8th. World Congress on Computational Mechanics (WCCM8)
5th. European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 – July 5, 2008
Venice, Italy

LARGE DEFLECTION OF RECTANGULAR PLATES USING OPTIMIZATION TECHNIQUE

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Key Words: *Structural Analysis, Finite Element Methods, Large Deflection, Nonlinear Programming Theory, Rectangular Plates.*

ABSTRACT

It is well known when the magnitude of deflection (out-of-plane displacement) becomes to the thickness of the thin plates subjected to lateral loads or bending moments, the effect of in-plane forces is not neglected. This phenomenon is called large deflection, and the geometrical nonlinearity of the out-of-plane displacement must be taken into account. This problem is studied historically by many researchers [1], [2]. Some of the methods are the energy method assuming the proper solutions in advance, and finite element approach introducing the geometrical nonlinear stiffness matrix [3]. However, in the previous methods, few researches are shown on the effect of the coupling between out-of-plane and in-plane displacements. Making use of the past methods, results of rectangular plates are dealt with because of its geometrical and useful shapes. And results of linear and nonlinear solutions of rectangular plates have been shown.

The optimization technique based on a mathematical programming theory is one of the most effective ways for the practical engineering problems such as optimal designs. Concerning with this method, Schmit, Bogner and Fox have studied the skillful numerical method for flat plates and shells [4].

In this study, large deflection of rectangular plates is dealt with as an optimization problem based on the nonlinear programming method. The displacement function is assumed using two-dimensional Hermite interpolation functions. In the relationship between strain and displacement, the geometrical nonlinear terms are taken account. Strain energy in an element is formulated for the rectangular plate element. Then, the total potential energy expressed in terms of in-plane and out-of-plane displacements. And it is

constructed with the regular quadratic, cubic and quartic terms. The cubic terms indicates the coupling term between out-of-plane and in-plane displacements. Hence, the large deflection becomes the minimization problem without constraint condition. The total potential energy is the objective function. This total potential energy is minimized by the Davidon-Fletcher-Powell method [5].

The application to the bending of rectangular plates is shown and evaluation of the effectiveness of this method is studied. The points of our study are:

(1) Formulation of isotropic homogeneous rectangular plate element and transformation into the matrix form.

(2) To show large deflection problem is solved in a single step loading.

(3) Examination of coupling effect due to the assumption of geometrical nonlinearity.

Numerical examples are presented for square plates with three boundary conditions subjected to the lateral uniform pressures on the plate surfaces. Three boundary conditions are clamped edge (Case 1), simply supported one (Case 2) and movable one simply supported one (Case 3), respectively. The loading steps are expressed with the dimensionless ratios P/P_0 for three cases. The basic load P_0 is defined as the load which yields the same out-of-plane displacement as the thickness of the plate for each case. The distributions of out-of-plane displacements, in-plane displacement and stress distributions in the x-axial section are given. To compare the precision of the method, result of clumped boundary condition of Case1 obtained by this method is shown with the previous analytical solutions. The difference of results between the square plates with simply supported edge of Case 2 and movable simply supported edge of Case 3 become evident with the increase of pressure loads. It is concluded that large deflection of rectangular plates under bending can be analyzed efficiently by this method in a single loading step without using the incremental process.

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