NUMERICAL ANALYSES OF CONCRETE SUBJECTED TO TRIAXIAL COMPRESSIVE LOADING

Peter Pivonka*, Roman Lackner, and Herbert A. Mang

Institute for Strength of Materials
Vienna University of Technology, Vienna, Austria
e-mail: Peter.Pivonka@tuwien.ac.at, web page: http://www.fest.tuwien.ac.at/

Key words: concrete, triaxal loading, confinement, Leon model, multi-surface plasticity, fictitious crack concept, pull-out analysis

Abstract. This paper deals with numerical analyses of concrete subjected to triaxial compressive loading. For this purpose, two different 3D-constitutive models for concrete are considered. The Leon model, a single-surface plasticity model previously formulated by Etse[10], is reformulated with respect to the ductility function for the description of ductile concrete behavior. This model accounts for the dependence of the concrete strength on the Lode angle. The second model is a multi-surface plasticity model consisting of three Rankine yield surfaces for the description of cracking of concrete and a Drucker-Prager yield surface for the description of compressive behavior of concrete. This model is reformulated to account for the dependence of both strength and ductility of concrete on confinement. Consideration of confinement is controlled by the major principal stress. The model behavior under different confinement levels is compared with experimental results reported in [12]. Within a finite element (FE) analysis of a pull-out test, the predictive capabilities of the models are investigated.
1 Introduction

Certain engineering applications, such as, e.g., the anchorage of tendons in prestressed concrete structures are characterized by highly concentrated loads. In the past, the underlying research for such engineering applications was performed experimentally. More recently, numerical tools such as the finite element method (FEM) are being used to reduce the experimental effort. A synthesis of both numerical analyses and experiments allows to minimize time and costs in the development process. As regards numerical simulations, realistic material models are required in order to quantify the material response correctly. Such models must be able to capture the material behavior under various loading paths, such as tensile, low and high confined compressive paths. The bulk of existing constitutive models of concrete, however, is designed and calibrated for the mechanical description of concrete under moderately large stresses.

The present paper deals with the development of two material models for plain concrete applicable to a wide range of different stress states. Both models are based on plasticity theory. The paper is organized as follows:

Section 2 contains a description of the material models. The performance of both models for different loading paths is investigated in Section 3. In Section 4 the model response on the structural level is studied by means of a pull-out analysis. Concluding remarks are given in Section 5.

2 Material models for plain concrete

The use of standard plasticity models such as the Mohr-Coloumb model or the Drucker-Prager model for the description of concrete is restricted to moderately large stresses. However, because of the relatively simple formulation these models are commonly used in numerical analysis beyond their original range of applicability. The need for realistic material models covering a larger response spectrum of concrete under various stress states and loading paths is evident. In many engineering applications, structures are subjected to different modes of loading. In the following, two kinds of material models are dealt with. They are referred to as single-surface and multi-surface models. They have been developed in order to gain realistic numerical results for concrete subjected to a wide range of triaxial stress states.

2.1 Single-surface plasticity model

From the single-surface models proposed in the open literature, the Extended Leon Model (ELM) [10] was chosen. The loading surface of the ELM was designed such that good agreement between numerical results and experimental data in a wide range of stress states was obtained. Figure 1 shows the loading surface of the ELM at different loading states.
Figure 1: Loading surface of the ELM in principal stress space for different loading states

2.1.1 General characteristics of the Extended Leon Model (ELM)

The overall concrete response of the ELM is divided into three regions, i.e., an initial linear elastic regime, a non-linear hardening pre-peak, and a non-linear softening post-peak regime. Unloading follows the initial elastic behavior. According to this model, the loading surface is formulated by means of the hydrostatic pressure $p$, the deviatoric radius $r$, the Lode angle $\theta$, and the stress-like internal variables $q_h$ and $q_s$:

$$f(p, r, \theta; q_h, q_s) = \left\{ \left(1 - \frac{\tilde{q}_h}{f_{cu}}\right) \left[ \frac{p}{f_{cu}} + \frac{rg(\theta, e)}{\sqrt{6} f_{cu}} \right]^2 + \sqrt{3}\frac{rg(\theta, e)}{2f_{cu}} \right\}^2 + \left(\frac{\tilde{q}_h}{f_{cu}}\right)^2 m(q_s) \left[ \frac{p}{f_{cu}} + \frac{rg(\theta, e)}{\sqrt{6} f_{cu}} \right] - \left(\frac{\tilde{q}_h}{f_{cu}}\right)^2 \frac{\tilde{q}_s}{f_{tu}} = 0, \quad (1)$$

with

$$\tilde{q}_h = f_{cy} - q_h \quad \text{and} \quad \tilde{q}_s = f_{tu} - q_s. \quad (2)$$

$f_{cu}$ and $f_{tu}$ denote the uniaxial compressive and tensile strength, respectively. $f_{cy}$ represents the elastic limit under compressive loading. The deviatoric shape of the loading surface is described by the elliptic function

$$g(\theta, e) = \frac{4(1 - e^2) \cos^2 \theta + (2e - 1)^2}{2(1 - e^2) \cos \theta + (2e - 1) \sqrt{4(1 - e^2) \cos^2 \theta + 5e^2 - 4e}}, \quad (3)$$

where the parameter $e = r_t/r_c$ is referred to as eccentricity. $r_t$ and $r_c$ denote the pressure-dependent deviatoric strength for $\theta = 0^\circ$ and $\theta = 60^\circ$, respectively. The parameter $m(q_s)$ is called frictional parameter. It defines the slope of the loading surfaces in the post-peak regime.

The elastic response is bounded by an initial loading surface, which grows isotropically with increasing inelastic deformations. The evolution of the loading surface is controlled by two stress-like internal variables, $q_h$ and $q_s$. In the pre-peak regime the material
is assumed to exhibit degrading stiffness without localized macro-defects. Degrading stiffness is modelled by means of strain-hardening resulting in an increasing compressive strength $\bar{q}_h$. During hardening, the stress-like softening parameter $q_s$ and the frictional parameter $m(q_s)$ remain unchanged:

$$\bar{q}_s = f_{tu} \quad \text{and} \quad m(q_s) = m_o = \frac{(f^2_{cu} - f^2_{tu})}{f_{cu} f_{tu}}.$$ (4)

Softening is initiated when micro-defects at peak characterized by $\bar{q}_h = f_{cu}$ coalesce into localized macro-defects, i.e., when the concrete starts cracking. Softening is characterized by a decrease of the tensile strength $\bar{q}_s$ resulting in an increase of the frictional parameter $m(q_s)$ (for details see Subsection 2.1.4).

