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A weakly over-penalized symmetric interior penalty method

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ABSTRACT

Let $\Omega \subset \mathbb{R}^2$ be a bounded polygonal domain, $f \in L_2(\Omega)$ and $\varphi \in H^2(\Omega)$. Consider the model problem of finding $u \in H^1(\Omega)$ such that

$$-\Delta u = f \qquad \text{in } \Omega, \tag{1a}$$
$$u = \varphi \qquad \text{on } \partial\Omega. \tag{1b}$$

In this talk we discuss a weakly over-penalized symmetric interior penalty (WOPSIP) method for (1) introduced in [1,2]. Let $\mathcal{T}_h$ be a simplicial triangulation of Ω and V_h be the discontinuous P_1 finite element space associated with $\mathcal{T}_h$, i.e.,

$$V_h = \{v \in L_2(\Omega) : \, v_T = v\big|_T \in P_1(T) \quad \forall\, T \in \mathcal{T}_h\}.$$

The WOPSIP method is to find $u_h \in V_h$ such that

$$a_h(u_h, v_h) = \int_\Omega f v \, dx + \eta \sum_{e \in \mathcal{E}_h^b} \frac{1}{|e|^3} \int_e \Pi_e^0 \, [\![\varphi]\!] \cdot \Pi_e^0 \, [\![v]\!] \; ds \qquad \forall\, v_h \in V_h, \tag{2}$$

where

$$a_h(w, v) = \sum_{T \in \mathcal{T}_h} \int_T \nabla w \cdot \nabla v \, dx + \eta \sum_{e \in \mathcal{E}_h} \frac{1}{|e|^3} \int_e \Pi_e^0 \, [\![w]\!] \cdot \Pi_e^0 \, [\![v]\!] \; ds,$$

$\mathcal{E}_h$ (respectively $\mathcal{E}_h^b$) is the set of edges (respectively boundary edges) of $\mathcal{T}_h$, Π_e^0 is the orthogonal projection from $[L_2(e)]^2$ onto $[P_0(e)]^2$ (the space of constant vectors on e), and $\eta > 0$ is a penalty parameter.

The WOPSIP method satisfies, for any choice of η, quasi-optimal *a priori* error estimates in both the L_2 norm and the energy norm $\|\cdot\|_h$ defined by

$$\|v\|_h^2 = \sum_{T\in\mathcal{T}_h} \|\nabla v\|_{L_2(T)}^2 + \sum_{e\in\mathcal{E}_h} |e| \| \{\{\nabla v\}\} \|_{L_2(e)}^2 + \eta \sum_{e\in\mathcal{E}_h} |e|^{-3} \|\Pi_e^0 [[v]] \|_{L_2(e)}^2,$$

where $\{\{\nabla v\}\}$ and $[[v]]$ denote the mean of the gradient of v and the jump of v across the edge e respectively. The WOPSIP method is also very easy to program and can handle meshes with hanging nodes. Therefore it has various advantages over other interior penalty methods [3,4,5].

We will sketch the *a priori* and *a posteriori* error analysis of the WOPSIP method and also discuss multigrid and adaptive algorithms for (2). Details can be found in [1,2,6,7].

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