NUMERICAL SIMULATION OF ASYMMETRICAL STEADY SHOCK WAVE INTERACTIONS

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Abstract. Various aspects of asymmetrical steady shock wave interactions between two wedges are investigated numerically. It is shown that a hysteresis phenomenon exists in transition between regular and Mach reflection of strong asymmetrical steady shock waves, similar to that observed earlier in symmetrical shock wave interactions. The transition angles are in close agreement with theoretical predictions. Special types of shock wave reflection, such as an inverse Mach reflection, predicted analytically by Li et al. [1] have been observed in the numerical computations. It has been also shown numerically that a unique shock wave reflection type with one weak and one strong reflected shocks exists at a certain combination of the wedges angles.
1 INTRODUCTION

In this numerical study we consider various aspects of an interaction of asymmetrical steady shock waves between two wedges installed in a supersonic stream at different angles of attack. In this case, as well as for the symmetrical steady shock wave interaction, two possible configuration exist: regular reflection (RR) and Mach reflection (MR). These are illustrated in Fig. 1. Regular reflection consists of two incident shock waves IS whose angles of incidence $\alpha_1, \alpha_2$ are determined by the wedges angles of attack $\theta_1, \theta_2$, two reflected shock waves RS, and a slipstream SS emanating from the reflection point. The configuration of MR is more complicated: in addition to the incident and reflected shocks there exist a Mach stem with a subsonic flow behind it, and two slipstreams emanating from the triple points T. Due to an interaction with the expansion fans EF, a length scale to this configuration is provided. The slipstreams form a “virtual nozzle” [2], in which the subsonic stream is accelerated to supersonic velocity. The height of the Mach stem is defined by the relation between the inlet and throat cross-sectional areas of the virtual nozzle.

Figure 1: Schematic of an asymmetrical regular (left) and Mach (right) shock wave interaction

A detailed theoretical analysis of the asymmetrical steady shock wave interactions was provided by Li et al.[1]. It was shown using a pressure-deflection shock polars technique that depending on the flow parameters and wedges angles of attack there exist some domains where different shock wave interactions might take place. These domains are shown in $(\theta_1, \theta_2)$-plane in Fig.2, which is a reproduction of Fig.7 of [1]. For the wedges
angles below the von Neumann line, which is analogous to the well-known von Neumann criterion for symmetrical shock waves interactions, only an RR is theoretically possible. For the angles $\theta_1, \theta_2$ combinations above the detachment line, which is analogous to the detachment criterion in symmetrical shock wave reflections, the only theoretically possible configuration is an MR. Between these two theoretical bounds, in a so-called dual solution domain, both types of reflection, RR and MR, are possible. The asymmetrical Mach reflection configuration, as was shown by Li et al.[1], may consist of two direct Mach reflections (DiMR-DiMR), one DiMR and one stationary Mach reflection (DiMR-StMR), and one DiMR and one inverse Mach reflection (DiMR-InMR).

![Figure 2: Domains of different solutions in asymmetrical case. M = 4.96](image)

The existence of different MR configurations is dependent on the $\theta_1, \theta_2$ angles. The theoretical bounds for them are indicated as dashed lines in Fig. 2. Along these lines a DiMR-StMR configuration is possible, with one of the slipstreams being parallel to the oncoming flow near the triple point. For higher $\theta_1$ or $\theta_2$ DiMR-DiMR configuration takes place when both slipstreams are directed to the inner portion of the flow. For lower $\theta_1$ or $\theta_2$ the configuration is DiMR-InMR. In this case one of the slipstreams is directed outwards. Such an MR with an inverse direction of a slipstream was observed in the experiments conducted by Li et al.[1].

The existence of a dual solution domain where RR and MR are theoretically possible implies the possibility of a hysteresis in transition between two types of reflection. The
hypothesis of the hysteresis phenomenon was proposed by Hornung et al.[3]. The hysteresis was indeed observed in transition between RR and MR for a symmetrical reflection of steady shock waves in experiments [4], [5] and computations [6]. Recently, a similar hysteresis has been recorded experimentally for the asymmetrical shock waves reflection in course of Li et al.[1] study and also by [7]: the RR → MR transition occurred significantly higher than the reverse transition (in [1] experiments the transition angles were close to the detachment line). When the angle of the wedge was decreased, the MR persisted down to the angles close to the von Neumann line. Due to technical limitations, all the sequence of events during the rotation of the wedge (successive transitions between the different shock wave configurations predicted analytically) were not recorded in those experiments.

