NON-LINEAR MODAL DYNAMIC ANALYSIS OF DOME STRUCTURE

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Abstract. The paper presents comparatively both linear and nonlinear modal analyses of the strength structure of a steel dome. The nonlinear modal analysis was preceded by a nonlinear static transient analysis. The result of the nonlinear transient analysis, the stiffness matrix of the structure containing the effects of the falling in the plastic domain, was introduced in the nonlinear modal analysis. The paper points out the way of determining of the eigenvalues and eigenvectors vibration characteristics of a dome structure with partial plastified elements.
1 INTRODUCTION

The dome is a special structure not only from architectural point of view but from both
designing and execution point of view.

The paper presents the nonlinear modal analysis of a steel dome with a 30 m opening and
a height of 8 m. The sections of the elements of strength are completely made of pipe.

As a reticular steel structure, the dome is characterized by a wide opening without
intermediate support.

2 CONSIDERATIONS REGARDING THE MODELING

The dome has been analyzed using a finite element method included in the computational
program COSMOS M/2.0, produced by SRAC.

The structure has been divided into 192 Beam 3D elements connected in 73 nodes.

3 CONSIDERATIONS REGARDING THE ANALYSIS

3.1 Linear modal analysis

The moving equation for an undamped system, expressed in matrix notation using the
above assumptions is:

\[ \begin{bmatrix} \mathbf{M} & \mathbf{K} \\ \mathbf{K} & \mathbf{M} \end{bmatrix} \begin{bmatrix} \mathbf{y} \\ \dot{\mathbf{y}} \end{bmatrix} + \begin{bmatrix} \mathbf{u} \\ \dot{\mathbf{u}} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix} \]

Figure 1 Mesh & force

3 CONSIDERATIONS REGARDING THE ANALYSIS

3.1 Linear modal analysis

The moving equation for an undamped system, expressed in matrix notation using the
above assumptions is:
\[
[M] \ddot{u} + [K] u = 0
\]  \hspace{1cm} (1)

where:
- \([M]\) - the structure mass matrix;
- \([K]\) - the structure stiffness matrix in elastic range.

For a linear system, free vibrations will be harmonic of the form:

\[
\{u\} = \{\phi\}_i \cos \omega_i t
\]  \hspace{1cm} (2)

where:
- \(\{\phi\}_i\) - eigenvector representing the mode shape of the \(i\)th natural frequency;
- \(\omega_i\) - \(i\)th natural circular frequency (radians per unit time);
- \(t\) - time.

Thus, equation (1) becomes:

\[
( -\omega^2 [M] +[K] ) \{\phi\}_i = \{0\}
\]  \hspace{1cm} (3)

This equation is satisfied if either \(\{Q\}_i = \{0\}\) or if the determinant of \(( -\omega^2 [M] +[K] )\) is zero. The first option is the trivial one and, therefore, is not of interest. Thus, the second one gives the solution:

\[
\left| [K] -\omega^2 [M] \right| = 0
\]  \hspace{1cm} (4)

This is an eigenvalue problem which may be solved for up to \(n\) values of \(\omega^2\) and \(n\) eigenvectors \(\{\phi\}_i\), which satisfy equation (3), where \(n\) is the number of DOF’s.

The eigenvalue and eigenvector problem needs to be solved for mode-frequency analyses.
It has the form of:

\[ [K] \{ \phi_i \} = \lambda_i [M] \{ \phi_i \} \]  

(5)

where:

\( \{ \phi_i \} \) - the eigenvector;
\( \lambda_i \) - the eigenvalue.

Applying this type of analysis it has been obtained the eigenvalues and eigenvectors of vibration of the structure. Due to the obtained first period of \( T_1 = 0.439 \) s, the dome is considered a rigid structure.

![Figure 3 Linear elastic mode shape 3; \( T_3=0.150 \) s](image)

3.2 Nonlinear static analysis

Using the determined eigenvectors we calculated the seismic load corresponding to a structure situated in an area of high seismic activity (8 degree on MSK).

The nonlinear static analysis was conducted considering the constant-gravitational load, increasing constantly the seismic load up to the values required by the standard. Attention was paid to comply with both resistance and stability conditions enforced by the international standards.

Increasing the seismic load level by 50\% over the standard value, the structure falls under plastic domain.

