

A NEW PROFILE-TEMPERATURE FULLY COUPLED MODEL FOR ICE SHEETS SIMULATION

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ABSTRACT

A new complex shallow ice coupled model which governs the thermal, hydrodynamic and mechanical processes in the dynamics of ice sheets is proposed and solved by means of efficient numerical techniques. More precisely, the novelty in this work, with respect to [1], is the introduction of a profile equation which takes into account the nonisothermal nature of the problem [2]. Thus, a nonlinear integro-differential equation with temperature dependent coefficients has to be solved. Furthermore, the positivity constraint on the ice sheet profile is taken into account by means of a complementarity formulation. For its numerical solution, a characteristic method for time discretization, a combination of an explicit treatment and numerical quadrature for the integral term, and duality methods for the nonlinear diffusive term and the unilateral constraint are proposed. All of them combined with piecewise linear Lagrange finite elements for the spatial discretization.

Once the profile and temperature equations are fully coupled, then a PDE system governing the upper ice sheet profile, the velocity field, the temperature and the basal magnitudes (velocity and stress) is posed. For solving the global coupled system a fixed point iteration which sequentially treats each sub-problem is developed [1]. In addition to the difficulties associated to the new profile equation, several techniques have been considered for the numerical solution of the temperature [3], velocity and basal magnitudes: upwind methods for time discretization, 2-d finite element discretizations, duality methods associated to monotone operators, Newton methods for nonlinear problems, numerical quadrature formulas for velocity computation, etc.

The novelties in the model have been included in an ice sheet simulation software toolbox [4].

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