Figures 2 and 3 show the loading surface of the ELM for the pre-peak (hardening) and the post-peak (softening) regime.

Figure 2: Different locations of the loading surface of the ELM in the pre-peak regime for an increase of the compressive strength from $\bar{q}_h = f_{cu}/10$ to $\bar{q}_h = f_{cu}$ in consequence of strain-hardening: (a) meridian plane and (b) deviatoric plane

2.1.2 Non-associative flow rule

In elasto-plasticity, the flow rule defines the evolution of the plastic strain tensor. In general, the direction of the plastic strain rates is assumed to be orthogonal to the yield surface (associate flow rule). Hence, the shape of the yield surface defines the plastic response. Smith, Willam, and Gerstle[22] have conducted comprehensive experiments to determine the direction of the incremental plastic strains during strain-driven triaxial tests
Figure 3: Different locations of the loading surface of the ELM in the post-peak regime for a decrease of the tensile strength $\bar{q}_s$ from $\bar{q}_s = f_{tu}$ to $\bar{q}_s = f_{tu}/10$ in consequence of strain-softening ($TP$: transition point): (a) meridian plane and (b) deviatoric plane

of concrete specimens. The results of this study have clearly shown that the assumption of associativeness is not valid for concrete subjected to triaxial states of stress. Therefore, in the framework of the Extended Leon Model a non-associated flow rule is used to define the direction of the plastic flow. A yield potential $Q$ is introduced to modify the yield function $f$ with respect to its volumetric part. The yield potential has the form

$$Q(p, r, \theta, m_Q; q_h, q_s) = \left\{ \begin{array}{l} 
(1 - \frac{q_h}{f_{cu}}) \left[ \frac{p}{f_{cu}} + \frac{rg(\theta, e)}{\sqrt{6f_{cu}}} \right]^2 + \frac{3}{2} \frac{rg(\theta, e)}{f_{cu}} \\
+ \left( \frac{q_h}{f_{cu}} \right)^2 \left[ \frac{m_Q}{f_{cu}} + \frac{rg(\theta, e)}{\sqrt{6f_{cu}}} \right] - \left( \frac{q_h}{f_{cu}} \right)^2 \frac{q_s}{f_{tu}} = 0 
\end{array} \right. \quad (5)$$

with the modified frictional parameter

$$m_Q = m_Q(p), \quad \frac{\partial m_Q}{\partial p} = D \exp \left( ER^2(p) \right) + F, \quad (6)$$

where

$$R(p) = \frac{p - f_{tu}/3}{2f_{cu}}. \quad (7)$$

The parameters $D$, $E$, and $F$ are calibrated from measurements of the plastic dilatancy obtained from three different experiments. One uniaxial tension test and two confined compression tests at low- and high-confinement levels are sufficient to determine these
parameters. With respect to the deviatoric section, the yield potential \( Q \) and the yield function \( f \) coincide. Hence, the deviatoric components of the plastic strain tensor are governed by an associative law. The gradient of the yield potential is obtained as

\[
\mathbf{m} = \frac{\partial Q}{\partial \mathbf{\sigma}} = \frac{\partial Q}{\partial \mathbf{p}} \frac{\partial \mathbf{p}}{\partial \mathbf{\sigma}} + \frac{\partial f}{\partial \mathbf{r}} \frac{\partial \mathbf{r}}{\partial \mathbf{\sigma}} + \frac{\partial f}{\partial \mathbf{\theta}} \frac{\partial \mathbf{\theta}}{\partial \mathbf{\sigma}}.
\]

(8)

The rate of the plastic strain tensor follows then as

\[
\dot{\varepsilon}^p = \dot{\gamma} \mathbf{m},
\]

where \( \gamma \) denotes the plastic multiplier.

### 2.1.3 Non-linear isotropic hardening law

The material behavior is assumed to be isotropic during the entire deformation history. Inelastic deformations occur when the elastic limit, which is defined by the initial loading surface \( \bar{q}_h = f_{cy} \), is exceeded. In consequence of strain-hardening, the compressive strength increases until \( \bar{q}_h = f_{cu} \). If \( \bar{q}_h = f_{cu} \), the failure surface is reached. The evolution of the loading surface from its initial location \( \bar{q}_h = f_{cy} \) to the failure surface \( \bar{q}_h = f_{cu} \) is controlled by strain-hardening based on the equivalent plastic strain \( \varepsilon^p \). The rate of the equivalent plastic strain, \( \dot{\varepsilon}^p \), is defined as the Euclidian norm of the tensor of the rate of plastic strains

\[
\dot{\varepsilon}^p = \sqrt{\dot{\varepsilon}^{p^T} \dot{\varepsilon}^p} = \dot{\gamma} \| \mathbf{m} \|.
\]

(10)

The evolution equation for the strain-like internal variable \( \alpha_h \) governing the hardening behavior is defined as [9]

\[
\dot{\alpha}_h = \frac{1}{x_h} \dot{\varepsilon}^p,
\]

(11)

where the confining pressure is accounted for by the ductility parameter \( x_h \). In order to capture concrete behavior under low and high confinement levels, two quadratic polynomials are employed for the definition of \( x_h \), yielding \( x_h \) as a continuous function of the hydrostatic pressure \( p \):

\[
x_h = x_h(p) = \begin{cases} A_h(p/f_{cu})^2 + B_h(p/f_{cu}) + C_h & \text{for low confinement}, \\ D_h(p/f_{cu})^2 + E_h(p/f_{cu}) + F_h & \text{for high confinement}. \end{cases}
\]

(12)

The six coefficients in Equation (12) are determined from five experiments at different levels of confinement, together with the continuity condition. The value of \( \alpha_h \) defines the actual compressive strength of concrete, \( \bar{q}_h \). The respective stress-like internal variable \( q_h \) is expressed by a monotonically decreasing function of \( \alpha_h \):

\[
q_h = q_h(\alpha_h) = \begin{cases} -(f_{cu} - f_{cy})\sqrt{\alpha_h(2 - \alpha_h)} & \text{for hardening } (\alpha_h < 1), \\ -(f_{cu} - f_{cy}) & \text{for softening } (\alpha_h \geq 1). \end{cases}
\]

(13)

