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3D NURBS-ENHANCED FINITE ELEMENT METHOD (NEFEM)

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ABSTRACT

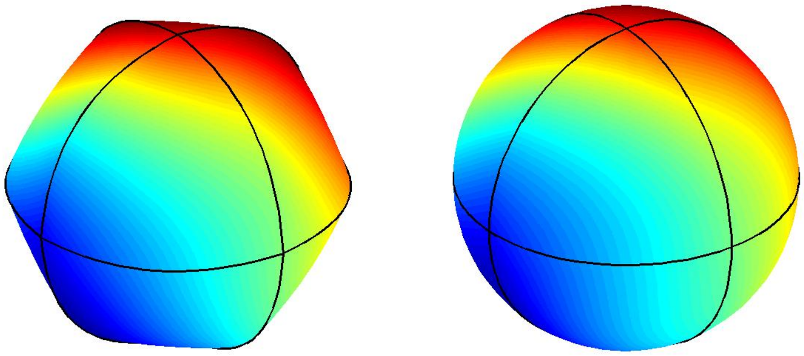
The importance of the geometrical model in finite element (FE) simulations has recently been pointed out by several authors, see [1,2,3] to name a few. In some applications, such as compressible flow problems, if a Discontinuous Galerkin (DG) formulation is adopted, an important loss of accuracy is observed when a linear approximation of the boundary is used, see [1]. Bassi and Rebay show that, in the presence of curved boundaries, a meaningful high-order accurate solution can only be obtained if the corresponding high-order approximation of the geometry is employed (i.e. isoparametric FE).

Maxwell equations are also very sensitive to the quality of the boundary representation. In reference [4] the error induced by the approximation of curvilinear geometries with isoparametric elements is studied. The 3D Maxwell equations are solved in a sphere with isoparametric FE and with an exact mapping of the geometry. The exact mapping provides more accurate results with errors differing by an order in magnitud. Thus, in some applications, an isoparametric representation of the geometry is far from providing an optimal numerical solution for a given spatial discretization.

Non-Uniform Rational B-Splines (NURBS, see [5]) are widely used for geometry description in CAD (Computer Aided Design). This fact has motivated new numerical methodologies considering an exact representation of the computational domain with NURBS, such as the *isogeometric analysis* [2] and the NURBS-Enhanced Finite Element Method (NEFEM) [3]. The *isogeometric analysis* considers the same NURBS basis functions for both the description of the entire geometry and for the approximation of the solution. NEFEM also considers an exact representation of the domain but it differs from the *isogeometric analysis* in two main points: the geometry is given by the NURBS description of the boundary (i.e. the information usually provided by CAD), and standard FE polynomial interpolation is considered for the approximation of the solution. Thus, in the large majority of the domain —for elements not intersecting the boundary— a standard FE interpolation and numerical integration is used, preserving the computational efficiency of classical FE techniques. Specifically designed piecewise polynomial interpolation and numerical integration is only required for those FE along the NURBS boundary.

In [3] NEFEM is presented in two dimensional simulations. Poisson and Maxwell problems demonstrate the applicability of the proposed method in both a continuous and discontinuous Galerkin frameworks. When the quantities of interest are defined on curved boundaries NEFEM is at least one order in magnitud more precise than the corresponding isoparametric FE. Moreover, the exact representation of the boundary allows to mesh the domain independently of the geometric complexity of the boundary whereas classical isoparametric finite elements need h-refinement to properly capture the geometry. Application to fluid mechanics can be found in [6], where the advantages of NEFEM for the simulation of compressible flow problems are shown for both linear and high-order approximations.

This work presents the extension of NEFEM to 3D domains. Numerical experiments show that, for the same FE interpolation, results are more accurate using NEFEM than isoparametric FE. Figure 1 shows FEM and NEFEM solutions of a Poisson problem in a tetrahedral mesh with only eight elements, using a polynomial approximation of degree p=2. It is important to note that not only the solution is captured with lower accuracy with isoparametric FE but also geometric errors are clearly observable in the piecewise quadratic approximation of the geometry.



(a) FEM (b) NEFEM

Figure 1: FEM and NEFEM solution of a Poisson problem with $p = 2$

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