NUMERICAL SIMULATION OF CONJUGATE FORCED CONVECTION AND HEAT CONDUCTION WITH SOLIDIFICATION OF WATER IN FOOD FREEZING


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Key words: Food freezing, Conjugate Solidification, Finite volume simulation.

Abstract. Conjugate forced convection in air and heat conduction with solidification of water content in food freezing is predicted. Continuity, linear momentum and energy equations are solved to predict the fluid dynamics and heat transfer around the food. Solidification of water is calculated by using a temperature dependent apparent specific heat. The method of finite volumes, developed in generalized body fitted coordinates is used along the SIMPLER algorithm to solve the discretized equations. Time dependent results for velocity and temperature distributions are presented for two cases: a plate shaped and a piece of a cross section of salmon meat being cooled in air.
1 INTRODUCTION

Food freezing is an industrial process used to preserve the quality of a variety of solid foods such as meats, fish, fruits and vegetables. A cold stream of air is usually forced to flow around the food. The heat flow in solid foods is by conduction and from the surface to the surroundings by forced convection.

nearities in the model. The presence of non linear terms in the mathematical model has motivated the use of either finite differences or finite elements methods \(^1\text{-}^4\). However, the uncertainties in the values of the heat transfer coefficient used in the boundary conditions of the third kind have been recognized to be the major error source in the numerical predictions \(^4,^5\).

The objective of this paper is to describe the numerical solution to predict the fluid mechanics and convective heat transfer in air around foods during freezing. Forced convection from the food to the surrounding air is calculated from continuity, linear momentum and energy equations conjugated to the heat diffusion equation inside the food. The liquid to solid phase transformation of the water content in the food is calculated by using a temperature dependent specific heat that includes the solidification enthalpy. A sequential procedure based on a semi-implicit method for pressure linked equations, SIMPLE, is used to solve the discretized basic equations. The system of algebraic equations is solved by using an hybrid algorithm that includes the Tri-Diagonal Matrix Algorithm and Gauss-Seidel with successive under-relaxation. Food with two different geometries are investigated: one with a plate shape and the second one with a cross section that changes in space in a way that represents the geometry of salmon. The physical domain is discretized by using a body fitted coordinates system around the food geometry. Results for transient evolution of the velocity vectors, stream function, pressure and temperature distributions in air are presented along to the temperature distribution in the food. Physical experiments have been performed to obtain cooling curves at selected locations inside pieces of salmon meat. Temperature measured with thermocouples and a data acquisition computerized system are used to validate the mathematical model and the numerical method used to solve the problem \(^6\).

2 MATHEMATICAL MODEL

The analysis consider a solid food at a temperature \(T_0\) surrounded by air at rest at \(T_0\). Suddenly a refrigeration unit moves the air around the food causing the temperature of the air stream to decrease. Figure 1 shows a schematic view of the physical situation with the system of coordinates.

The approximations used in the analysis include: air is a Newtonian, incompressible fluid with constant properties, the flow is unsteady, laminar and two-dimensional. The food is a binary mixture of solids and water that can be either in liquid phase, when \(T>T_{cr}\), or partially in solid phase, as ice, with density, specific heat and thermal conductivity that are changing.
with temperature. The governing equations for the forced convection heat transfer in air flow around the food are: continuity, Navier-Stokes and energy equations:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

(1)

\[
\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \frac{\partial p}{\partial x}
\]

(2)

\[
\rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \frac{\partial p}{\partial y}
\]

(3)

\[
\rho c_p \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)
\]

(4)

Inside the food the heat diffusion equation describes the heat conduction:

\[
\frac{\partial}{\partial t} \left( \rho_f c_{p_f} T \right) = \frac{\partial}{\partial x} \left( k_f \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k_f \frac{\partial T}{\partial y} \right)
\]

(5)

The properties of the air are assumed to remain constant during the process: \( \mu = 1.8224 \times 10^{-5} \) kg/(m.s); \( \rho = 1.3 \) kg/m\(^3\); \( k = 0.0251 \) W/(m\(^\circ\)C) and \( c_p = 1.01 \) kg/(kg\(^\circ\)C). Thermal properties of the food have been calculated from a thermodynamic model that assumes an ideal binary solution of solids and pure water. The phase change of the water content in the food has been incorpored into the mathematical model (eq. 5) by the use of an apparent specific heat that depends on the enthalpy of solidification of the water, \( h_{fs} \)

\[
c_{p_f} = (m_f c_p)_{H_2O} + (m_f c_p)_{H_2O} + (m_f c_p) + h_{fs} \frac{dnf_{H_2O}}{dt}
\]