Another interesting phenomenon hypothesized in Li et al.[1] theoretical analysis is a strong solution, which is possible in a reflection of asymmetrical shock waves at some combination of the wedges angles. This phenomenon is of special interest because of its uniqueness.

In this study we are aimed at verifying numerically all above mentioned theoretical predictions.

2 NUMERICAL TECHNIQUES

To simulate the interaction of asymmetrical shock waves, we solve numerically two-dimensional unsteady Euler equations

$$\frac{\partial Q}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} = 0,$$

here

$$Q = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ e \end{pmatrix}, \quad F = \begin{pmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ u(e + p) \end{pmatrix}, \quad G = \begin{pmatrix} \rho v \\ \rho uv \\ \rho v^2 + p \\ v(e + p) \end{pmatrix}.$$  

A perfect gas model with $\gamma = 1.4$ was used. The equation of state is given by

$$p = (\gamma - 1) \left( e - \rho \frac{u^2 + v^2}{2} \right) = \rho T/\gamma M^2$$

The fourth-order MUSCL reconstruction formula developed by Yamamoto & Daiguji [8] is utilized to reconstruct the cell face values of the primitive variables (i.e., density, pressure and the velocity components) from cell averaged ones. Thus, the cell face values are computed as

$$q_{i+1/2}^L = q_i + (Dq_i^L + 2Dq_i^R)/6,$$

$$q_{i+1/2}^R = q_{i+1} - (Dq_{i+1}^R + 2Dq_{i+1}^L)/6,$$
Numerical flux at a cell interface proximate Riemann solver [9] which is very robust for modeling high-speed flows. The numerical fluxes are calculated with the HLLE (Harten—Lax—van Leer—Einfeldt) scheme by Shu & Osher [10].

Parameters $b, b_1$ are chosen as $b = 4, b_1 = 2$. Here the minmod slope limiter is

$$\text{minmod}(x_1, \ldots, x_n) = \begin{cases} x_k, & \text{if } \text{sgn}(x_1) = \text{sgn}(x_2) = \ldots = \text{sgn}(x_n), \\ 0, & \text{otherwise} \end{cases}$$

\[ x_k = \min(|x_1|, \ldots, |x_n|) \]

Numerical fluxes are calculated with the HLLE (Harten—Lax—van Leer—Einfeldt) approximate Riemann solver [9] which is very robust for modeling high-speed flows. The numerical flux at a cell interface $\hat{F}_{i+1/2}$ computed with HLLE solver is given by

$$\hat{F}_{i+1/2} = \frac{a^+ F^L - a^- F^R + a^+ a^- (q^R - q^L)}{a^+ - a^-},$$

where

$$a^- = \min\{0, u_n^L - c^L, \bar{u}_n - \bar{c}\},$$

$$a^+ = \max\{0, u_n^R + c^R, \bar{u}_n + \bar{c}\}$$

Here $u_n$ — is a normal to the boundary velocity component, $c = \sqrt{\gamma p/\rho}$ — is the speed of sound, and a tilde refers to the Roe average between the states $q^L$ and $q^R$.

The time integration was accomplished by the third-order explicit TVD Runge-Kutta scheme by Shu & Osher [10].

The flow around two wedges immersed in a uniform supersonic stream was simulated. The wedges (see Fig. 3), which were shaped as right-angle triangles, had an angle of $15^\circ$ at their leading edge. During the course of each series of computations the angle of attack of one wedge was fixed while the other wedge was rotated around its trailing edge. The distance between the trailing edges of the two wedges, $2g$, was kept constant and equal to 0.84w (where $w$ is the length of the wedge hypotenuse) in most of the computations. The first computation of each series started from a uniform flow filling the entire computational domain. Each subsequent computation of the series started from the converged flow field of the preceding computation. The computational domain was divided into four zones (Fig. 3), the total number of quadrilateral cells in the four zones, in most of the computations, was approximately 80,000. A uniform flow with a given Mach number was specified on the left (inflow) boundary. The right (outflow) boundary
was situated far enough downstream of the wedges so that the flow was supersonic on it. The flow variables were extrapolated from within the domain on the outflow as well as on the bottom and top boundaries. The use of the extrapolation on the bottom and top boundaries is similar to imposing some kind of non-reflective conditions and results only in a very weak artificial reflection from these boundaries. Inviscid solid wall conditions were imposed on the wedge surface.