The constitutive low of steel Von Mises behavior and yield criterion with associated rule and kinematics hardening has been used throughout this analysis\(^2\).
The equivalent stress is therefore:

\[ \sigma_e = \left[ \frac{3}{2} (\{s\} - \{a\})^T [M](\{s\} - \{a\}) \right]^{1/2} \] (6)

where:
\[ \{s\} - \text{the deviatoric stress vector}; \]
\[ \{s\} - \{\sigma\} - \sigma_m [1 \ 1 \ 1 \ 0 \ 0 \ 0]^T \] (7)

where:
\[ \sigma_m = \frac{1}{3} (\sigma_x + \sigma_y + \sigma_z) \] (8)
\[ \sigma_m - \text{the mean or hydrostatic stress}; \]
\[ \{a\} - \text{the yield surface translation vector}. \]

Note that since the equation (6) is dependent on the deviatoric stress, yielding is independent of the hydrostatic stress state. When \( \sigma_e \) is equal to the uniaxial yield stress, \( \sigma_y \), the material is assumed to yield. The yield criterion is therefore:

\[ F = \left[ \frac{3}{2} (\{s\} - \{a\})^T [M](\{s\} - \{a\}) \right]^{1/2} - \sigma_y = 0 \] (9)

The associated flow rule yields:

\[ \left\{ \frac{\partial Q}{\partial \sigma} \right\} = \left\{ \frac{\partial F}{\partial \sigma} \right\} = \frac{3}{2 \sigma_e} (\{s\} - \{a\}), \] (10)

so the increment in plastic strain is normal to the yield surface. The associated flow rule with the von Mises yield criterion is known as the Prandtl – Reuss flow equation.

The yield surface translation is defined as:

\[ \{a\} = 2G \{\varepsilon_{sh}\} \] (11)

where:
\[ G - \text{the shear modulus}; \]
\[ E - \text{the Young’s modulus}. \]

The shift strain is computed analogously to equation (12)

\[ \{\varepsilon_{sh}^{n}\} = \{\varepsilon_{sh}^{n-1}\} + \{\Delta \varepsilon^{sh}\} \] (12)
where:
\[ \{ \Delta \varepsilon^{sh} \} = \frac{C}{2G} \{ \Delta \varepsilon^{sh} \} \]  \hspace{1cm} (13)
\[ C = \frac{2}{3} \frac{EE_T}{E - E_T} \]  \hspace{1cm} (14)
where:
\( E_T \) - the tangent modulus from the bilinear uniaxial stress-strain curve.
The yield surface translation \( \{ \varepsilon^{sh} \} \) is initially zero and changes with subsequent plastic straining.
The equivalent plastic strain is dependent on the loading history and defined to be:
\[ \hat{\varepsilon}^{pl}_n = \hat{\varepsilon}^{pl}_{n-1} + \Delta \hat{\varepsilon}^{pl}_n \]  \hspace{1cm} (15)
where:
\( \hat{\varepsilon}^{pl}_n \) - the equivalent plastic strain for this time point;
\( \hat{\varepsilon}^{pl}_{n-1} \) - the equivalent plastic strain from the previous time point.
The equivalent stress parameter is defined to be:
\[ \hat{\sigma}^{pl}_n = \sigma_y + \frac{EE_T}{E - E_T} \hat{\varepsilon}^{pl}_n \]  \hspace{1cm} (16)
where:
\( \hat{\sigma}^{pl}_n \) - the equivalent stress parameter.
Note that if there is no plastic strain (\( \hat{\varepsilon}^{pl} = 0 \)) then, \( \hat{\sigma}^{pl}_n \) is equal to the yield stress.
\( \hat{\sigma}^{pl}_n \) only has meaning during the initial, monotonically increasing portion of the load history.
If the load were to be reversed after plastic loading, the stress and therefore \( \sigma_e \) would fall below yield but \( \hat{\sigma}^{pl}_n \) would register above (since \( \hat{\varepsilon}^{pl}_n \) is non-zero).
The equation system has been solved using an iteration procedure: the Newton – Raphson method.
The finite element discretization process yields a set of simultaneous equations:
\[ [K] \{ u \} = \{ F^a \} \]  \hspace{1cm} (17)
where:
\([K]\) - the structure stiffness matrix in plastic range;
\( \{ u \} \) - vector of unknown DOF (degree of freedom) values;
\( \{ F^a \} \) - vector of applied loads.
If the structure stiffness in plastic range \([K]\) is itself a function of the unknown DOF
values (or their derivatives) then equation (17) is a nonlinear equation. The Newton - Raphson method is an iterative process of solving the nonlinear equations and can be written as:

$$[K^T_i]{\Delta u_i} = \{ F_i^r \} - \{ F_i^m \}$$  \hspace{1cm} (18)

$$\{u_{i+1}\} = \{u_i\} + \{\Delta u_i\}$$ \hspace{1cm} (19)

where: $$[K^T_i]$$ - the Jacobian matrix (tangent matrix);

$$i$$ - subscript representing the current equilibrium iteration;

$$\{F_i^r\}$$ - vector of restoring loads corresponding to the element internal loads.