Figure 4 shows the dependence of the evolution of compressive strength, \( \bar{q}_h = f_{cy} - q_h \), on confinement represented by \( x_h \).
2.1.4 Non-linear isotropic softening law

The most important consequence of material instability in the form of cracking is the localization of deformations. Localization occurs suddenly at a certain point of the loading history when the entire additional deformation is confined in narrow band-shaped parts of the body, while the remaining parts of the body exhibit unloading. Localization is usually accompanied by a decrease of the load-carrying capacity after reaching the peak load. Such a gradual decrease of stiffness and load-carrying capacity with the increase of deformation imposed on the body is called softening. Within the smeared-crack concept, commonly employed in the context of the FEM, softening is described in terms of the respective stress-strain relations. However, this concept is characterized by lack of objectivity of the numerical results with respect to the element size. In order to ensure objectivity, the following regularization techniques were suggested:

- fracture energy concept [11] [12] [2] [15],
- non-local (integral) models [19],
- higher-order strain gradient models [6] [17],
- micropolar (Cosserat) formulation [5].

The ELM is formulated on the basis of the smeared-crack approach regularized by the fracture energy concept. The residual loading surface in the softening regime, $f_r$, is given by

$$
\begin{align*}
\tilde{q}_h (\alpha_h (\epsilon^p, x_h)) &= \frac{3}{2} \left( \frac{rg(\theta, \epsilon)}{\sqrt{6} f_{cu}} \right)^2 + m_r \left( \frac{p}{f_{cu}} + \frac{rg(\theta, \epsilon)}{\sqrt{6} f_{cu}} \right) = 0.
\end{align*}
$$

$f_r$ is obtained from the loading surface given in Equation (1) by setting $\tilde{q}_h = f_{cu}$, $\tilde{q}_s = 0$, and $m = m_r$. The evaluation of the residual frictional parameter $m_r$ is based on the location of the so-called transition point (TP) of brittle to ductile failure [23]. The
location of $TP$ is assumed to remain fixed in the stress space. This assumption leads to an increase of the frictional parameter ($m \rightarrow m_r$) for a decreasing tensile strength $\bar{q}_s$ in consequence of softening. The employed function of the friction parameter is given as

$$m = m(q_s) = \begin{cases} m_o & \text{for hardening (}$\alpha_h < 1$), \\ m_r - (m_r - m_o)\bar{q}_s/f_{tu} & \text{for softening (}$\alpha_h \geq 1$). \end{cases} \tag{15}$$

with $\bar{q}_s = f_{tu} - q_s$. Based on this definition, an intermediate state of the softening surface is defined as

$$f(p, r, \theta; q_s) = \frac{3}{2} \left( \frac{rg(\theta, e)}{\sqrt{6}f_{cu}} \right)^2 + m(q_s) \left( \frac{p}{f_{cu}} + \frac{rg(\theta, e)}{\sqrt{6}f_{cu}} \right) - \frac{\bar{q}_s}{f_{tu}} = 0. \tag{16}$$

The evolution of the softening surface form its initial location ($\bar{q}_s = f_{tu}$) to its residual location ($\bar{q}_s = 0$) is controlled by strain-softening. Therefore, the equivalent plastic strain $\dot{\epsilon}^s$ is introduced. It can be viewed as the damage metric. Rates of $\dot{\epsilon}^s$ should only be monitored if the existing micro-cracks are activated. Mathematically, this situation may be expressed by requiring that the tensor of plastic strain rates, $\dot{\epsilon}^p$, has at least one positive eigenvalue [16]. Considering the principal components of $\dot{\epsilon}^p$ and introducing the McAuley operator $\langle \bullet \rangle = (\bullet + |\bullet|)/2$, the equivalent plastic strain rate can be expressed as

$$\dot{\epsilon}^s = \| \langle \dot{\epsilon}^p \rangle \| = \sqrt{\langle \dot{\epsilon}^p \dot{\epsilon}^p \rangle} = \gamma \| \langle m \rangle \|. \tag{17}$$

Similar to the isotropic hardening formulation, a strain-like internal variable is introduced in the form:

$$\dot{\alpha}_s = \frac{1}{x_s(p)} \dot{\epsilon}^s, \tag{18}$$

where $x_s$ accounts for the ductile behavior of concrete in softening. $x_s$ is defined by a polynomial function of the hydrostatic pressure $p$:

$$x_s = x_s(p) = A_s \bar{R}^4(p) + B_s \bar{R}^2(p) + 1, \tag{19}$$

with $\bar{R}(p) = (p - f_{tu}/3)/f_{cu}$. The value for $x_s$ is computed from the hydrostatic pressure at peak load and kept constant for the remaining part of the loading path. Hence, for uniaxial tensile loading characterized by $p = f_{tu}/3$ at peak load, $x_s$ becomes equal to one. The parameters $A_s$ and $B_s$ are calibrated from low-confined and high-confined compression tests.

### 2.1.5 Calibration of the ELM in the context of the fracture energy concept

Softening material behavior leads to a decrease of the tensile strength $\bar{q}_s$. The respective stress-like internal variable, $q_s$, is expressed by an exponential function of $\alpha_s$:

$$q_s = q_s(\alpha_s) = \begin{cases} 0 & \text{for hardening (}$\alpha_h < 1$), \\ f_{tu} [1 - \exp[-(\alpha_s/\alpha_u)^n]] & \text{for softening (}$\alpha_h \geq 1$). \end{cases} \tag{20}$$
where according to [12], \( n = 1 \). However, \( n = 1 \) leads to a discontinuous slope for the transition from the hardening regime (\( \alpha_h < 1 \)) to the softening regime (\( \alpha_h \geq 1 \)), which was not confirmed by experimental data. The exponent \( n = 2 \) is characterized by a continuous slope. For this reason, \( n = 2 \) was chosen for the present work. The decrease of the tensile strength \( \bar{q}_s = f_{tu} - q_s \) with increasing \( \epsilon^s \) is illustrated in Figure 5 for different levels of confinement. In Equation (20), \( \alpha_u \) denotes a calibration parameter. It is computed from the fracture energy of mode-I cracking \( G_{Ic} \), representing the energy released as one crack is opening. It is considered as a material parameter. \( G_{Ic} \) is obtained from

\[
G_{Ic} = \int_0^{\infty} \sigma du_s,
\]

