LARGE ROTATION OF SUPERELEMENT FOR MULTIBODY ANALYSIS

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Abstract. This paper deals with large rotation of superelement for nonlinear analysis. All the equations are written in a frame, which follows the superelement. With respect to previous works 1,2,3, the difference is in the way the moving frame is computed. Here, we keep as independent variables the rotation and the center of the frame and we introduce 6 kinematic constraints with Lagrange multipliers in order to define the frame. The constraints are orthogonality conditions between the local displacements and the rigid body modes in the superelement frame. The method can be applied on superelement, which has no rotational degree of freedom and only translational degree of freedom.
1 INTRODUCTION

In order to introduce flexibility in a multibody analysis, one can use either classical finite elements or superelements. If the structure undergoes only small deformation but large rotations, the CPU and the size of the files will be smaller with the superelement approach. Superelements are also used in structural analysis when for a large model, only a small part has a nonlinear behavior.

The superelements we are concerned with are of the Craig and Bampton type. In the nonlinear analysis, we only need the reduced stiffness and mass matrices. The strain and kinetic energy are defined in a frame, which follows the superelement. So we use a corotational approach. With this definition, it is possible to find the internal and inertia forces. The key point is on the way the moving frame is defined. With respect to previous works \(^1,^2,^3\), this is the main difference. We also give the geometric stiffness linked to the superelement, which is necessary if an eigenvalue analysis around a deformed shape is performed.

2 NOTATION

\begin{itemize}
  \item \(x_B\) deformed nodal position in the basic frame
  \item \(X_B\) initial nodal position in the superelement frame
  \item \(Y_B\) deformed nodal position in the superelement frame
  \item \(u_B\) nodal displacement in the superelement frame
  \item \(x_0\) origin of the superelement frame
  \item \(R_0\) rotation matrix linked to the superelement frame
  \item \([a]=\tilde{a}\)
\end{itemize}

3 TREATMENT OF KINEMATIC CONSTRAINTS

In order to introduce kinematic constraints, an augmented Lagrangian\(^1\) method is used. We add to the strain energy \(U\) the kinematic constraints \(\phi\) as follow:

\[ U^* = U + k \lambda \phi + \frac{1}{2} p \phi^2 \]

The internal forces are computed from the first variation and are equal to:

\[ \delta U^* = \left[ \delta q^T \delta \lambda \right] B^T \left( k \lambda + p \phi \right) \]

\[ B = \frac{\partial \phi}{\partial q} \]

The iteration matrix is computed from the second variation and is given by:
\[
\delta dU^\dagger = \left[ \delta q^T \delta \lambda \right] \frac{pB^T B}{kB} \frac{d\lambda}{dq} + \delta q^T \frac{\partial^2 \phi}{\partial q^2} (k\lambda + p\phi) dq
\]

The last term depends on the Lagrange multiplier and gives the geometric stiffness.

4 STRAIN ENERGY

The relation between the current position in the basic frame and the displacement in the superelement frame is given for one boundary node \( B \) by:

\[
x_B = x_0 + R_0 \left( X_B + u_B \right) = x_0 + R_0 Y_B
\]

From this relation, we can compute the relative displacements in the superelement frame. We can also compute the relative rotations between the node and the superelement frame:

\[
u_B = R^T_B (x_B - x_0) - X_B
\]

\[
R_{rel,B} = R_0 R_B = R(\Psi_{rel})
\]

The strain energy is computed from the stiffness matrix \( K \) of the superelement:

\[
U = \frac{1}{2} \left[ \begin{array}{c}
u_{b1} \\
\Psi_{rel,B1}^T \\
u_{b2} \\
\Psi_{rel,B2}^T \\
\vdots
\end{array} \right]^T K \left[ \begin{array}{c}
\Psi \Psi_{rel,B1}
\Psi
\Psi_{rel,B2}
\vdots
\Psi
\end{array} \right] = \frac{1}{2} q_{rel}^T K q_{rel}
\]

The internal forces of the superelement will be computed from the first variation of the strain energy. In order to compute this first variation, we need to compute the variation of the relative displacements and rotations. After some easy computation, we find:

\[
\delta u_B = R_B^T (\delta x_B - \delta \Psi_B) + \bar{Y}_B \delta \theta_0
\]

\[
\delta \Psi_{rel,B} = \delta \Psi - R_{rel,B}^T \delta \theta_0
\]

\[
\delta \Psi_{rel,B} = T_{rel,B}^T (\delta \Psi_B - R_{rel,B}^T \delta \theta_0) = T_{rel,B}^T \left( R_{rel,B}^T T_B \delta \Psi_B - R_{rel,B}^T T_B \delta \Psi_0 \right)
\]

Or, in matrix form:

\[
\delta q_{rel} = B \delta q
\]

The internal forces are equal to:
The tangent stiffness matrix is computed from the derivative of the internal forces:

\[ F_{int} = B^T K_{rel} \]

The tangent stiffness matrix is computed from the derivative of the internal forces:

\[ K_T = B^T K_B + (d B^T) K_{rel} \]

the first part is classical, the second part is a geometric stiffness and will be developed later.