![Figure 3: Schematic of the computational domain](image)

3 RESULTS

The results of the Euler numerical simulations of the transition between regular and Mach reflection are presented in Fig. 4 and Fig. 5. The angle of the upper wedge $\theta_1$ was constant in each set of the computations: $\theta_1 = 18^\circ$ and $\theta_1 = 28^\circ$, respectively. The angle of the lower wedge $\theta_2$ was varied during computations from the value below the von Neumann line (see Fig. 2) to the value above the detachment line, and then back to the initial value. The computed density isolines are shown on the right in Fig. 4 and Fig. 5. Each one of the horizontal rows is at a certain angle $\theta_2$, the corresponding shock polar solution is given on the left.

The hysteresis during RR $\leftrightarrow$ MR transition is quite evident in Fig. 4. The computations start with an RR at $\theta_2 = 22^\circ$, which is below the von Neumann line (Fig. 2). When $\theta_2$ is increased RR still exists in the dual solution domain. At $\theta_2$ higher than the detachment line regular reflection is theoretically impossible, and the resulting configuration is an MR (see frame at $\theta_2 = 36^\circ$), which consists of one DiMR and one InMR (note the direction of the upper slipstream). The DiMR-InMR configuration persists in the dual solution domain when $\theta_2$ is decreased, and two different solutions (RR and MR) are obtained at the same lower wedge angle $\theta_2$. At $\theta_2$ below the von Neumann line MR configuration
is theoretically impossible, and, as a result, MR → RR transition occurs. A similar hysteresis loop is obtained at $\theta_1 = 28^\circ$ and is shown in Fig. 5. The RR is maintained throughout the dual solution domain when increasing $\theta_2$. At $\theta_2 = 30^\circ$, that is above the detachment line, the MR solution is obtained, which consists of two DiMR. The DiMR-DiMR configuration is maintained at decreasing $\theta_2$ (see the frame at $\theta_2 = 24^\circ$). In accordance with theoretical predictions, at $\theta_2 = 21^\circ$ the DiMR-StMR configuration is obtained (near the lower triple point the slipstream is parallel to the oncoming flow). At $\theta_2 = 18^\circ$ this configuration transforms into the DiMR-InMR. The MR → RR transition occurs when we go back to the initial value $\theta_2 = 12^\circ$.

![Figure 4: Results of numerical computations of the hysteresis loop (right) and corresponding shock polar solutions (left). $M = 4.96$, $\theta_1 = 18^\circ$](image)

It is evident that all the shock wave configurations predicted analytically in asymmetrical shock wave interactions are perfectly reproduced in the numerical simulation. The hysteresis phenomenon in MR → RR transition is definitely observed in our computations, and the transition angles correspond to those prescribed theoretically.
As mentioned earlier, Li et al. [1] hypothesized, in the course of their analytical study, that a shock wave configuration that consists of one weak and one strong reflected shock, i.e., wRS-sRS, can be obtained in the reflection of asymmetrical shock waves. The enlarged view of the pressure-deflection diagram, which is given in Fig. 6, shows the shock polars of the upper and lower reflected shock waves for $M = 4.96$, $\theta_1 = 35^\circ$ and a few values of $\theta_2$. In this figure, $S$ and $D$ are the sonic point and the maximum deflection point of the $R_1$ shock polar of the upper reflected shock wave, respectively, and $E$ is the point of tangency of two polars that corresponds to the detachment criterion for the reflection of asymmetrical shock waves. If the $R_2$, polar of the lower reflected shock wave, intersects the $R_1$-polar below point $S$ (e.g., the $A$-polar in Fig. 6) then the resulted shock wave configuration is wRS-wRS with a supersonic flow behind the two reflected shock waves, i.e. a regular reflection. However, if the intersection point lies between points $S$ and $D$ (e.g., the $B$-polar), the shock wave configuration is again a wRS-wRS but, unlike in the previous case, the flow behind the upper reflected shock wave is subsonic. It should be mentioned that in both of these cases the polars intersect the $R_1$-polar at two points, the second one (which is not shown in the figure) corresponds to a wRS-sRS. In practice however, the solution with lower pressure, i.e. wRS-wRS actually takes place. The most interesting situation occurs when both the intersection points lie on the strong branch of the $R_1$-polar: the first point is between points $D$ and $E$, and the second point is above point $E$ (e.g., the $B'$-polar). Even if it is assumed, as earlier, that the solution with the lower pressure is the one that actually materializes, it corresponds now to a wRS-sRS wave configuration. Hence, there exist two solutions with equal flow deflection angles, $\theta$, and different pressure ratios, $p/p_0$, behind the reflected shock waves (e.g., the $A$- and $A'$-polars in Fig. 6). Therefore, the upper reflected shock wave corresponding to the intersection of $R_1$-polar with $A$-polar is the weak solution and that corresponding to the intersection with $A'$-polar is the strong solution.