where \( u_s \) denotes the crack opening displacement and \( \sigma \) is the applied tensile stress. In the context of the smeared-crack approach (see Figure 6), the integration over \( u_s \) is replaced by the respective plastic strain measure \( \epsilon^s \). Rewriting Equation (21) and setting \( \sigma \) equal to the tensile strength \( \bar{q}_s \), yields

\[
G_{Ic} = \ell_t \int_0^{\infty} \bar{q}_s \ell_t d\epsilon^s = \ell_t \int_0^{\infty} (f_{tu} - q_s) d\epsilon^s,
\]

where \( \ell_t \) represents the width of the crack band in the context of the smeared-crack approach. Within the FEM, \( \ell_t \) is related to the element size (see, e.g., [15]). Inserting Equation (20) into (22) and considering \( \dot{\alpha}_s = \dot{\epsilon}^s \) (see Equation (18)) for uniaxial tensile loading, gives

\[
G_{Ic} = \ell_t \int_0^{\infty} f_{tu} \exp[-\alpha_s/\alpha_u] d\alpha_s.
\]

Integration, followed by re-arranging terms, finally yields

\[
\alpha_u = \frac{2g_{Ic}}{\sqrt{\pi f_{tu}}}, \quad \text{with} \quad g_{Ic} = G_{Ic} / \ell_t.
\]
Figure 6: Homogenization of localization of deformations in consequence of cracking by means of the smeared-crack approach: (a) discrete and (b) smeared representation of a crack.

The extension of the fracture energy concept from uniaxial tensile loading to triaxial confined stress states is accounted for by \( x_s \). Replacing \( 1/x_s \) in Equation (18) by \( G_f / G_{II}^f \), where \( G_{II}^f \) denotes the fracture energy in mixed mode cracking, one gets

\[
\dot{\alpha}_s = \frac{1}{x_s(p)} \dot{\epsilon}^s = \frac{G_f^I}{G_{II}^f(p)} \dot{\epsilon}^s. \tag{25}
\]

Hence, \( x_s \) can be identified as the increase of the fracture energy in consequence of increasing confinement (see, e.g., [8] for a similar identification). It represents the number of opening cracks, which for uniaxial tensile loading \( (G_{II}^f(p) = G_f^I) \) is equal to one and grows with increasing confinement.

### 2.2 Multi-surface plasticity model

This subsection contains the reformulation of a multi-surface plasticity model described in [14]. It consists of a Drucker-Prager (DP) yield surface for the description of concrete subjected to compressive loading and three Rankine (RK) surfaces for the description of the tensile behavior of concrete (see Figure 7). Generally, the Drucker-Prager criterion is calibrated by means of a uniaxial and a biaxial compression test. Based on this mode of calibration the simulation of confined compression tests shows poor agreement with experimental data (see Section 3.1). Hence, a modification of the Drucker-Prager model is proposed accounting for the influence of confinement on the hardening/softening behavior. Confinement is represented by the major principal stress (see, e.g., [20]).
2.2.1 Yield surfaces

Cracking of concrete is modelled within the framework of the smeared-crack concept. The maximum tensile stress criterion (Rankine criterion) is used to determine the tensile strength of concrete for 3D states of cracks. In the principal stress space, the failure criterion is described as

\[ f_{RK,A}(\sigma_A, q_{RK}) = \sigma_A - \bar{q}_{RK}, \quad \text{with} \quad \bar{q}_{RK} = f_{tu} - q_{RK}, \]

where the subscript "A" (A=1,2,3) refers to one of the three principal axes and \( q_{RK} \) is an isotropic stress-like internal variable.

The ductile material behavior of concrete subjected to a multiaxial state of compressive stresses is accounted for by a hardening/softening Drucker-Prager model reading

\[ f_{DP}(\sigma, q_{DP}) = \sqrt{J_2} - \kappa_{DP} I_1 - \frac{\bar{q}_{DP}}{\beta_{DP}} \quad \text{with} \quad \bar{q}_{DP} = f_{cy} - q_{DP}, \]

where \( f_{cy} \) represents the elastic limit of concrete. The parameters \( \kappa_{DP} \) and \( \beta_{DP} \) are calibrated from the peak strengths of uniaxial and biaxial loading. For \( f_{cb}/f_{cu} = 1.16 \), \( \kappa_{DP} = -0.07 \) and \( \beta_{DP} = 1.97 \).

2.2.2 Evolution equations

The increase of the strain-like internal variables \( \alpha_{RK} \) and \( \alpha_{DP} \) is controlled by means of associate evolution equations, reading

\[ \dot{\alpha}_{RK} = \sum_{A=1}^{3} \dot{\gamma}_{RK,A} \frac{\partial f_{RK,A}}{\partial q_{RK}}, \quad \dot{\alpha}_{DP} = \dot{\gamma}_{DP} \frac{\partial f_{DP}}{\partial q_{DP}}. \]
where $\gamma_{RK,A}$ represents the plastic multiplier related to the Rankine surface $f_{RK,A}$, and $\gamma_{DP}$ is the plastic multiplier related to the Drucker-Prager yield surface $f_{DP}$. In the context of multi-surface plasticity, plastic loading is characterized by $\dot{\gamma}_k > 0$ and $f_k = 0$. For elastic loading, $\dot{\gamma}_k = 0$ and $f_k < 0$. The flow rule for the evolution of the plastic strain tensor $\varepsilon^p$ in the context of multi-surface plasticity is given by [13]

$$\dot{\varepsilon}^p = \sum_{A=1}^{3} \dot{\gamma}_{RK,A} \frac{\partial Q_{RK,A}}{\partial \sigma} + \dot{\gamma}_{DP} \frac{\partial Q_{DP}}{\partial \sigma},$$

(29)

with $Q_{RK,A}$ and $Q_{DP}$ denoting the yield potentials for the Rankine and the Drucker-Prager criterion, respectively. Whereas an associate flow rule is chosen for the Rankine surfaces, i.e., $Q_{RK,A} = f_{RK,A}$, a non-associate flow rule is employed for the Drucker-Prager criterion. It is characterized by modification of the Drucker-Prager yield function with respect to its volumetric part, reading

$$Q_{DP}(\sigma, q_{DP}) = \sqrt{J_2 - \bar{\kappa}_{DP} I_1} - \frac{\bar{q}_{DP}}{\beta_{DP}} = 0.$$

(30)

The gradient of the yield potential is obtained as

$$m = \frac{\partial Q_{DP}}{\partial \sigma} = \frac{\partial \sqrt{J_2}}{\partial \sigma} - \bar{\kappa}_{DP} \mathbf{I},$$

(31)

where $\mathbf{I}$ represents the second-order identity tensor.

