity simulated oedometric test; stress-strain path](image-url)
The hypoplasticity theories which have been modified with time history functions and the intergranular strain approach are more appropriate for simulating the cyclic behaviour of the granular material. The hardening effect in case of reloading is accounted for and the energy dissipation can be simulated (Fig. 4 and 5).

Figure 4: Stress-strain paths of an oedometric tests; hypoplasticity theory including the time history function

Figure 5: Stress-strain paths of an oedometric tests; intergranular strain approach
The simulation of the triaxial tests showed that the hypoplasticity and the modified version of Bauer could not be used for modelling this experiment. The displacement capacity cannot be used as a criterion for selecting the correct branch when the load direction changes. The energy criterion is able to distinguish between loading and reloading. The stress-strain path is described by a hysteresis loop and is tantamount to an asymptotical approach for the case of monotonically increasing loads (Fig. 6).

Figure 6: Simulation of Stress-strain path of a triaxial test, by hypoplasticity modified with the energy criterion

The intergranular strain can also simulate the cyclic loaded triaxial test. The results of these tests show that the hypoplasticity theory modified by the energy criterion and the intergranular strain approach are in a position to model cyclic behaviour adequately. The difference between the two material descriptions is shown in a small strain test. The experimental set-up is identical with the oedometric test apparent except that the load change is small. The results for two different time load functions (Fig. 7) are shown.
The hysteresis loops of the hypoplastic material law which has been modified by the time history function show the influence of the function $f_m$ on limiting the compressibility. The intersection points of the hysteresis loops move towards the area of higher stresses (Fig. 8).
In the intergranular strain theory, less strain is accumulated than according to the hypoplasticity theory and it also behaves less dissipatively. The intergranular strain approach describes a nearly elastic material behaviour for small amplitudes (Fig. 9 and 10).

![Figure 9: Small-strain test](image)

![Figure 10: Blow up region of the small-strain test](image)
The problem with the ratcheting effect seems to be solved or at least under control. The test with reduced amplitudes shows this effect explicitly. Only a blow up of the region around the reversal point shows that the material did not behave elastically (Fig. 11 and 12).

![Figure 11: Small strain test with reduced amplitudes](image1)

![Figure 12: Blow up region of the small strain test with reduced amplitudes](image2)
5 CONCLUSIONS AND FURTHER RESEARCH

The comparison of the different descriptions for the cyclic behaviour of granular material shows that the basic version of the hypoplasticity theory is not able to describe the cyclic behaviour well enough. The modified hypoplasticity theory is able to simulate cyclic oedometric tests and the modified version with the energy criterion based material law is able to simulate triaxial tests as well. The intergranular strain approach yields the most effective material law. The small-strain test shows that the ratcheting effect can be controlled.

The next step has to be the simulation of silos under earthquake excitation utilise the two material laws.

6 REFERENCES