8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

Influence of Contact Force Models on the Global and Local Dynamics of Drifting Impact Oscillator

Olusegun Ajibose[1], * Marian Wiercigroch[2], Ekaterina Pavlovskaia[3] and Alfred Akisanya[4]

Centre for Applied Dynamic Research, School of Engineering,
Kings College, University of Aberdeen, Aberdeen
AB24 3UE, UK
[1]o.k.ajibose@abdn.ac.uk, [2]m.wiercigroch@abdn.ac.uk, [3]e.pavlovskaia@abdn.ac.uk, [4]a.r.akisanya@abdn.ac.uk

Key Words: *Nonlinear dynamics, Impact Oscillator, Contact Mechanics.*

ABSTRACT

The effect of the contact forces on the global and local dynamics of the drifting impact oscillators introduced in [1] is examined in this paper. The oscillator has been extensively studied in our earlier investigations. In particular, the drift was seperated from the bounded dynamics [2] and the five dimensional flow was reduced to one dimensional iterative map [3].

The current work builds upon previous studies by Pu̇st and Peterka [4] and Muthukumar *et al* [5]. In [4] the nonlinearity of restoring forces between solid bodies is modelled as function of the deformation and velocity for non-drifting impact oscillators where the free and forced vibration of systems with Hertz contact were considered, while [5] uses a Hertz contact force model, which incorporates non-linear hysteresis damping, to simulate pounding, a phenomenon which occurs during the collision of building structures in earthquakes.

Three models are considered in the current study, namely the Kelvin-Voigt (KV), the Hertz stiffness (HS) and nonlinear contact stiffness and damping (NSD) models. The Kelvin-Voigt model was studied extensively in our previous work (e.g. [1–3]) and is a reference for the current two models. In the HS model, the contact force is a sum of spring force obeying the Hertz's law and a linear damping force. The NSD model presents the contact forces as a combined effect of Hertz's spring and a nonlinear hysteresis damping element. The equations of motion of the different models are written concisely in terms of dimensionless displacements and time:

$$x' = y, \tag{1}$$
$$y' = a\cos(\omega\tau+\varphi) + b - P_1P_2(1-P_3)L_1(z,v,z') - P_1P_3, \tag{2}$$
$$z' = P_1 y - (1-P_1)L_2(z,v)/2\xi, \tag{3}$$
$$v' = P_1P_3P_4(L_3(z,v)/2\xi + y). \tag{4}$$

where x, z and v are the displacement of the mass, slider top and bottom respectively. y is the mass velocity, g the initial gap between the mass and top slider, a is the amplitude of the dynamic force, b is the static force and $P_1,\ P_2,\ P_3$ and P_4 are Heaviside functions defined as:

$$P_1 = H(x-z-g), \qquad P_2 = H(L_2(z,v)), \qquad P_3 = H(L_2(z,v)-1), \qquad P_4 = H(v').$$

	KV	HS	NSD
$L_1(z,v,z')$	$2\xi z' + z - v$	$2\xi z' + (z-v)^{3/2}$	$(2\xi z' + 1)(z-v)^{3/2}$
$L_2(z,v)$	$z - v$	$(z-v)^{3/2}$	1
$L_3(z,v)$	$z - v - 1$	$(z-v)^{3/2} - 1$	$1 - (z-v)^{-3/2}$

Table 1: Definition of functions $L_1,\ L_2$ and L_3.

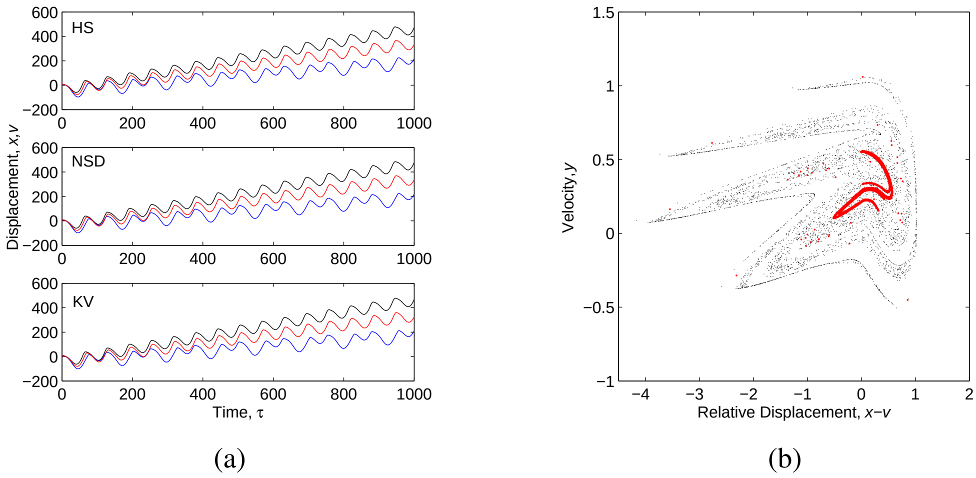

Figure 1: (a) Time histories computed for $a = 0.3, \xi = 0.05, \omega = 0.1, g = 0.02$ and $b_1 = 0.1$ (blue), $b_2 = 0.125$ (red), and $b_3 = 0.15$ (black). (b) Poincaré sections constructed for $a = 0.3, \xi = 0.1, \omega = 1.4, g = 0.02$ and $b = 0.1$ for HS (red) and NSD (black) models.

Functions $L_1(z,v,z'),\ L_2(z,v)$, and $L_3(z,v)$ are defined in Table 1 for each contact force model.

The nonlinear dynamics analysis reveals very different local and global behaviour. Fig. 1(a) shows the time histories for three different values of static force b. It is evident that the simplest, i.e. KV model adequately predicts the short-term (local) dynamics as all the three sets of dynamic responses are almost indentical. The long-term behaviour is however dependent on the type of model as shown in Fig. 1(b) where the Poincaré maps for the HS and NSD model are compared. We note that the attractor for the KV model is topologically the similar to that of the HS model.

REFERENCES

[1] E. Pavlovskaia , M. Wiercigroch and C. Grebogi. "Modeling of an impact system with a drift." *Phys. Rev. E*, Vol. **64**, 0562244, 2001.

[2] E. Pavlovskaia and M. Wiercigroch. "Analytical drift reconstruction for visco-elastic impact oscillators operating in periodic and chaotic regimes." *Chaos Solitons & Fractals*, Vol. **19**, 151–161, 2004.

[3] E. Pavlovskaia and M. Wiercigroch. "Low dimensional maps for piecewise smooth oscillators ." *Journal of Sound and Vibration*, Vol. **305**, 750–771, 2007.

[4] L. Půst and F. Peterka. "Impact oscillator with hertz's model of contact." *Meccanica*, Vol. **38**, 99–114 , 2003.

[5] S. Muthukumar and R. DesRoches. "A Hertz contact model with nonlinear damping for pounding simulation." *Earthquate. Engng Struct. Dyn.*, Vol. **35**, 811-828, 2006.