NUMERICAL SIMULATION OF SUPersonic AND HYPERSONIC FLOW AROUND DELTA WINGS IN COMPARISON TO EXPERIMENTAL RESULTS

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Abstract. This paper reviews results selected from recent numerical and experimental investigations of supersonic and hypersonic flow around a thick delta wing. The flow was simulated with a numerical solution of the parabolized Navier-Stokes equations which offer the advantage, that high Reynolds number flows can properly be dealt with in terms of sufficient grid resolution, and that the leeward vortex system can be computed without additional assumptions. The numerical solution, written for curvilinear coordinates, is constructed with the space marching technique for steady super- and hypersonic flow. Because of the inherent non-linearities in the equations the solution has to be iterated. In the momentum equations the convective terms are discretized with the second-order flux-vector splitting of Schwane and Hänel, and the terms describing the Stokes and Reynolds stresses with central differences. In the subsonic region of the boundary layer the parabolized conservation equations for steady flow become elliptic; however, for non-separating flows, the space-marching technique can be adapted by extrapolating the flow variables to the neighboring downstream points with second-order accuracy. Surface flow, separation and reattachment lines, cross-flow vortex patterns, and surface pressure distributions, obtained with the solution of Henze will be compared with recent experimental results for free-stream Mach numbers 2 and 4.
1 INTRODUCTION

The flow around a thick delta wing with round leading edge represents an idealization of the flow around an aerospace plane or of a space transportation system. Such vehicles are presently under development since one of the future requirements on space transportation systems will be a substantial reduction of overall costs for space travel. With regard to the necessary improvements of economic conditions for near-earth transportation, preliminary studies show, that the costs can be reduced to one tenth if new technologies are introduced. The implementation of new technologies seems to be overdue, since the Space Shuttle was developed over twenty five years ago. Another important aspect to be considered is the increasing proliferation of space debris. If non-recoverable transportation systems will continuously be used in future missions, the probability of endangering satellites, orbiting space-transportation systems or space stations will some day exceed acceptable limiting safety margins.

The introduction of new concepts will require the construction and testing of launch vehicles, designed with new technological strategies, as for example, multiple reusability of transportation systems as the primary goal; the generation of lift for take off by aerodynamic forces rather than rocket-propelled vertical take off; the reduction of weight by the use of air-breathing propulsion systems, and, one of the most important aspects, their integration into the vehicle; the reduction of stages, and last but not least, the introduction and development of high-temperature metallic and non-metallic materials.

The technological validation of such concepts will also require interdisciplinary fundamental investigations, involving aerodynamic considerations but to a much larger extent solutions of interactive problems, as for example, the interaction of three-dimensional shock waves with each other, with expansion waves, boundary layers, separated regions, and longitudinal vortices. For high flight Mach numbers, the influence of chemical reactions taking place in the air passing by the vehicle, in the combustion process and in the expansion in the exhaust nozzles must be taken into account. Because of the extreme heat loads the structures are exposed to, intensive theoretical and experimental investigations for high-temperature materials will be necessary, including structural dynamics. In the solution of such problems numerical techniques will play an important role. For that reason, their application will be demonstrated in the following for a particular case, the hypersonic research configuration ELAC 1, studied at the RWTH Aachen in a long-range research program.

The ELAC 1 configuration is a lifting body, its name standing as an abbreviation for the German ‘Elliptische Auftriebs-Configuration 1’. Its cross-section is composed of two semi ellipses of equal major but different minor axes. For the forebody, the ratio of the semi axes is $1 : 4$ for the upper side, and $1 : 6$ for the lower. The sweep angle is $75^\circ$. They then decrease again linearly to the trailing edge, which is
slightly elevated to give room for the propulsion system. The configuration is equipped with two fins. The details of the geometry, depicted schematically in Fig. 1, may be found in articles by Decker et al. [1], and Limberg and Stromberg [2,3].

