

## LIMIT ANALYSIS BASED UPSCALING OF STRENGTH PROPERTIES AND INVESTIGATION OF FAILURE MODES OF TWO-PHASE MATERIALS

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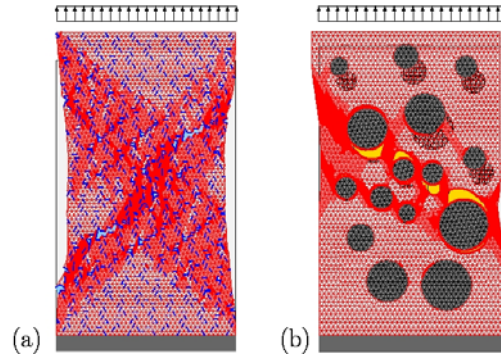
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### ABSTRACT

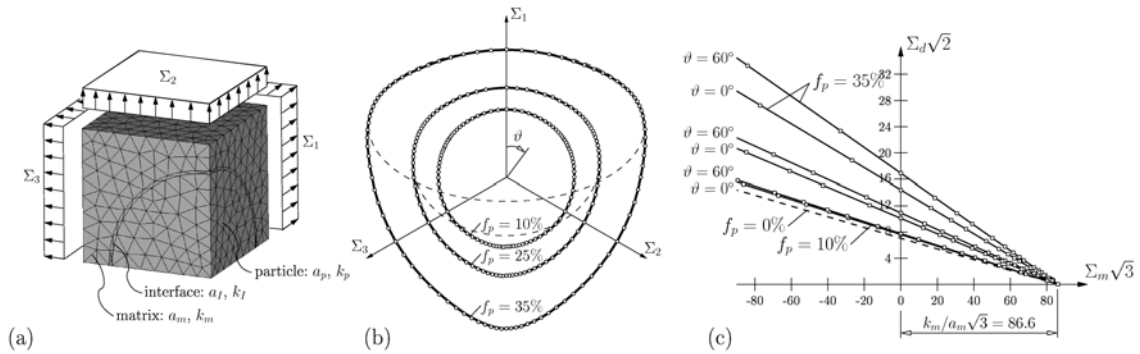
In the most general case, composites composed of two materials, exhibiting a matrix-particle morphology, can be described by three phases: the matrix phase, the particle phase, and the interface between the particle and the matrix. In order to relate effective material properties to the matrix-particle behaviour and the morphology, analytical and/or numerical schemes may be employed. As regards effective strength properties, averaging schemes as e.g. used for upscaling of elastic and viscoelastic properties are not able to capture the localized mode of material failure and do not provide information about the failure modes within the material.

In this work, the application of numerical limit analysis formulations [1,2] to upscaling of strength properties is proposed, giving access to the respective material strength and the corresponding failure modes [3]. Hereby, special boundary conditions allow the evolution of distinct failure modes while periodicity is retained. The solution of the underlying optimization problem arising from the limit analysis formulation is based on second-order-cone-programming (SOCP).

Deterioration processes resulting in the formation of microcracks and debonding between particles and the matrix can be taken into account by decreasing/deactivating the strength properties at discontinuities (see Figure 1(a)) and interfaces (see Figure 1(b)), respectively. Moreover, by varying the loading situation on 3D volume elements (see Figure 2(a)), microstructure-based effective failure surfaces are obtained for different material morphologies and different states of degradation of the material system (see Figure 2(b) and (c)). The evolution of these failure surfaces is controlled by internal variables with a clear physical meaning, such as the crack density in case of microcracking and the area of the weakened interface zone in case of interface debonding.



**Figure 1:** Failure modes obtained from upper-bound limit-analysis formulation considering (a) microcracks and (b) degraded interfaces between the particles and the matrix



**Figure 2:** (a) Discretization of the considered RVE for the application of the upper-bound theorem. Upper bound of effective failure surface in (b) deviatoric plane at hydrostatic pressure  $\Sigma_m = 0$  and (c) in the plane along the hydrostatic axis for different Lode angle  $\vartheta$  for different particle content  $f_p$  (strength parameter of matrix:  $a_m = 0.1$ ,  $k_m = 5$ , and ‘rigid’ particle)

## REFERENCES

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