Both $$[K^T_i]$$ and $$\{F_i^m\}$$ are evaluated based on the values given by $$\{u_i\}$$. The right-hand side of equation (18) is the residual or out-of-balance load vector; i.e., the amount the system is out of equilibrium. Single solution iteration is depicted graphically in fig. 4 for a one DOF model. In a transient analysis, $$[K^T_i]$$ is the effective matrix and $$\{F_i^m\}$$ is the effective applied load vector, which includes the inertia and damping effects.

As seen in the following figures nr. 4 and 5, more than one Newton – Raphson iteration is needed to obtain a converged solution. The general algorithm proceeds as follows:

1. Assume $$\{u_0\}$$. $$\{u_0\}$$ is usually the converged solution from the previous time step. On the first time step, $$\{u_0\} = \{0\}$$.

2. Compute the updated tangent matrix $$[K^T_i]$$ and the restoring load $$\{F_i^m\}$$ from configuration $$\{u_i\}$$.

3. Calculate $$\{\Delta u_i\}$$ from equation (10).

4. Add $$\{\Delta u_i\}$$ to $$\{u_i\}$$ in order to obtain the next approximation $$\{u_{i+1}\}$$ (eq. (19)).

5. Repeat steps 2 to 4 until convergence is obtained.

The solution obtained at the end of the iteration process would correspond to load level
The final converged solution would be in equilibrium, such that the restoring load vector \( \{F_{nr}^i\} \) (computed from the current stress state) would equal the applied load vector \([F_a]\) (or at least to within some tolerance). None of the intermediate solutions would be in equilibrium.

If the analysis included path-dependent nonlinearities (such as plasticity), then the solution process requires that some intermediate steps be in equilibrium in order to correctly follow the load path. This is accomplished effectively by specifying a step-by-step incremental analysis; i.e., the final load vector \([F^a]\) is reached by applying the load in increments and performing the Newton - Raphson iterations at each step:

\[
[K_{n,i}^{ij}](\Delta u_i) = \{F_a^n\} - \{F_{nr,i}^n\}
\]

where:
- \([K_{n,i}^{ij}]\) - the tangent matrix for time step \(n\), iteration \(i\);
- \([F_a^n]\) - the total applied force vector at time step \(n\);
- \([F_{nr,i}^n]\) - the restoring force vector for time step \(n\), iteration \(i\).

The Newton - Raphson procedure guarantees convergence if and only if the solution at any iteration \(\{u_i\}\) is “near” the exact solution.

The nonlinear model analysis was conducted in order to determine the vibration period of the structure in an advanced state of plastification.

Applying the nonlinear static analysis, the stiffness matrix of structure has been obtained, a matrix, which includes also effects due to the presence of the structure in the plastic domain. This stiffness matrix has been afterwards inserted in the dynamic moving equilibrium...
equations. Thus it has resulted the eigenvalues and eigenvectors of vibration due to both nonlinear physical constitutive behavior and nonlinear geometry of structure of dome.

The nonlinear first period of the structure is $T_1 = 0.96\, \text{s}$, with $217\%$ greater than the linear elastic first period.

![Figure 6 Nonlinear mode shape 1](image)

For the same level of stress the influence of geometrical nonlinear analysis is low, approximate $2\%$, because the dome behaves as a rigid structure (with important vertical loads in comparison with horizontal loads).

4 CONCLUSIONS

The first period of vibrations of structure increase in the nonlinear plastic range, in comparison with the elastic period. In most of cases, the dome is covered by curtain walls. In order to design is necessary to know the vibration frequency of the carrier structure. Designing glasses in order not to have accidents (breaking and falling from great height) due to the falling of the structure under plastic domain (the structure has not its collapse yet) is a sensitive problem, which has not been sufficiently studied yet.

REFERENCES

