

ENERGY-RESIDUAL-BASED APPROACH TO GRADIENT PLASTICITY

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ABSTRACT

The “energy-residual-based approach” mentioned in the title consists in a thermodynamically consistent procedure for the formulation of a phenomenological plasticity model of either strain gradient, or nonlocal (integral) type. The authors have developed this procedure on the last ten years, see refs. [1] to [9]. It seems therefore appropriate to present an update of this theory at this forum. For brevity we shall limit ourselves to strain gradient plasticity.

The classical plasticity model taken on as a reference counterpart is the well-known generalized standard elastic-plastic material of Halphen and Nguyen (1975). This is endowed with internal variables, say (ξ, χ) , a hardening potential $\Psi(\xi)$, and admits a *finite* (i.e. *nondifferential*) hardening law as $\dot{\xi} = \mathbf{H}(\xi) \cdot \dot{\xi}$, where $\mathbf{H} = \partial^2 \Psi / \partial \xi^2$ is the hardening moduli tensor. In contrast, for a strain gradient plasticity model, while the flow rule remains formally unchanged, the hardening law changes into one shaped as a PDE system, that is:

$$\dot{\chi} = \mathcal{L} \dot{\xi}, \quad \text{in } V, \quad \mathcal{B} \dot{\xi} = \mathbf{0} \quad \text{on } B = \partial V, \quad (1)$$

where \mathcal{L} is some differential operator and \mathcal{B} some relevant boundary operator. The form taken on by these operators essentially depends on which kind of ξ gradients are chosen in order to capture size effects. If the gradient of ξ up to the n -th order, $n \geq 1$, are used for this purpose, eq.(1) proves to be a $2n$ -th order PDE system with n (higher order) boundary conditions upon $\dot{\xi}$ and its gradients up to the $(n - 1)$ -th order, either kinematic-type, or static type.

In general, however, the plastic deformation mechanism is active only in a subregion $V_p \in V$, hence the boundary conditions of eq.(1) hold only for the external boundary of V_p , say $B_{p(ext)}$, where for the moving elastic/plastic boundary, say $B_{p(int)}$, due to the C^n -continuity of ξ across the latter boundary the following $n + 1$ boundary conditions hold:

$$\dot{\xi} = \mathbf{0}, \quad \nabla \dot{\xi} = \mathbf{0}, \quad \dots, \quad \nabla^{(n)} \dot{\xi} = \mathbf{0}, \quad \text{on } B_{p(int)}. \quad (2)$$

The extra boundary condition in eq.(2) with respect to $(1)_2$ is required to determine the instantaneous location of $B_{p(int)}$. A central issue is how to obtain eqs.(1) and (2) in a manner consistent with the thermodynamics principles. The energy-residual-based gradient plasticity theory mentioned above provides simple and effective means to do that. Indeed, this theory is builded upon a thermodynamic framework characterized as in the following:

1. The thermodynamics principles of the local action does not hold since the material particles interact not only by contact stresses and heat condition, but also by long distance energy interactions. These are described by a scalar variable (the energy residual), say P , giving the power density transmitted to the particle by all other particles in V . Since the intere collection of nonsimple material particles is *globally simple* (i.e. constitutively insulated), then the (global) *insulation condition* is satisfied, i.e. the integral of P over V is zero. In addition, since in the absence of strain gradients the material behaves as a simple one, P must vanish correspondingly (locality recovery condition).
2. The energy balance of the first thermodynamics principle is affected by the energy residual P , which in fact adds to the strain power, say $\sigma : \dot{\epsilon}$. In fact standard strain power is to be replaced by $\sigma : \dot{\epsilon} + P$ in the energy balance equation.
3. The second thermodynamics principle (the entropy production density is nonnegative for every material particle) and the Onsager reciprocity principle (the plastic dissipation power is a bilinear form in terms of independent fluxes and related affinities) are accepted in their classical forms.

With the aid of this theoretical framework, a standard procedure of constitutive equation theory enables us to derive all the pertinent restrictions upon the constitutive equations. These include, besides the elasticity law, the constitutive expressions of the dissipation density and of the residual. as well as the hardening law in the shape of PDE system with related higher order boundary conditions, including those on the moving elastic/plastic boundary

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