8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

ANALYSIS OF BLOCK GAUSS-SEIDEL METHODS WITH REFERENCE TO FLUID-STRUCTURE INTERACTION IN BIOMEDICAL APPLICATIONS

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Key Words: *Fluid-Structure Interaction, Block Gauß-Seidel Method, Biomedical Engineering.*

ABSTRACT

A fluid-structure interaction problem (FSI) features a fluid domain and a structural domain, which are separated by an interface. An efficient strategy for the computer simulation of FSI should allow for the employment of different numerical discretization techniques with non-matching grids or meshes for the fluid and solid domains. In any case, the resolution of the strong coupling between the fluid flow and the deformation of the structure poses a major problem, and the strategy adopted to address this problem typically determines the computational cost of the overall algorithm.

In this work, we investigate the convergence behaviour of the block Gauß-Seidel solution strategy, which is widely used for the resolution of the strong coupling between the fluid and the solid phases (see *e. g.* [1,2] and references therein). The methodology is also referred to as Dirichlet-Neumann coupling (see [3]). The solution strategy corresponds to the classical Gauß-Seidel scheme for the iterative solution of systems of coupled equations. The reason for the popularity of the strategy lies in its high degree of modularity. It allows for the relatively straightforward combination of existing fluid and solid solvers without requiring significant modifications or even the full understanding of the two solvers. The strengths and weaknesses of the Gauß-Seidel strategy are generally well established. However, in this work, we use a simple model problem composed of point masses, springs and dashpots to highlight some aspects which arise in the context of the application of the block Gauß-Seidel methods to FSI problems (see also [3,4]). The limitations of block Gauß-Seidel methods have led to the development of a variety of alternative partitioned iterative strategies, namely "exact" and "inexact" block Newton methods (see [5,6] for the former and [7,8,9] for the latter and references therein). Other types of strongly coupled algorithms are based on monolithic approaches. However, at present, most of these strategies have been implemented only in research codes and, due to their appealing simplicity, block Gauß-Seidel methods are regarded as competitive and will continue to be used extensively in the modelling of FSI.

In this work, we use the simple model problem to show that the fluid/solid mass ratio is critical for the rate of convergence of block Gauß-Seidel methods. We demonstrate the beneficial effect of relaxation on convergence and we establish critical and optimal values for the relaxation parameter. Furthermore, we investigate the performance of the block Gauß-Seidel method in the presence of physical constraints

and nonlinearities. In particular, we show that a simple constraint imposed on the solution of a basic model problem generally deteriorates the rate of convergence. We also consider a nonlinear model problem and investigate the performance of the Aitken acceleration method (see [10]), which is often used in the simulation of FSI.

These studies are highly relevant for the modelling of FSI in the human vascular system. A constraint commonly encountered in such problems is the assumed incompressibility of the blood. Nonlinearities include the convective acceleration in the balance of momentum of the fluid and the nonlinear behaviour of the solid structure. The latter is very typical for realistic simulations of the vascular system.

We conclude the work by presenting numerical results of incompressible fluid flow through a flexible pipe composed of nonlinear solid material. The efficient and accurate simulation of this problem is a requirement for any numerical strategy, which is intended for realistic large scale simulations of, for instance, the human heart. In this work, we provide a comparison of the performance of block Gauß-Seidel strategies with that of an inexact block Newton method.

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