8th. World Congress on Computational Mechanics (WCCM8)
5th. European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 – July 5, 2008
Venice, Italy

ADAPTIVE hp-FEM FOR THE STABILIZED NAVIER-STOKES EQUATIONS

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Key Words: *hp-FEM, incompressible flow, Least Square FEM, hierarchical FEM.*

ABSTRACT

In this work a new semi-implicit stable and stabilized hp-FEM for the solution of incompressible Navier-Stokes (NS) equations is introduced. The explicit procedures used by Karniadakis and Sherwin (2004) has time limitation of h/up^2 where h is the grid size, u is the velocity and p is the order of the polynomial. To use longer time step, an implicit stabilizing procedure suggested by de Sampio et al (1993) based on Galerkin Least Square (GLS) is proposed. The stabilizing parameter used in this work is the time step Δt used for integration. Previously Gervasio and Saleri (1998) used two stabilizing parameters for unsteady NS equations and one of them is similar to h/up^2. Gerdes and Schotzau (1999) suggested $0.1h^2/p^4$ as the parameter for solving Stokes problem. These stabilizing parameters provide stability and at the same time they all introduce high numerical diffusion in hp-FEM. Further research is needed on the stabilizing parameters in hp-FEM. In this work the performance of the new procedure is evaluated by studying the flow over circular cylinder at Re=200.

The Labotto and Legendre polynomials are used as modal shape functions in a quadrilateral element. The Euler Backward method is used to approximate in time and equal order interpolation is used. Since the procedure is based on GLS the matrices are symmetric and hence preconditioned conjugate gradient solution techniques are used to solve the equations. Of the three equations to be solved at each time, the pressure equation took more computer time and hence the matrices formed using Legendre polynomials are very useful. Once an initial grid is selected, further refinements are made by increasing the order of the polynomial p. The error is estimated at all Gauss points by comparing the higher order to lower order as discussed in Selvam and Qu (2002).

RESULTS: An initial grid of 20x32 (NN=664 & NE=640) is used around the cylinder with a smallest grid spacing of 0.008D where D is the diameter. The order of the polynomial p to start with 2 or 3 and then it is increased by adaptive procedure up to 6. Refinements are done when the maximum error in vorticity is beyond 10%. The error is estimated at every 5 non-dimensional time. The computed Strouhal number ($St = D/(TU_\infty)$) of 0.191 (4% error) for Re=200 is in good comparison with 0.199 from experimental measurement. When the time step Δt is varied from minimum grid CFL= 0.9, 0.9/2 and 0.9/4, the St varied to 0.179, 0.182, and 0.191 respectively. Thus longer

time step introduces higher numerical diffusion. The order of the polynomial, error in the domain and the computed vorticity at the end of 90 time units are plotted in Fig. 1 and Fig. 2.

CONCLUSIONS: An efficient adaptive hp-FEM procedure which is based on GLS technique is introduced. In this procedure the stabilization parameter considered is Δt. In Selvam and Qu (2002) unequal order interpolation with fractional step method is used to solve NS equations and this leads to 5 equations (u*, v*, p, u and v) to be solved at each time step. Here only 3 equations (u, v and p) need to be solved. The implicit procedure can use Δt in the range of grid CFL number (h/u) rather than h/up^2 and the computed results are in good comparison with experimental results. Further work is underway to investigate the effect of Crank-Nicolson and other improved time approximations as well much stringent tolerance in error reduction.

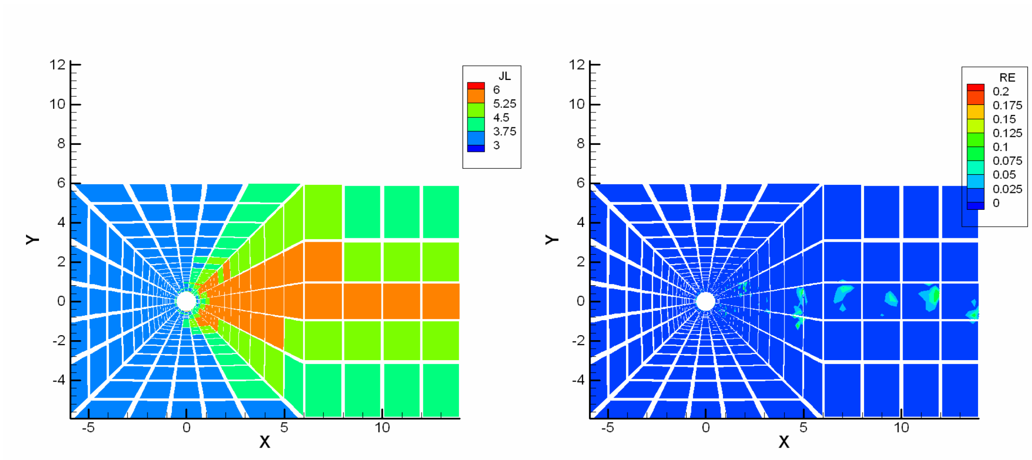

Figure 1. Contours of the polynomial order p in the computational domain due to adaptive procedure and the corresponding error

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