Our attempts to simulate this unique situation numerically have succeeded. In contrast to other computations of this study, the number of grid points was increased to approximately 160,000 in this case in order to improve the resolution near the reflection point. The distance between the wedges $2g$, in this calculation, was changed to $0.6w$. A wRS-sRS configuration was obtained in our simulations for $M = 4.96$, $\theta_1 = 35^\circ$ and $\theta_2 = 15.98^\circ$ (see Fig. 7), which corresponds to the $A'$-polar in Fig. 6. A shock wave configuration that corresponds to polar $A$ and consists of two weak regular reflections, i.e., wRS-wRS, across which the flow deflection was identical to the above-mentioned wRS-sRS case, was obtained for $M = 4.96$, $\theta_1 = 35^\circ$, and $\theta_2 = 14.58^\circ$, (see Fig. 8). Therefore, the numerical simulation indeed resulted in two different shock wave configurations, a wRS-sRS and a wRS-wRS behind which the flow deflections are the same but the pressures are different: $(P_{\text{low}}/P_0 = 40.7$ for the wRS-wRS case, and $P_{\text{low}}/P_0 = 46.0$ for the wRS-sRS case). Furthermore, while the flow was supersonic in the wRS-wRS case, it was subsonic behind the strong shock wave in the wRS-sRS case. This is illustrated in Fig. 9 where the subsonic region behind the upper reflected shock wave in the case of
Figure 5: Results of numerical computations of the hysteresis loop (right) and corresponding shock polar solutions (left). $M = 4.96, \theta_1 = 28^\circ$.

strong reflection is shown. Further downstream the flow is accelerated to a supersonic velocity due to the influence of the expansion fan emanating from the trailing edge of
the wedge. It should be noted that the above mentioned pressure rise \( P_{\text{high}}/P_0 = 46.0 \) in the wRS-sRS case is somewhat lower than the analytically predicted value for this case (i.e., 46.8). This discrepancy is probably associated with the upstream influence of the expansion fan through the subsonic zone, which cannot be accounted of in an analysis based on the shock polar technique. The existence of this subsonic region implies that the shock reflection configuration (the size of subsonic region, and variation of the reflected shock angle, and pressure along the subsonic zone) may depend on the geometry of the problem, i.e. on normalized distance between the wedges \( 2g/w \). That is, in contrast to usual regular reflection, the strong reflection contains a characteristic length scale. The numerical simulations were also performed for the conditions that match the \( R_1-B \) and \( R_1-B' \) shock polar combinations in Fig. 6. The difference of this case from the one considered above is that in addition to the subsonic flow behind the strong reflected shock wave of the wRS-sRS wave configuration, the flow is subsonic also behind one of the weak reflected shock waves of the wRS-wRS wave configuration. The computations revealed the closed (bounded) subsonic zones similar to that shown in Fig. 9, for both the strong and the weak reflections.

![Figure 6](image)

Figure 6: Enlarged view of the reflected shock polar intersection. \( M = 4.96, \theta_1 = 35^\circ, \theta_2 = 14.58^\circ \left(R_2 \text{ polar } A\right), 15.98^\circ \left(A'\right), 15.07^\circ \left(B\right), 15.90^\circ \left(B'\right) \).
The surprising possibility of the existence of the strong solution, which was realized in the course of the present computations, is a unique feature of the reflection of asymmetrical shock wave. As explained by Li et al.[1], the presence of a complementary weak regular reflection provides the mechanism to maintain the pressure behind the reflected shocks high enough to support the strong reflected shock wave.
4 CONCLUSIONS

Asymmetrical interaction of steady shock waves between two wedges is simulated numerically with Euler high-order shock-capturing code. It has been shown that a hysteresis phenomenon in transition between regular and Mach reflection similar to that observed in symmetrical shock wave reflections is also present in the asymmetrical case. Special types of Mach reflection configuration, such as direct, stationary and inverse Mach reflection, have been observed in the numerical computations. The existence of a strong solution in asymmetrical shock wave reflection hypothesized in analytical study of Li et al.[1] has been documented in the numerical computations. Two solutions, regular with two weak reflections and a reflection of a special type with one weak and one strong reflection, are obtained for the same deflection angle. The pressures behind the reflected shock differ significantly in these two cases, and the flow behind the strong reflected shock is subsonic.

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