(6)

where the specific heats of water in solid (ice) and liquid phases, and of the solid in the food are changing continuously with temperature. The mass fractions of each components can be calculated with the known initial values of water content in the food. In the mathematical model the temperature dependence of each thermal property of salmon meat calculated with the thermodynamic model has been implemented in terms of fifth degree polynomial functions,
\[ f(T) = \alpha_0 + \alpha_1 T + \alpha_2 T^2 + \alpha_3 T^3 + \alpha_4 T^4 + \alpha_5 T^5 \]  

(7)

where \( f \) can be the density, specific heat or thermal conductivity. A full detailed description of this procedure has been previously published \(^6\).

The initial conditions consider that air is at rest, at a uniform temperature \( T_0 \) and in the thermal equilibrium with the food,

\[ t < 0 : \; u = v = 0 \; \; ; \; T = T_0 \]  

(8)

The boundary conditions for the air flow are:

\[ \text{At} \; x = 0 : \; u = V_{\text{in}} \; ; \; v = 0 \; \; ; \; T = \alpha - b \ln(t) + c \sin(dt) \]  

(9)

where temperature of air is changing with time \( t \), first with a logarithm dependence and later in a periodic way. The last term describes the variation of air temperature caused by a thermostat that initiates and shut down the refrigeration compressor unit. Constants \( a, b, c \) and \( d \) are calculated after the air temperature measured with thermocouples is adjusted by least square fitting.

\[ \text{At} \; x = x_{\text{out}} : \; v = 0 \; ; \; \frac{\partial u}{\partial x} = \frac{\partial T}{\partial x} = 0 \]  

(10)

\[ \text{At} \; y = 0 \text{ and } y = Y : \; u = 0 \; ; \; v = 0 \; ; \; \frac{\partial T}{\partial y} = 0 \]  

(11)

3. COMPUTATIONAL METHOD

A body system coordinates grid generation scheme was developed. The computational domain in cartesian coordinates \((x-y)\) was transformed into a curvilinear trajectories of the coordinates \(^7\) that was derived from solving two Poisson equations with stretching functions of the type \(^8\)

\[
\frac{\partial^2 \varepsilon}{\partial x^2} + \frac{\partial^2 \varepsilon}{\partial y^2} = P_\varepsilon \; ; \; \frac{\partial^2 \eta}{\partial x^2} + \frac{\partial^2 \eta}{\partial y^2} = P_\eta
\]  

(12)

where \( P_\varepsilon \) and \( P_\eta \) are the Poisson sources for each curvilinear coordinate.

The system of partial differential governing equation was transformed into the new system of coordinates. The finite volume method was used to generate the system of algebraic discretized equations to calculate temperature and pressure at the grid point, and velocity components and heat fluxes at the control volume surfaces, with a staggered curvilinear grid. A discretized equation for pressure was derived from the continuity equation.
and a sequential scheme was implemented to solve the velocity from the momentum equation. Based on the SIMPLER algorithm\(^9\).

Second order derivatives describing diffusion viscous forces in momentum equation and heat conduction in the energy equation, were calculated by using linear interpolation profiles, between nodes, calculated by the power law interpolating scheme\(^9\). Unsteady terms were evaluated in an implicit way by a normal backward finite difference scheme. An iterative procedure was implemented to solve the system of equations. The nonlinear systems of equations was linearized by using values of the thermal properties (\(\rho, k, cp\)) calculated in the previous iteration and by using the Crank-Nicolson procedure to evaluate the convective terms. An hybrid scheme that combined the tri-diagonal matrix algorithm, TDMA and the iterative Gauss-Seidel method with successive overrelaxation was used to solve the system of discretized equations. The relaxation factors for pressure, temperature and each velocity components were: 0.85, 0.75 and 0.65. Convergence for each dependent variable (p, T, u, v) was obtained with a relative deviation of 7 x 10\(^{-3}\) after 1,000 iterations. A non-dimensional variable time step was used to solve the unsteady problem in the initial stage (when \(t \leq 10s\)),

\[
\Delta \tau(t) = \frac{\Delta t_{max}}{t_{ref}} \begin{cases} 
0.01 + 0.197 t - 0.0098 t^2 & ; 0 \leq t \leq 10s \\
1 & ; t > 10s
\end{cases}
\]

(13)

The maximum time step \(\Delta t_{max}\) was equal to 10 s.