### 2.2.3 Isotropic hardening/softening laws

Within the framework of the Rankine criterion the softening behavior is accounted for by an exponential softening law (see Figure 8(a)), reading

$$q_{RK} = q_{RK}(\alpha_{RK}) = (f_{tu} - f_{tr}) \left[1 - \exp \left(-\frac{\alpha_{RK}}{\alpha_{RK,u}}\right)\right],$$

(32)

with $f_{tu}$ and $f_{tr}$ denoting the peak and the residual tensile strength, respectively. Based on the fracture energy concept, the parameter $\alpha_{RK,u}$ is adjusted to the element size represented by the characteristic length $\ell_t$ and to the fracture energy in mode-I cracking, $G_f^I$, yielding

$$\alpha_{RK,u} = \frac{g_f^I}{f_{tu} - f_{tr}}, \quad \text{with} \quad g_f^I = \frac{G_f^I}{\ell_t}.$$

(33)

The hardening/softening law for the Drucker-Prager criterion is expressed as (see Figure 8(b))

$$q_{DP}(\alpha_{DP}) = \begin{cases} (f_{cy} - f_{cu}) \sqrt{\frac{(2\alpha_{DP,m} - \alpha_{DP})}{\alpha_{DP,m}^2}} & \text{for} \ \alpha_{DP} \leq \alpha_{DP,m}, \\ f_{cy} - (f_{cu} - f_{cr}) \exp \left[-\left(\frac{\alpha_{DP} - \alpha_{DP,m}}{\alpha_{DP,u}}\right)^2\right] - f_{cr} & \text{for} \ \alpha_{DP} > \alpha_{DP,m}, \end{cases}$$

(34)
where \( f_{cu} \) and \( f_{cr} \) are the peak and the residual compressive strength, respectively. The parameter \( \alpha_{DP,m} \) accounts for the ductile behavior. \( \alpha_{DP,u} \) is a calibration parameter for the softening regime. It is adjusted to the element size represented by the characteristic length \( \ell_t \) and the fracture energy \( G_{II}^f \). \( G_{II}^f \) represents the fracture energy obtained from uniaxial compression tests. It is computed from (see Figure 8(b))

\[
G_{II}^f = -\ell_t \int_{\varepsilon_m}^{-\infty} (q_{DP} - f_{cr})d\varepsilon^p = -\ell_t \int_{\varepsilon_m}^{-\infty} (f_{cy} - q_{DP} - f_{cr})d\varepsilon^p, \tag{35}
\]

where \( \varepsilon_m \) represents the strain at peak stress, with \( \varepsilon_m < 0 \). For the uniaxial case the flow rule and the evolution law for the strain-like internal variable \( \alpha_{DP} \) can be expressed as

\[
\dot{\varepsilon}^p = \dot{\gamma} \frac{\partial Q_{DP}}{\partial \sigma_1} = \dot{\gamma} \left( -\frac{1}{\sqrt{3}} - \kappa_{DP} \right) \quad \text{and} \quad \dot{\alpha}_{DP} = \dot{\gamma} \frac{\partial f_{DP}}{\partial q_{DP}} = \dot{\gamma} \frac{1}{\beta_{DP}}, \tag{36}
\]

yielding

\[
\dot{\varepsilon}^p = -\bar{c} \dot{\alpha}_{DP} \quad \text{with} \quad \bar{c} = \beta_{DP}(1/\sqrt{3} + \kappa_{DP}). \tag{37}
\]

Inserting Equation (34) into Equation (35) and using (37) leads to

\[
G_{II}^f = \bar{c} \ell_t \int_{\alpha_{DP,m}}^{\infty} (f_{cu} - f_{cr}) \exp \left[ -\left( (\alpha_{DP} - \alpha_{DP,m})/\alpha_{DP,u} \right)^2 \right] d\alpha_{DP}. \tag{38}
\]

Finally, the required calibration parameter is obtained as

\[
\alpha_{DP,u} = \frac{2}{f_{cu} \sqrt{\pi}} \frac{g_{II}^f}{\bar{c}}, \quad \text{with} \quad g_{II}^f = G_{II}^f / \ell_t. \tag{39}
\]

Figure 8: Employed hardening/softening formulation for (a) the Rankine criterion and (b) the Drucker-Prager surface.
2.2.4 Consideration of confinement

The influence of confinement on the peak and residual compressive strength of concrete, $f_{cu}$ and $f_{cr}$, is considered by means of the major principal stress $\sigma_1$, with $\sigma_1 \geq \sigma_2 \geq \sigma_3$, yielding $f_{cu} = f_{cu}(\sigma_1)$ and $f_{cr} = f_{cr}(\sigma_1)$. In the following, the dependence of $f_{cu}$ and $f_{cr}$ on the major principal stress $\sigma_1$ is investigated by means of experimental data. Therefore, results obtained from triaxial tests conducted by Hurlbut[12] and Smith[21] are employed. These tests are characterized by constant lateral stresses, i.e., $0 \geq \sigma_1 = \sigma_2$.

As regards the influence of confinement on the peak strength, experimental data [21][12] show a linear dependence of the axial peak stress $\sigma_3$ on confinement represented by the lateral stress $\sigma_1$ (see Figure 9(a)):

$$\frac{\sigma_3}{f_{cu,0}} = \frac{\sigma_3}{f_{cu,0}}(\sigma_1) = -1 + c \frac{\sigma_1}{f_{cu,0}}, \quad (40)$$

where $f_{cu,0}$ denotes the uniaxial compressive strength obtained from unconfined test, i.e., $\sigma_1 = \sigma_2 = 0$. $c$ is the confinement parameter. Regression analysis performed on the basis of the considered experimental data yields $c=4.41$. The experimental data together with the regression curve are shown in Figure 9(a).

