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BIFURCATION, LOCALIZATION AND ILL-POSEDNESS FOR THE TIME DISCRETIZED BOUNDARY-VALUE PROBLEM

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ABSTRACT

The issues of bifurcation, localization and ill-posedness for *rate-dependent* solids are investigated here for the finite step problem obtained by discretization in time of the associated initial-value problem. Bifurcation is investigated in the framework of Hill's general theory while localization and ill-posedness are related to the loss of ellipticity of the discrete field equations.

Critical time steps may exist beyond which these phenomena may occur. For a sufficiently small time step however, they are ruled out and uniqueness is guaranteed in general although situations may exist where this time step is too much a small. These phenomena occur in general for implicit algorithms and in presence of strain softening, strain-rate softening and/or non-associative flow. We furnish a method to compute the critical values of the time step.

The results are related both to the continuum and incremental problems associated to the underlying rate-independent problem. It is indeed shown that the finite step problem, parameterized by the time step, resembles the incremental problem for a rate-independent material and softening or negative strain-rate sensitivity for instance translate to softening in the discretization process lead to loss of ellipticity, ill-posedness and associated pathological numerical consequences when the time step is greater than the critical value.

They are also related to the stability properties of the continuous initial boundary value problem.

Finally, the results are also put in the light of those obtained for coupled problems such as for elastic-plastic porous saturated media and or for thermo-mechanical processes. In the two situations, a diffusion equation (Darcy's law for fluid flow in the former and Fourier's law for heat condition on the latter) enters the formulation and introduces some rate effects. However if one considers rate-independent behaviour for the skeleton and for the material under thermo-mechanical couplings, it is found that these diffusion phenomena do not rule ill-posedness observed for the rate-independent behaviour when considered in its own. The difference with the situation considered here is that no critical time step is found to insure well-posedness. In fact the time integration algorithm is found to be not consistent. Consideration of rate effects for the skeleton

improves indeed the situation provided the time step restriction discussed above is taken into account.

All the results are illustrated for a simple associative elastic-viscoplastic J_2 flow theory and applied later to elastic-viscoplastic materials exhibiting dynamic strain ageing and Portevin-Le Châtelier effect.