

A Homogenization Based Spectral Stochastic Computational Approach for the Analysis of Multi-phase Periodic Materials with Random Material Properties

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ABSTRACT

In cases when the problem at hand involves multiple scales, solving for local variations of field variables and taking uncertainty into account at the same time becomes crucial since direct simulation methods become computationally expensive and sometimes infeasible on the physical domains that are very finely discretized. This brings in the need for devising approaches that are able to efficiently and accurately handle the process of uncertainty quantification parallel to resolving for multiscale character of the solutions. In this work a homogenization based spectral computational scheme is proposed that is able to capture the local variations of quantities of interest and the propagation of uncertainties across scales at the same time. To solve for the multiscale variation of field variables a computational method that is based on the homogenization theory built into a Finite Element framework is used where the full probabilistic characterization of the solutions to a set of local problems defined on the period cell is first sought [1]. The macroscale problem is then provided with data obtained by solving the forgoing set of local problems leading to a stochastic description for the homogenized properties. Each one of the problems in the set of local problems, known as the cell problems, is actually a stochastic partial differential equation. Among all possible boundary conditions the periodic boundary condition which gives rise to a better estimation of homogenized properties is chosen to be used. To propagate the uncertainty through the solution of the local problems the approach uses Spectral Stochastic Finite Elements (SS-FEM), where functions from a suitable complete product space are used to represent the response [2]. After obtaining the homogenized properties the whole media is then analyzed to get the macroscopic descriptions of field variables which are then used, through a localization procedure, to solve for a full probabilistic characterization at microscopic scale.

REFERENCES

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