Until now the flow around the research configuration ELAC 1 was simulated for super- and hypersonic free stream conditions with a numerical solution of the parabolized Navier-Stokes equations. The parabolized system was solved with a space marching technique, which is ideally suited for the simulation of high Reynolds number flows, as long as the flow in the direction of the main stream does not separate. The integration yielded surface pressure and heat-transfer distributions, vortex structures, and forces. The data were compared with experimental results ranging from hypersonic flow conditions to low-speed flows [4]. In the present paper some of the important results obtained for supersonic flow and for the onset of hypersonic flow at \(Ma_\infty = 4\) will be compared with experimental data as described by Krause et al. [5]. In passing it is mentioned here that the high speed experiments were carried out in several supersonic tunnels and in a shock wave tunnel. Low-speed flows were also investigated in several wind-tunnels up to a Reynolds number \(Re_\infty = 40 \times 10^6\). The latter experiments were carried out with a 6\(m\) model in the German Dutch Windtunnel (DNW).

Prior to the comparison of the numerical results with the experimental data a brief description of the method of solution as developed by Henze [6] and reported elsewhere [7,8] will be given. The paper will close with an outlook on future work.

2 THE PARABOLIZED CONSERVATION EQUATIONS

2.1 Simplification of the Navier-Stokes equations

The conservation equations are simplified in the process of parallelization by dropping the terms with second derivatives in the streamwise direction of the Stokes stresses and retaining the highest derivatives in the direction normal to the wall. For high free-stream Mach numbers \(Ma_\infty\) and relatively low Reynolds numbers \(Re_\infty\) an order of magnitude analysis results in the following non-dimensionalized form of the momentum equations:

\[
\rho (uu_x + vu_y) = -p_x/(\gamma Ma_\infty^2) + (\mu u_y)_y \quad (1)
\]

\[
\rho (uv_x + vv_y) = -p_y(Re_\infty)/((\gamma Ma_\infty^2) + (4/3)(\mu v_y)_y - (2/3)(\mu u_x)_y + (\mu u_y)_x \quad (2)
\]

Equations (1) and (2) show that for

\[
\gamma Ma_\infty^2 \to \infty, \quad Re_\infty/(\gamma Ma_\infty^2) \to 1, \quad \text{and} \quad p_x = O(1), \quad p_y = O(1) \quad (3)
\]

the influence of the pressure gradient in the direction of the main stream \(p_x\) on solution is greatly reduced, and \(p_x\) can be neglected [9]. For supersonic Mach numbers and high Reynolds numbers the various terms in the above equations attain different magnitudes, and the previous statement no longer holds; instead the order of magnitude of the second
momentum equation reduces to $O(1/Re_\infty)$ in the boundary layer. Outside of the boundary layer, neither one of the pressure gradients $p_x$ nor $p_y$ can be neglected, while near the wall in the subsonic part of the boundary layer the pressure gradient $p_y$ reduces to the order of magnitude stated above, namely $O(1/Re_\infty)$. If both pressure gradients are retained in the subsonic part of the boundary layer, the conservation equations change their type in that region. This is evident from the sub-characteristics. If one eliminates pressure, density, and temperature from the Euler equations, and if for simplicity, only two-dimensional flows are considered, i.e.

\begin{align*}
u u_x + v u_y + p_x / \rho &= 0 \quad (4) \\
u v_x + v v_y + p_y / \rho &= 0 \quad (5) \\
(pu)_x + (pv)_y &= 0 \quad (6) \\
u T_x + v T_y - (up_x + vp_y) / (c_p \rho) &= 0 \quad (7)
\end{align*}

the gasdynamic equation for isentropic flow is obtained:

\[(u^2 - a^2) \cdot u_x + (v^2 - a^2) \cdot v_y + 2uv u_y = 0. \quad (8)\]

The slopes of the Mach lines

\[dy/dx = [uv \pm a(u^2 + v^2 - a^2)^{1/2}] / (u^2 - a^2) \quad (9)\]

become imaginary for $u < a$, signalizing that the system of conservation equations becomes elliptic, when the velocity is less than the local speed of sound. A numerical solution of the parabolized Navier-Stokes equations for supersonic flow must therefore be constructed in such a way, that in the subsonic part of the boundary layer the influence of the pressure gradient in the direction normal to the wall does not perturb the numerical solution. A discussion of the complete characteristic polynomial of the parabolized Navier-Stokes equations is given by Henze [10]. Here the simplification of the governing equations was demonstrated with an order of magnitude consideration and with a discussion of the sub-characteristics of the parabolized equations to keep the derivation as short as possible.