![Figure 9: Relation between the axial stress $\sigma_3$ and the lateral stress $\sigma_1$ for triaxial experiments obtained from Hurlbut[12] and Smith[21]: (a) $\sigma_3 =$ peak stress and (b) $\sigma_3 =$ residual stress](image)

The peak strength $f_{cu}$ corresponding to an axial peak stress $\sigma_3$ is obtained from the yield criterion using $\sigma^T = [\sigma_1, \sigma_1, \sigma_3(\sigma_1)]$:

$$f_{DP}(\sigma) = \sqrt{J_2(\sigma_1)} - \kappa_{DP}I_1(\sigma_1) - \frac{f_{cu}(\sigma_1)}{\beta_{DP}} = 0. \quad (41)$$
Reformulation of Equation (41) yields the peak strength of concrete as a function of the confinement represented by the lateral stress $\sigma_1$:

$$f_{cu}(\sigma_1) = \beta_{DP} \left( \sqrt{J_2(\sigma_1)} - \kappa_{DP} I_1(\sigma_1) \right).$$  \hspace{1cm} (42)

The residual strength $f_{cr}$ denotes the final strength of concrete in the post-peak regime. The post-peak regime is characterized by strain-softening. Experimental results show that softening only occurs at low levels of confinement. At high levels of confinement continuous hardening is observed. In order to distinguish both modes of failure, brittle softening and continuous hardening, the transition point ($TP$) is introduced. According to Willam et al. [23], the transition point lies on the loading path defined by $\sigma_3/f_{cu,0} = 8\sigma_1/f_{cu,0}$. Intersection of this loading path with the regression curve for the peak stress given in Equation (40),

$$\sigma_3 = -1 + c \frac{\sigma_1}{f_{cu,0}} := 8 \frac{\sigma_1}{f_{cu,0}},$$  \hspace{1cm} (43)

yields the lateral stress $\sigma_1$ at the transition point:

$$\frac{\sigma_1}{f_{cu,0}} \bigg|_{TP} = \frac{1}{c - 8} = -0.28.$$  \hspace{1cm} (44)

Since no softening occurs for stress paths characterized by $\sigma_1/f_{cu} \leq -0.28$, the following considerations concerning the residual strength $f_{cr}$ are restricted to $\sigma_1/f_{cu} > -0.28$. Figure 9(b) contains the residual axial stress of triaxial compression tests reported in [12] [21]. For the numerical simulation, the experimental data are approximated by the following regression function:

$$\frac{\sigma_3}{f_{cu,0}}(\sigma_1) = a \left( -\frac{\sigma_1}{f_{cu,0}} \right)^b \quad \text{for} \quad 0 \geq \frac{\sigma_1}{f_{cu,0}} > -0.28,$$  \hspace{1cm} (45)

where $a$ and $b$ are calibration parameters. They are computed from the condition of a smooth transition from the regression curve for the residual stress (45) to the regression curve for the peak stress (40) at the transition point $TP$ (see Figure 10(a)), yielding

$$b = \frac{c}{8} \quad \text{and} \quad a = 8 \left( -\frac{\sigma_1}{f_{cu,0}} \bigg|_{TP} \right)^{1-b}.$$  \hspace{1cm} (46)

For a given confinement represented by the lateral stress $\sigma_1$, with $\sigma_1 < 0$, the residual strength $f_{cr}$ is obtained from the yield criterion using $\sigma^T = [\sigma_1, \sigma_1, \sigma_3(\sigma_1)]$:

$$f_{cr}(\sigma_1) = \beta_{DP} \left( \sqrt{J_2(\sigma_1)} - \kappa_{DP} I_1(\sigma_1) \right).$$  \hspace{1cm} (47)

Figure 10(b) contains the regression functions employed for the approximation of the peak and residual values of the axial stress $\sigma_3$ in consequence of confinement. Remarkably, up
Figure 10: Assumed relations between the axial stress $\sigma_3$ and the lateral stress $\sigma_1$: (a) regression function for the residual stress and (b) combination with the regression function for the peak stress

to now, only one additional parameter, namely $c$, was required for the consideration of confinement.

Apart the increase of the peak and the residual strength in consequence of confinement, also an increase of the ductility of concrete in the pre-peak regime is observed. In general, the parameter $\alpha_{DP,m}$, which defines the ductile behavior of concrete, is computed from the strain at the peak load, $\varepsilon_m$, using Equation (37)

$$\alpha_{DP,m} = \frac{-\varepsilon^p_m}{\bar{c}} = \frac{-(\varepsilon_m - \varepsilon^e_m)}{\bar{c}} = \frac{-(\varepsilon_m + f_{cu}(\sigma_1)/E_c)}{\bar{c}},$$

(48)

where $E_c$ denotes Young’s modulus of concrete. The dependence of $\varepsilon_m$ on confinement represented by the lateral stress $\sigma_1$ is shown in Figure 11. A linear relation of the form

Figure 11: Ductility function: dependence of the peak-strain $\varepsilon_m$ on confinement represented by the lateral stress $\sigma_1$
\[ \varepsilon_m = \varepsilon_m(\sigma_1) = \varepsilon_{m,0} + d \frac{\sigma_1}{f_{cu,0}}, \]  

is chosen, where \( \varepsilon_{m,0} = -0.0022 \) (see CEB-FIB [4]) represents the peak-strain for no confinement. The parameter \( d \) is adjusted to experimental data, yielding \( d = 0.021 \).

3 Re-analysis of experimental test results

This section deals with re-analysis of experimental results contained in Hurlbut[12]. These results refer to triaxial compression tests performed on cylindrical concrete specimen of 107.95 mm height and 53.98 mm diameter. Confinement was applied axisymmetrically ranging from 0 to 69 N/mm\(^2\). After application of confinement, the axial load was increased.

For the purpose of the verification of the proposed models, five compression tests with confinement levels of \( p = 0 / -0.69 / -3.44 / -6.89 / -13.79 \) N/mm\(^2\) and one tension test are considered. The numerical simulations are performed by means of a constitutive driver. The boundary conditions for the different tests are considered by prescribed values for the respective strain and stress components.