4. RESULTS

The mathematical model solved with the finite volume method was applied to describe the unsteady fluid dynamics and heat transfer for food freezing. External forced convection in air around a food was coupled to conduction with solidification of the water content inside the food.

Two examples of food freezing in air were studied. The food geometries were assumed to be as a plate shape, as it is shown in figure 1 (a) and in a profile similar to the cross section of a salmon, figure 1 (b).
The physical domain including the plate shaped food and the surrounding air was discretized with a non-uniform staggered grid of 29 nodes in y direction and 81 nodes in the x direction, as it is shown in figure 2 a. Figure 2b depicts the mesh with 81 by 51 nodes used to define the computational domain for the freezing of the cross section of a salmon. In both cases, it can be seen that a local refinement has been used in the vicinity between the solid food and the air nearby, where the velocity and temperature gradients are assumed to be large.

The effect of grid size on the numerical solution for the fluid dynamics for the flow around a plate shaped food is shown in figure 3. The velocity at a dimensionless axial location \( x^* = 1 \) calculated with five different grids, from 71 x 29 up to 71 x 91 is shown in figure 3 a to be almost the same in all computations on the top surface of the food calculated with five different grids. Results are seen to be almost independent of the grid when 79x91 nodes are used.
Fig. 3. Effect of grid size on fluid dynamics results for flow around a plate shaped food.

Fluid dynamics results obtained from the computational simulation the velocity vectors for around the plate shaped food time equal to 0.75 s and 2.25 s are shown in figure 4 a. The forced convection has been calculated for a laminar flow with a Reynolds number Re=5.650, for a food with a length L to height T ratio L/T=6.58 with a grid of 29 x 81 nodes. Figure 4 b shows for the same case a comparison between experimental results for temperature measured with thermocouples at the geometrical center of the shaped food and the cooling curve predicted with the conjugate model, with three different grids. It is shown that the overall agreement obtained with the grid with 131x 51 nodes is excellent.

Fig. 4. Freezing of plate shaped food (a) velocity vectors; (b) Cooling curve
Figure 5 (a) shows the fluid dynamics of the air flow around the salmon cross section for two time instants (0.5 s and 1.0 s). Numerical simulation results show that a pseudo steady state of the fluid dynamics is reached after a time of 1 second. Therefore, from that instant of time the coupled transient solution of continuity and linear momentum equations calculated with a time step of $10^{-3}$ s can be finished. After 1 second only the energy equations for the fluid around the food and inside the food must be solved, with a larger time integration step, that was found to be equal to 0.25 second.

Figure 5 (b) describes the results of a comparison between calculated and measured transient temperature at the center and near the tip of the cross section of a piece of salmon meat. The numerical simulation was accomplished by using a grid with a mesh of 81x 51 nodes, for a flow with a Reynolds number equals to 9,420, that described the physical experiment in which the uniform intet velocity was equal to 3.3 m/s. The overall trend of the predicted cooling curves is seen to agree well with the experimental results during the cooling and freezing processes.
5. CONCLUSIONS

A mathematical model that includes continuity, linear momentum and energy equations for forced convection and heat diffusion with solidification of water content inside food has been developed to predict food freezing processes.

The finite volume method, with the SIMPLER algorithm, has been implemented in curvilinear coordinates to solve iteratively the unsteady conjugate fluid dynamics of non isothermal flows around solids with internal heat conduction and solidification. The computational simulation of bidimensional freezing problems can be accomplished with non uniform curvilinear grids with about 2,500 nodes.

Integration in time of the full model can be accomplished with time steps of 0.001 s for the first 2 seconds. At this time a pseudo steady state was found for the fluid mechanics problems and the time step was increased to 0.25 seconds to solve the unsteady energy equation in air and the heat diffusion equation with solidification of water in the food. Experimental measured temperature with thermocouples have been used to validate the simulation for a plate shaped and a piece of the cross section of salmon meat.

REFERENCES


**Acknowledgements:** The authors gratefully acknowledge the CONICYT-Chile for financial support through grant FONDECYT 1000207 and DICYT/USACH.