## 2.2 Comments on the numerical solution of the parabolized Navier-Stokes equations

If it can be assumed that flow separation does not occur in the main flow direction, the conservation equations can be simplified by keeping only the leading order terms of the Stokes and Reynolds stresses, respectively. Because of the non-linearity of the equations the solution must be obtained in an iteration procedure. In the solution derived by Henze [5] the iteration is carried out with a time-like operator. For three-dimensional flows the parabolized conservation equations in dimensionless form may then be written in curvilinear coordinates $\eta$, $\zeta$, and $\xi$ as

\[Q_t + E_\zeta + F_\eta + G_\xi = (1/Re_\infty) \cdot S_\eta . \quad (10)\]
The quantity $Q$ is the solution vector of the conservative variables $\rho$, $\rho u$, $\rho v$, $\rho w$, and $\rho E$, and the quantities $E$, $F$, $G$, and $S$ represent the fluxes in the three coordinate directions, each multiplied by the inverse of the Jacobian of the metric transformation, as defined in reference [5]. The above equation represents the balance of mass, momentum, and energy for an infinitesimal volume element. The system is parabolic in time and space. The space marching technique is therefore well suited for constructing a solution steady super- and hypersonic flow. In subsonic regions, as in the boundary layer very close to the wall, this approximation fails, since the equations for steady flow become elliptic there, as pointed out before. In the solution obtained by Henze [6], this problem was circumnavigated by extrapolating the flow variables to the neighboring downstream points with a second-order extrapolation without any further assumption for the pressure gradient.

The time-like operator $Q_t$ is discretized with an 5-step Runge-Kutta scheme, and since this term is used for the iteration only, local time steps can be used to speed up the rate of convergence. For the spatial discretization a node-centered scheme is employed [5] with the MUSCLE extrapolation of van Leer [11] for each of the three spatial directions. To avoid oscillations near shocks the flux limiter of Albada [12] is used in reference 5. The spatial derivatives of the terms on the left-hand side of equ. (10) are approximated with the second-order flux-vector splitting of van Leer [13], and for higher Mach numbers with the flux-vector splitting of Schwane and Hänel [14]. For the extrapolation in the direction of the main flow in the subsonic region a second-order approximation is used. Thereby the difficulties with the change of type of the governing equations can be overcome in a simple manner. The terms on the right-hand side, which represent the Stokes and Reynolds stresses, and the corresponding terms in the energy equation, are discretized with central differences. The initiation of the space marching procedure in the x direction requires initial conditions. They are obtained with the solution procedure just outlined by assuming conical flow.

The grid used in the numerical integration is generated with a numerical solution of the Poisson equation for $\xi$ and $\eta$ with independent variables $x$ and $y$. The clustering and the orthogonality of the grid can be influenced by the source terms in the Poisson equation; the discretized equations are solved with a point Gauss-Seidel iteration, for details see reference 5. The third transformed coordinate $\zeta$ does not have to be considered as for the computation of the flow only two adjacent cross-sections are needed. After convergence is obtained for one cross-section the integration proceeds to the next station in the main flow direction and uses the values just computed as new initial data.

