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Hardening plasticity for geomaterials: micromechanical roots and alternative formulations

***Itai Einav[1], Giang Nguyen[2]**

[1] School of Civil Engineering
University of Sydney
i.einav@civil.usyd.edu.au

[2] School of Civil Engineering
University of Sydney
G.Nguyen@civil.usyd.edu.au

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ABSTRACT

Hardening plasticity has a long history in constitutive modelling, with many applications in geomechanics. Mathematical models of non-hardening plasticity involve the definition of a single stationary yield surface in stress-space. Examples include the conventional models of elasto-plasticity, often based on the yield criteria of Tresca, von Mises, and Mohr-Coulomb. Within the yield surface the constitutive behaviour is kept elastic, while the onset of yielding manifests inelastic deformations, accompanied by energy dissipation. These models are mathematically convenient, but geomaterials present a much more complicated behaviour. Stress-strain investigations using elementary material tests reveal the following trends: (1) in cyclic shear tests hysteresis behaviour is evident, and (2) the yield threshold is ever increasing under continuously increasing isotropic pressure. To *implicitly* cover these aspects, traditional constitutive modelling of geomaterials involves, respectively: (1) multi-surface kinematic hardening, and (2) isotropic hardening.

The purpose of this presentation is to motivate simple micromechanically driven explanations to the roots of these phenomena, and to question whether the traditional modelling concepts of plasticity are adequate. We will present alternative modelling concepts that *explicitly* adopt the micromechanical roots of the hardening phenomena. The alternative formulations are equally simple, can describe the same phenomenological aspects *explicitly* rather than *implicitly*, and provide additional information that relates to meaningful microscopically-based variables and physical parameters. Cutting-edge research involves refined microscopical understanding, which can assists to the ever-demanding industrial applications. For these reasons, the alternative formulations look advantageous. Our treatment of the hardening phenomena is based on two different formulations, but both of which are founded on micro-, thermo-, and statistical- mechanics principles. The first formulation describes general elasto-plastic random material. The second formulation is called Breakage Mechanics, enabling to model confined comminution (the process of grain size reduction) within the framework of continuum mechanics. In this case, quite related to the first formulation, the statistics is mainly related to the evolving grain size distribution (gsd) in brittle granular material.

Multi-surface kinematic hardening: A typical starting point in multi-surface kinematic hardening formulations is the well-known Iwan-Mroz model. As a 1D element model the stress-strain behaviour is simulated by a mechanical system of Hookean spring element placed in a series of sliding elements, each one placed in parallel to an additional spring. Upon extensions to 3D, the series of sliders are represented by a nested series of kinematically moving yield surfaces in stress space. An alternative 1D element model defines the mechanical system of Hookean spring that is placed parallel to a system of Hookean springs, each placed in a series with a sliding element. Although this is a quite popular model in its 1D form (the so-called Masing-Iwan model), the extension to 3D was made available only recently[1] and revealed an equally simple formulation. This time the sliders are replaced by a nested series of stationary yield surfaces, each of which bounds a single 'micro' stress, so that the overall 'macro' behaviour leads to hysteresis behaviour since the macro-stress is the average of the micro-stresses. We therefore see that 'kinematic hardening' is not necessarily related to kinematically translating yield surfaces. Furthermore, the internal variables and parameters posses a clearer physical meaning, related to the statistics of yielding within a representative volume element[2].

Isotropic hardening: Traditional Critical State Soil Mechanics (cssm) approach uses the experimentally-based normal compression curve to drive the modelled hardening process. This way of modelling neglects the underlying microscopical processes which in fact govern the hardening behaviour of the material, and most importantly differ between different geomaterials. As a consequence, ad hoc parameters which are difficult to experimentally determine or assign physical meanings must be used in the constitutive modelling to capture some observed features of the macroscopic hardening processes. For breakable granular materials, the shifting of gsd due to comminution is the dominant governing mechanism of the isotropic hardening phenomenon observed on macroscopic scale. The use of statistical homogenization in a thermodynamically-based Breakage Mechanics approach[3,4] helps to relate the evolving gsd to the initial and ultimate ones using the breakage measure *B*, as a thermo-mechanical internal variable for the underlying grain breaking process: $p(d) = p_0(d)(1-B) + p_u(d)B$. In a thermodynamic context the thermodynamic force associated with *B*, termed the breakage energy E_B, is related to the energy release rate due to grain breaking. Analogy with fracture mechanics can readily be shown. In addition, the fact that E_B appears in the yield function and therefore drives the isotropic hardening process allows to capture this process in a natural way, without having to introduce any ad hoc parameters.

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