8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

Schur-Complement Preconditioning in Two-Phase Incompressible Flow with Extended Finite Elements

Jörg Grande[1]

[1] RWTH Aachen University
Lehrstuhl für Numerische Mathematik
Templergraben 55, 52056 Aachen, Germany
grande@igpm.rwth-aachen.de, http://www.igpm.rwth-aachen.de/grande

Key Words: *Krylov method, saddle point preconditioning, incompressible Navier-Stokes*

ABSTRACT

We consider a non-stationary, two-phase Navier-Stokes-problem for a levitated fluid drop at low to medium Reynolds-numbers, in which we have discontinuous material-parameters and surface-tension effects. For small discontinuities and surface-tension we use the standard Hood-Taylor finite-elements. The pressure ansatz-functions are in H^1, which allows the use of a Cahouet-Chabard-type preconditioner for the Schur-complement. For Stokes-flow, the robustness of this preconditioner with respect to the problem parameters is analyzed in [1].

At higher surface-tensions we introduce extended finite-elements for the pressure, which have a discontinuity at the interface. Thus, we recover optimal approximation-properties of the spatial discretisation, [2]. The preconditioner for the Schur-complement must be adapted to the additional discontinuous ansatz-functions. Following the framework in [1] we replace the pressure-Laplacian by $BM^{-1}B^T$, where B is the discrete divergence-operator and M is of mass-matrix-type. Explicitly calculating $BM^{-1}B^T$ is too expensive. A few CG-iterations are used to approximate the action of $(BM^{-1}B^T)^{-1}$. We discuss, how we precondition this Krylov-method.

These numerical building-blocks are combined with a Navier-Stokes-solver, a level-set-technique and a standard θ-time-stepping-scheme in the software package Drops, [3]. The performance of the resulting method is studied by numerical simulations of the levitated drop.

REFERENCES

[1] Maxim A. Olshanskii, Jörg Peters, Arnold Reusken. "Uniform preconditioners for a parameter dependent saddle point problem with application to generalized Stokes interface equations". *Numer. Math.*, Vol. **105**, 159–191, 2006.

[2] Arnold Reusken. "Analysis of an Extended Pressure Finite Element Space for Two-Phase Incompressible Flows". *Comp. Vis. Sci.*, accepted.

[3] A. Reusken et al. "Drops-Homepage". *http://www.igpm.rwth-aachen.de/drops*