The iteration necessary for obtaining convergence for each cross-section is facilitated with the time-integration of the term $Q_t$, as described above. The integration of the parabolized system of conservation equations reduces the computation time for three-dimensional flows markedly and enables a rapid simulation of flows around delta wings. Some of the results so obtained will be discussed here.
2.3 Experimental conditions and measuring techniques

The experiments to be referred to in the comparison with the numerical data were carried out in a small low-speed wind tunnel and a supersonic suction wind tunnel of the Aerodynamisches Institut of the RWTH Aachen, and in a supersonic blow down wind tunnel of the Institute for Theoretical and Applied Mechanics of the Russian Academy of Sciences in Novosibirk [5]. This tunnel can be operated at free-stream Mach numbers between \(1.75 \leq Ma_{\infty} \leq 6\); it has a rectangular cross-section of \(0.6 \times 0.6 \, m^2\). The unit Reynolds numbers of the flow in the test section can be varied between \(5 \times 10^6 \leq Re_{\infty} \leq 56 \times 10^6\). The tunnel of the Aerodynamisches Institut can be operated at subsonic and supersonic speeds with free-stream Mach numbers \(0.2 \leq Ma_{\infty} \leq 3.6\) and Reynolds numbers \(Re_{\infty} \leq 13 \times 10^6\). The test chamber has a cross-section of \(0.4 \times 0.4 \, m^2\).

The model ELAC 1 as described by Limberg and Stromberg [2], and Stromberg and Limberg [3] is a lifting body of triangular plan form with a sweep angle \(\varphi = 75^\circ\). The aspect ratio is \(AR = 1.1\), and the cross-section of the model is formed by two semi ellipses. The ratio of the semi axes of the ellipse generating the surface on the upper side is \(1 : 4\), and on the lower side \(1 : 6\) [1]. The rounding of the leading edges is formed by the junction of the two semi ellipses. The model is shown in Fig. 1, together with the three cross-sections for which the pressure measurements were obtained. There are 24 taps for measuring the static pressure in the three cross-sections located at \(x/l = 0.3, 0.45,\) and \(0.6\), measured from the tip of the wing, as indicated in Fig. 1, together with their locations. The Reynolds numbers of the various test conditions were computed with the length of the model \(l = 0.3 \, m\).

3 RESULTS OF SIMULATION AND COMPARISON WITH EXPERIMENTS

3.1 Visualization of the flow patterns

Delta wings generate a vortex system on the leeward side of the wing, which enhances the lift at high angles of attack. Although this problem has been studied in numerous investigations, the various flow modes observed for example for supersonic flow, cannot completely be understood from experimental considerations alone. It was, however, found out in extended combined numerical and experimental studies, that the vortex structures can be determined uniquely and that the accuracy of the numerical results can be verified by comparing the pressure surface distributions with experimental data. The numerical solution employed in these studies is the space marching technique outlined above for the solution of the parabolized Navier-Stokes equations for supersonic and hypersonic free-stream conditions. Fig. 2 shows an example of vortex formation in the cross-flow on the leeward side of a delta wing, here the ELAC 1 configuration, as obtained with the solution of reference 5 and reported by Stromberg et al. [4].

As mentioned before the wing has sweep angle \(\varphi = 75^\circ\) and a rounded leading edge. The
free-stream conditions correspond to a Mach number $Ma_\infty = 2.0$, a Reynolds number $Re_\infty = 4 \times 10^6$, and an angle of attack $\alpha = 24^\circ$. Shown are the streamlines of the cross-flow at a position of 61 percent chord. At this relatively high angle of attack, five vortices can be identified, as computed with the method of reference 5 under the assumption that the flow is laminar such that closure assumptions did not have to be introduced for the Reynolds stresses and the turbulent heat transfer. Since it is also not known, under what conditions the flow undergoes transition to turbulent flow, it was felt best, to simulate the flow in a first attempt for laminar conditions only, and then conclude from the comparison of the numerical data with the experimental data on the influence of the turbulent momentum and energy transport.