The material properties for the experiments conducted in [12] are listed in Table 1:

<table>
<thead>
<tr>
<th>material properties</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus ( E_c )</td>
<td>19305.32 N/mm(^2)</td>
</tr>
<tr>
<td>Poisson’s ratio ( \nu )</td>
<td>0.2</td>
</tr>
<tr>
<td>uniaxial compressive strength ( f_{cu} )</td>
<td>22.063 N/mm(^2)</td>
</tr>
<tr>
<td>uniaxial tensile strength ( f_{tu} )</td>
<td>2.758 N/mm(^2)</td>
</tr>
<tr>
<td>tensile fracture energy ( G_{fI} )</td>
<td>0.035 – 0.050 Nmm/mm(^2)</td>
</tr>
<tr>
<td>compressive fracture energy ( G_{fII} )</td>
<td>4.938 Nmm/mm(^2)</td>
</tr>
<tr>
<td>characteristic length ( l_t )</td>
<td>107.95 mm</td>
</tr>
</tbody>
</table>

Table 1: Material properties corresponding to the experimental data given in [12]

3.1 Model parameters

For the single-surface model, the uniaxial compressive strength \( f_{cu} \) and the uniaxial tensile strength \( f_{tu} \) are employed to determine the failure surface at \( \theta = 0^\circ \) and \( \theta = 60^\circ \). The initial elastic region is defined by the elastic limit \( f_{cy} \). The model parameters of the described non-associate flow rule \( D, E, \) and \( F \) are obtained from [8]. The hardening law for low and medium confinement is specified by the model parameters of ductility \( A_h, B_h, \) and \( C_h \). For high confinement, the model parameters \( D_h, E_h, \) and \( F_h \) are required. The softening behavior is controlled by two ductility softening parameters \( A_s \) and \( B_s \).

The Drucker-Prager failure surface of the multi-surface model is determined by the uniaxial and the biaxial compressive strength, \( f_{cu} \) and \( f_{bc} \), respectively, yielding \( \kappa_{DP} \) and
The elastic limit, \( f_{cy} \), defines the initial elastic region. The dependence of the peak and the residual strength with increasing confinement is described by the confinement parameter \( c \), whereas the ductile behavior is described by the peak strain \( \varepsilon_{m,0} \) and the ductility parameter \( d \). The non-associated flow rule is controlled by the value of \( \bar{\kappa}_{DP} \).

The model parameters are listed in Table 2.

<table>
<thead>
<tr>
<th>model parameters</th>
<th>single-surface model</th>
<th>multi-surface model</th>
</tr>
</thead>
<tbody>
<tr>
<td>elastic limit:</td>
<td>( f_{cy} = 0.1 , f_{cu} )</td>
<td>( f_{cy} = 0.4 , f_{cu} )</td>
</tr>
<tr>
<td>pre-peak:</td>
<td>( A_h = -0.000425 ) ( D_h = 0.002210 )</td>
<td>( \varepsilon_{m,0} = -0.0022 )</td>
</tr>
<tr>
<td></td>
<td>( B_h = -0.004950 ) ( E_h = -0.007388 )</td>
<td>( d = 0.021 )</td>
</tr>
<tr>
<td></td>
<td>( C_h = 0.000212 ) ( F_h = -0.008870 )</td>
<td>( c = 4.429 )</td>
</tr>
<tr>
<td>post-peak:</td>
<td>( A_s = 12.51717 ) ( B_s = 118.767 )</td>
<td>( f_{ir} = 0.2 , \text{N/mm}^2 )</td>
</tr>
<tr>
<td>flow rule:</td>
<td>( D = 8.675 + 5.115 \exp[e] )</td>
<td>( \bar{\kappa}<em>{DP} = 0 ) and ( \kappa</em>{DP} )</td>
</tr>
<tr>
<td></td>
<td>( E = -14.956 + 6.736 \exp[e] )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( F = -6.3 ) with ( e = -5(1 - \tilde{q}/f_{cu}) ) acc. to [8]</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Model parameters for the single-surface and the multi-surface plasticity model, respectively, used for re-analysis of experimental test data [12]

Figure 12 contains the numerical result obtained from the proposed single-surface model. The axial stress - axial strain curves for no confinement and low confinement,
$p = 0$ and -0.69 N/mm$^2$, respectively, show good agreement with the respective experimental results. For high confinement, the axial peak stress is underestimated by the single-surface model. In accordance with experimental data, no softening is observed for stress paths corresponding to medium to high confinement. Such stress paths are characterized by ideally plastic behavior after the failure surface ($\alpha_h = 1$) has been reached. As regards the numerical response of the lateral strains, Figure 12 shows good agreement with experimentally obtained results. This fact reflects the high quality of the employed yield potential $Q$ (see Equation (5)), which controls the evolution of the plastic strain tensor, and hence, the size of the lateral strains.

Figure 13 contains the numerical results obtained from the multi-surface model characterized by disregard of confinement. As expected, only the axial stress - axial strain curve for the unconfined test shows good agreement with the respective experimental result. For confined tests, however, the axial peak stress as well as the post-peak behavior show high deviations to the experimental results.

![Figure 13: Re-analysis of triaxial compression tests: stress-strain curves obtained from the multi-surface model disregarding confinement ($c = 0$, $d = 0$) and using an associate flow rule ($\bar{\kappa}_{DP} = \kappa_{DP}$)](chart)

The results obtained from the multi-surface model considering confinement are shown in Figure 14. They indicate good agreement of the experimentally obtained axial stress in the pre-peak as well as in the post-peak regime. As regards the lateral strain, the underlying associate flow rule ($\bar{\kappa}_{DP} = \kappa_{DP}$) leads to underestimation of the lateral deformation for low confinement and to overestimation for high levels of confinement.

Use of a non-associate flow rule $\bar{\kappa}_{DP} = 0$ results in a decrease of the lateral strains (see Figure 15) and, hence, in an improvement of the numerical results for high levels of confinement. However, for low confinement, the lateral deformation is still underestimated.
For a proper representation of lateral deformations the proposed yield potential given in Equation (30) has to be modified. Such a modification must account for the dependence of the volumetric plastic strains on the actual state of stress. In the analyses presented

Figure 14: Re-analysis of triaxial compression tests: stress-strain curves obtained from the multi-surface model considering confinement \((c = 4.429, d = 0.0021)\) and using an associate flow rule \(\bar{\kappa}_{DP} = \kappa_{DP}\)

Figure 15: Re-analysis of triaxial compression tests: stress-strain curves obtained from the multi-surface model considering confinement \((c = 4.429, d = 0.0021)\) and using a non-associate flow rule \(\bar{\kappa}_{DP} = 0\)
Figure 16: Re-analysis of a uniaxial tension test: stress-strain curves obtained from (a) a single-surface and (b) a multi-surface model.

in Figures 14 and 15 the evolution of the volumetric plastic strain was assumed to be constant ($\bar{\kappa}_{DP} = \kappa_{DP}$) and zero ($\bar{\kappa}_{DP} = 0$), respectively.

