CFL conditions for Runge-Kutta and multi-step discontinuous Galerkin methods

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ABSTRACT

In this talk we discuss CFL conditions for the stability of Runge-Kutta (RK) and multi-step (MS) discontinuous Galerkin (DG) methods for time-dependent hyperbolic equations. A family of explicit strong-stability-preserving (SSP) RK and MS methods are analyzed for the time discretization of the semidiscrete DG equations. Among our findings, we demonstrate that improvements in CFL conditions for explicit RK and MS DG methods can be obtained by using SSP methods with a greater number of stages/steps than theoretically required for a given order. These improvements in CFL conditions are, in general, great enough to offset the additional computation and/or storage requirements introduced by the use of an increased number of stages or steps. We also derive CFL conditions for the stability of DG methods in two-dimensions on structured triangular meshes, and we show how these two-dimensional CFL conditions can be (approximately) related to the one-dimensional CFL conditions through an appropriately defined mesh parameter h and a constant that is dependent on the polynomial degree p of the DG spatial discretization. Parallel implementation of the methods for the solution of the two-dimensional shallow water equations will also be discussed and some numerical results will be presented.