Since it was not possible to visualize the cross-flow at a free-stream Mach number $Ma_\infty = 2.0$ in the supersonic wind tunnel of the Aerodynamisches Institut, an experimental visualization was attempted with the same model in incompressible flow in a low-speed wind tunnel [4]. The picture taken with the Laser-light sheet technique is shown in Fig. 3. The primary, secondary, tertiary and the shear-layer vortex are clearly recognized. The quaternary vortex can also be seen in the original photo, but is only vaguely visible in the picture of Fig. 3. The incompressible vortex structures are very similar to those computed for supersonic flow; differences are noted in the cross-sectional shape of the individual vortices. It seems that the number of vortices formed is not influenced by the density variation in the supersonic flow, but only the cross-sectional shape of the individual vortices.

Flow visualizations were also carried out for angles of attack $\alpha = 6^\circ$ and $\alpha = 10^\circ$ and free-stream Mach numbers $Ma_\infty = 2.02$ up to $Ma_\infty = 6$ in the supersonic tunnel in Novosibirsk. Within these limits the flow around the model changed its local characteristics substantially [5]. The photograph in Fig. 4 shows the experimentally determined oil-film patterns for an angle of attack of $\alpha = 10^\circ$ at a free-stream Mach number $Ma_\infty = 2.02$, and the experimentally determined pressure distribution for the stations $x/l = 0.3$, $x/l = 0.45$ and $x/l = 0.6$. As can be seen, the flow separates at the leading edge and under the primary vortex. The reattachment line of the primary vortex (divergence line $R_1$) and the secondary separation line (convergence line $S_2$) can clearly be identified, as can the tertiary vortex.

Elements of the computed wall streamlines are shown in Fig. 5 for $Ma_\infty = 2.02$ and $\alpha = 10^\circ$. Although the comparison of Figs. 4 and 5 shows that the computation yields almost the same patterns as the experiment, complete agreement can only be observed on the left side of the model [5]. The difference seems to indicate, that the flow orientation with regard to the model was not entirely symmetric during the test, as a close inspection of the experimental visualizations confirm. Similar results were obtained for the other free-stream Mach numbers investigated. The quality of the comparison also shows that joint numerical-experimental studies greatly benefit from the two-side approach, as the
results on one hand confirm the data but also increase the capability of flow investigations.

3.2 Comparison of computed with measured surface pressure distributions

In the investigations reported here the measured wall pressures were also compared with computed data. In Fig. 6 an example for the accuracy of the numerical simulation is shown for the spanwise pressure distribution obtained for \( Ma_\infty = 2, \ Re_\infty = 4 \times 10^6 \), and an angle of attack \( \alpha = 10^\circ \) for seventy, eighty, and ninety percent chord. The data shown here were chosen from several experimental campaigns, which, on the whole, are in equal agreement with the numerical results within the limits of the error bounds. The details of this comparison are reported by Henze [9].

The pressure distributions, given in Fig. 6 in the form of dimensionless pressure coefficients as a function of the dimensionless spanwise coordinate \( y/s \), are completely smooth for small angles of attack. The onset of vortex formation on the leeward side of the wing can be noted in the relatively steep change in the pressure distribution for \( 0.4 \leq y/s \leq 0.6 \). With increasing angle of attack, the step-like decrease in the pressure develops a pressure minimum, clearly recognized in the upper two diagrams of Fig. 6. The location of the pressure minimum indicates the position of the core of the primary vortex. The symbols show the pressure coefficients of the experiment and the solid lines the numerical results, which completely duplicate the former for all three stations. The close agreement of the data signalizes that the displacement effect of the boundary layer is adequately simulated with the numerical solution.

For the free-stream Mach number \( Ma_\infty = 4 \), vortex formation seems to be less pronounced, as may be concluded from Fig. 7. The decrease of the pressure on the upper side of the wing is barely noticeable in comparison to the pressure distributions obtained for \( Ma_\infty = 2 \). This behavior is also confirmed by the numerical results, which are shown for the angles of attack \( \alpha = 3^\circ, 6^\circ, 10^\circ \). As can be seen in Fig 7, the pressure on the lower surface varies substantially and increases with increasing angle of attack. For an angle of attack \( \alpha = 3^\circ \), when the surface is almost parallel to the direction of the oncoming velocity vector, the surface pressures are close to free-stream values, with a small negative gradient towards the leading edge, observed in all three cross-sections for all Mach numbers investigated.