5 FRAME DEFINITION

A first choice is to take as reference the frame linked to one node of the superement, the first one for instance. In this case, the results change if the node to which the frame is attached change.

A second choice is to take a mean value of the rotation of the nodes. We need superelement with rotational dof. In this kind of formulation, it can be easily seen that the \( B \) matrix is nearly full as \( x_0 \) and \( \psi_0 \) depend on all the dof of the superelement. In order to compute the tangent stiffness, we have 2 matrix products between matrices, which have the size of the superelement. For a superelement with 2500 dof, the number of operation in order to compute the tangent stiffness is equal to \( 2 \times 2500^3 \).

Here, we keep as independent variables \( x_0 \) and \( \psi_0 \). We introduce kinematic constraints in order to link these variables to the unknowns at the boundary nodes.

If we look at the definition of the relative displacements and rotations, we see they depend on \( x_0 \) and \( \psi_0 \). If \( x_0 \) and \( \psi_0 \) are slightly different, it means we will add a small rigid body motion to the relative displacements and rotations. The idea is to minimize in the relative dof the part, which depends on the rigid body motion. We introduce a constraint in order to have the relative dof orthogonal to the rigid body motion (defined in the superelement frame). As scaling factor, we introduce the mass matrix. The stiffness matrix cannot be taken, as the product stiffness rigid body mode is equal to 0. So we impose:

\[ \phi \equiv q_{rig}^T M q_{rel} = 0 \]

The 6 rigid body modes in the superelement frame are given by:

\[ q_{rig} = \begin{bmatrix} 1 & -\tilde{X}_{31} \\ 0 & I \\ 1 & -\tilde{X}_{32} \\ 0 & I \\ \vdots & \vdots \end{bmatrix} \]

We use 6 Lagrange multipliers in order to take the constraints into account. The 6 constraints are written:
The RIG matrix is assumed constant during the analysis. This is consistent with the linear behavior of the superelement in its frame.

The internal forces linked to the constraints are classically computed. They are computed from:

$$\delta q^T RIG^T (k \lambda + px) + \delta \lambda^T k \phi = \begin{pmatrix} \delta q^T & \delta \lambda^T \end{pmatrix} \begin{pmatrix} B^T RIG^T (k \lambda + px) \end{pmatrix}$$

The iteration matrix linked to the constraints is computed from:

$$\begin{pmatrix} \delta q^T & \delta \lambda^T \end{pmatrix} \begin{pmatrix} pB^T RIG^T RIG B & kB^T RIG^T kRB \end{pmatrix} \begin{pmatrix} dB^T RIG^T (k \lambda + px) \\ dB^T RIG (k \lambda + px) \end{pmatrix}$$

The first part is classical and the second part is a geometric stiffness, which will be described later.

With this choice, the B matrix is nearly diagonal as \(x_0\) and \(\psi_0\) are independent variables. We take into account the 0 when we compute the product \(B^T K B\). In fact, we compute \(B^T (K + pRIG^T RIG) B\).

If the size of the superelement is large, the CPU for the element computation will be much smaller than the one where the frame is defined by mean values. By comparison to a linear superelement where the frame is kept constant, the CPU time is increased by a factor, which is proportional to the number of dof and not to the cube of dof. But, there are 12 more unknowns per superelement.

The frame can be computed even if there is no rotational dof in the superelement. It can be seen that \(x_0\) and \(\psi_0\) are the position and rotation of the center of gravity of the superelement.

### 6 GEOMETRIC STIFFNESS

The geometric stiffness is linked to:

$$\delta q^T (dB^T) (Kq_{rel} + RIG^T (k \lambda + px))$$

We write this relation in the following way:
So boundary node “i” is only coupled with himself and with node 0 linked to the frame of the superelement.

When we compute the geometric stiffness, we take the following assumption:

\[
dT = -\frac{1}{2} d\bar{\psi} \quad T = I
\]

So during the iteration, the iteration matrix is not tangent. At a converged point, as \( \psi \) is equal to 0, the matrix is a tangent one.