Figure 16 contains the numerical results for a uniaxial tension test. The different definitions of the post-peak behavior (compare Equations (20) and (32)) are reflected by the obtained axial stress - axial strain curves for the single-surface model (Figure 16 (a)) and the multi-surface model (Figure 16 (b)).

4 Pull-out analysis

This section contains a report about the numerical simulation of an anchor bolt in concrete subjected to tensile loading, referred to as pull-out test. The focus of this investigation is on the influence of the underlying material model for concrete on the ultimate load. Therefore the main parameters of both the single- and the multi-surface model are varied.

4.1 Geometric dimensions and material properties

Figure 17 contains the experimental setup of the considered pull-out test (Round-robin test [7]). It shows the geometric dimensions as well as the support conditions of the specimen. Further, the material properties of concrete and steel are given. As regards the model parameters for the single- and the multi-surface model, the same parameters as used in Section 3 are employed (see Table 2).

4.2 Numerical study

In the numerical study, the material behavior of the steel bolt is assumed to be linear elastic. For the description of concrete, the two previously described material models are used.

Because of symmetry conditions, the problem is solved by means of axisymmetric
analyses. For this mode of analysis, special attention must be paid to the definition of the characteristic length $\ell_t$ used in the context of the fictitious crack concept. In here, an anisotropic formulation for $\ell_t$ accounting for a constant number of cracks in the circumferential direction is employed. According to this formulation, $\ell_t$ related to circumferential cracks increases with the distance from the symmetry axis. For cracks opening in the axisymmetric plane, $\ell_t$ is computed from $\ell_t = \sqrt{A_e}$, where $A_e$ represents the area of the finite element. According to [18], four cracks are assumed in the present study to open in the circumferential direction. The FE mesh used in the analyses is shown in Figure 18(a).

Two calculations were performed on the basis of the described single-surface model. They are characterized by the use of the associate and the non-associate flow rule, respectively. The obtained ultimate loads are listed in Table 3. The reason for the overestimation of the ultimate load by the analysis based on the associate flow rule is the development of highly confined stress states. They stem from volumetric plastic deformations which are significantly reduced by means of the non-associate flow rule.

As regards the plastic deformations obtained from the multi-surface model, small deviations between the results based on the associate and the non-associate flow rule, respectively, were observed in the previous section. The same situation occurred for the pull-out

**concrete:**
Young’s modulus: $E_c=30000$ N/mm²
Poisson’s ratio: $\nu_c=0.2$
uniaxial compressive strength: $f_{cu}=40$ N/mm²
uniaxial tensile strength: $f_{tu}=3$ N/mm²
fracture energy: $G_f'=0.1$ Nmm/mm²
fracture energy: $G_{f'}^H=50G_f'$

**steel:**
Young’s modulus: $E_s=210000$ N/mm²
Poisson’s ratio: $\nu_s=0.3$

---

**Figure 17:** Pull-out analysis: geometric dimensions and material properties

dimensions:
- $d=150$ mm  $a=150$ mm
- $t=15$ mm  $c=22.5$ mm
- $\phi=45$ mm  $\phi_r=24$ mm

---

---

---

---
Figure 18: Pull-out analysis: (a) FE discretization and (b) load-displacement curves obtained from the multi-surface model

analysis yielding almost the same values for the ultimate load for the considered flow rules. Further, the influence of confinement on the load-carrying behavior was investigated. For this purpose, each analysis was performed with and without consideration of confinement. The obtained results show a significant influence of confinement on the ultimate load (see Figure 18(b)). Consideration of confinement leads to an increase of the ultimate load by 19%.

Table 3 contains the values for the ultimate loads obtained from the analyses.

<table>
<thead>
<tr>
<th></th>
<th>single-surface model*</th>
<th>multi-surface model</th>
</tr>
</thead>
<tbody>
<tr>
<td>associate flow rule:</td>
<td>510 kN</td>
<td>347 kN (confined)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>292 kN (unconfined)</td>
</tr>
<tr>
<td>non-associate flow rule:</td>
<td>369 kN</td>
<td>342 kN (confined)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>292 kN (unconfined)</td>
</tr>
</tbody>
</table>

* initial value of $\alpha_h$ was set equal to $\alpha_h = 1$, i.e., $f_{cy} = f_{cu}$

Table 3: Ultimate loads obtained from single- and multi-surface model, respectively

5 Conclusions and outlook on future work

In the present paper, two material models for plain concrete were presented. One of them is a single-surface model whereas the other one is a multi-surface model. Their
potential was demonstrated by means of re-analyses of triaxial compressive tests and a uniaxial tensile test. The analysis of a pull-out test served as the vehicle for showing the predictive capabilities of the models on the structural level.

From the performed numerical simulations the following conclusions can be drawn:

• As regards re-analysis of triaxial compression tests, the evolutions of the axial stress at different levels of confinement, obtained by means of the single-surface and the multi-surface model, respectively, showed good agreement with the experimental results. As regards the lateral deformations, the rather simple formulation for the non-associate flow rule employed in the multi-surface model provided good results for medium confinement. However, for low and high levels of confinement, considerable deviations between the numerically and experimentally obtained lateral deformations were observed. The proposed yield potential used in the context of the single-surface model led to good agreement between the experimentally and numerically obtained lateral deformations at all levels of confinement.

• As regards the pull-out analyses, the difference between the ultimate loads obtained from the single- and the multi-surface model was 7 %. Use of the associate flow rule in the context of the single-surface model led to a significant overestimation of the ultimate load. The overestimation was caused by the development of volumetric plastic strains controlled by the associate flow rule. These strains led to confinement and, hence, to an increase of the result for the load-carrying capacity. For the multi-surface model, however, almost similar yield potentials were used for the associate and the non-associate flow rule. Hence, low deviations of the ultimate load were observed for the considered flow rules. Consideration of confinement within the Drucker-Prager criterion was found to have a significant influence on the numerical results.

Future research will include further improvements of the presented material models such as:

• an improvement of the non-associate flow rule employed in the multi-surface model in order to reduce the observed deviations between numerical and experimental results for the lateral deformations, and

• the extension of both models towards simulation of oedometric and hydrostatic stress paths (see [1] [3] for respective test results). Therefore, the presented multi-surface plasticity model needs to be extended by an additional yield surface in order to control the material response for high compressive stress states.

6 Acknowledgement

Partial financial support by the Austrian Foundation for the Promotion of Scientific Research (FWF) under contract P11337-ÖPY is gratefully acknowledged.
References