4 CONCLUSIONS

Results selected from a series of flow visualization studies and surface pressure measurements obtained with a model of a hypersonic research configuration for supersonic and hypersonic free-stream Mach numbers in wind tunnels of different size and type were reported. The results of the experiments were compared with numerical data obtained with a solution of the parabolized Navier-Stokes equations.

The comparison confirms the reliability of the predictions, as long as the questions of
transition and the description of the Reynolds stresses and of turbulent heat transfer do not have to be raised. For the flows investigated the free-stream Reynolds numbers ranged from \( Re_\infty = 4 \times 10^6 \) to \( Re_\infty = 56 \times 10^6 \), and the free-stream Mach numbers were varied between \( Ma_\infty = 2 \) and 4. In the experiments the angles of attack covered the range \(-3^\circ \leq \alpha \leq 10^\circ\). For these conditions the comparison of the data showed, that the surface flow patterns, the separation and reattachment lines, generated in the cross flow by the leeward vortex structures can reliably be predicted. The computations also correctly predicted the generation of a primary, secondary, tertiary, and quaternary vortex. There are, however, also indications of recent flow calculations and comparison with experimental data not addressed in this paper, that substantial differences between numerical predictions and experimental results may be noted, if, for example the location of transition is not known or if the Reynolds stresses and the turbulent heat transfer are not correctly described.

Aside from these problems, the accuracy of future numerical predictions will strongly depend on the arrangement of the computational grid and its necessary local refinements. If the leeward vortex structures, the shocks and their interactions with them are to be correctly predicted, a-priory information about the location of the shocks may be needed. Here, the flow classifications, mainly obtained from previous experimental investigations might be a useful tool for proper grid generation and necessary refinement. The results presented here also indicate, that numerical flow simulation techniques will play an important role in future design work of space transportation systems.
REFERENCES


5 Figures

Figure 1: The ELAC 1 Configuration: Geometry and measuring stations [1,2,3].
Figure 2: Formation of leeward vortices on a delta wing with a round leading edge; sweep angle $\varphi = 75^\circ$; geometry specified in reference 1. $Ma_\infty = 2.0$, $Re_\infty = 4 \times 10^6$, angle of attack $\alpha = 24^\circ$; shown are computed cross-flow streamlines at 61 percent chord [4].
Figure 3: Primary, secondary, tertiary, quaternary, and shear-layer vortex observed in experiments in incompressible flow [4].
Figure 4: Oil flow patterns on the leeward side of the model of the ELAC 1 configuration for a free-stream Mach number $Ma_{\infty} = 2.02$ and angle of attack $\alpha = 10^\circ$; data are from reference [5].
Figure 5: Computed wall streamlines for $Ma_\infty = 2.02$ and $\alpha = 10^\circ$. Shown are the velocity vectors in the immediate vicinity of the surface.
Figure 6: Comparison of numerically and experimentally determined spanwise surface pressure distributions for a delta wing with a rounded leading edge and a sweep angle $\varphi = 75^\circ$. Shown are the dimensionless pressure coefficients for $Ma_{\infty} = 2.0$, $Re_{\infty} = 4 \times 10^6$, an angle of attack $\alpha = 10^\circ$, at seventy, eighty, and ninety percent chord. The symbols indicate the experimental data [15], the solid lines the numerical results [10]. Data are from reference 10.
Figure 7: Measured and computed pressure distribution on model of ELAC 1 configuration for a free-stream Mach number $Ma_\infty = 4$; plotted is the dimensionless pressure coefficient $c_p$ versus the dimensionless spanwise coordinate for $\alpha = 3^\circ, 6^\circ$, and $10^\circ$. 

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