After some computation, we find the geometric stiffness linked to \( F_1 \):

\[
\left( \delta X_{b1}^T - \delta X_0^T \right) R_0 \left( -\bar{F}_1 \right) \bar{d}\psi_0 + \delta \psi_0^T \left( \bar{F}_1 \right) R_0 \left( dX_{b1} - dX_0 \right) + \delta \psi_0^T \left( \bar{F}_1 \bar{Y}_{b1} + \frac{1}{2} \left[ \bar{Y}_{b1} F_1 \right] \right) \bar{d}\psi_0
\]

and the geometric stiffness linked to \( M_1 \):

\[
\delta \psi_0^T \frac{1}{2} R_{rel,b1} \left( \bar{M}_1 \right) \bar{d}\psi_{b1} - \delta \psi_0^T \frac{1}{2} \left( \bar{M}_1 \right) R_{rel,b1} \bar{d}\psi_0
\]

Of course, the geometric stiffness is symmetrical, as it is the second derivative of the strain energy.

7 INERTIA FORCES

The kinetic energy of the superelement is taken equal to:

\[
T = \frac{1}{2} \left( \dot{\bar{x}}_{b1}^T R_0 \Omega_{b1} \dot{y}_{b1} + \Omega_{b1} \dot{y}_{b1}^T \right) \bar{M} \left( \dot{\bar{x}}_{b1} \ \Omega_{b1} \ \dot{y}_{b1} \right)
\]

where we use the rotational velocities defined in the material frame and not the one in the reference frame. So we take the choice, which, according to Cardona\(^3\), gives more accurate results.

The inertia forces and iteration matrix are computed from the Hamilton principle and are similar to the one of Cardona\(^3\). The only difference is on the way \( \Omega_0 \) and \( \delta \psi_0 \) are computed. In
one case, they depends on all the variables of the superelement, in the other case they are independent variables.

8  EXAMPLES

The formulation has been introduced in Samcef-Mecano\(^4\). The time integration scheme is a Hilber-Hugues-Taylor algorithm adapted for flexible mechanism\(^5\). In order to handle large rotations, the rotational vector is used.

8.1  Hinged beam

We take as first example the hinged beam used by Cardona\(^3\). The physical properties of the beam are: length 141.42, density \(7.8 \times 10^{-3}\), area 9.0, inertia 6.75, Young modulus \(2.1 \times 10^6\) and Poisson coefficient 0.3. The beam is hinged on one end, where a torque in function of time is applied (see Fig. 1)

![Fig. 1. Hinged beam](image)

The superelement is built from 5 beam elements and 4 internal vibration modes are included. As results, we look at the angular velocity at the base of the beam (Fig. 2). The results are closed to the one of Cardona\(^3\). The CPU time and the number of iterations are of the same order of magnitude.
8.2 Centrifugal effect

In the second example, the same beam is used. An imposed rotational velocity of 1 rad/sec is imposed at the hinge point and is constant in function of time. An additional mass of 12 is introduced at the free end. An imposed transversal force is imposed in function of time (Fig 3). As results, we show the transversal displacement obtained with the initial beam model, with Cardona’s superelement and with the present one. In order to see the influence of the geometric stiffness (due to the axial force created by the centrifugal effect), we also plot the displacement when the beam is not rotating.

We can see (Fig. 4) that the two superelements give a displacement a little bit larger than the beam model but are closed to each other. We can also observe that the geometric stiffness is well taken into account even if the superelement has a linear behavior in a frame, which is linked to it. The CPU time and the number of iterations needed by the two superelements are closed to each other.
8.3 Hard landing of an helicopter

The third example is an industrial example made for Eurocopter. It is a hard landing of a helicopter. In the model, there are seven superelements, the two largest have around 2500 dof. In this case, the CPU time needed for one iteration is very different if we use the present approach or the one described in 3. There is an order of magnitude of difference. Moreover, the convergence property is much better with the present approach on this problem.

Some results will be shown during the oral presentation.

9 CONCLUSION

The present way to define the moving frame gives results similar to the one of previous approach 3 on small models. The number of iterations and the CPU time are comparable. However, on a large model with irregular superelements, the convergence behavior is better and the CPU time is much lower.

Just recall, that even if the superelement has a linear behavior in the moving frame, some global geometric stiffness are taken into account as shown in the second example. This is true with this approach and the previous one 3.
10 REFERENCE